COMPARING GNG3D AND QUADRIC ERROR METRICS METHODS TO SIMPLIFY 3D MESHES

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Keywords: GNG3D model, Quadric Error Metrics, mesh simplification, Growing Neural Gas, surface reconstruction.

Abstract: The primary objective of this paper is to carry out a performance comparison of two models to generate simplified meshes representing 3D objects, the GNG3D model and the Quadric Error Metrics (QEM) model. The QEM model constitutes a well-known classical algorithm based on interactive contraction of vertex pairs and represents perhaps the best balance yet between speed, fidelity, and robustness of the proposed models and algorithms for mesh simplification in the last years. The GNG3D model is based on a neural network algorithm and a reconstruction phase of the object. For the purpose of comparison, several error measurements are proposed and motivated in order to evaluate the quality of the approximations that both models produce. It is justified with numerical results that the GNG3D model exhibits better performance for several 3D objects with different topologies and geometric properties.

1 INTRODUCTION

Mesh simplification has emerged as a critical step for handling such huge meshes. A great work for a neural network based approach related to the problem of mesh simplification has been performed during the last years (Fritzke, 1994; Ivrissimtzis, Jeong & Seidel, 2003). We have developed the GNG3D model (Alvarez, Noguera, Tortosa & Zamora, 2007) to simplify any three-dimensional mesh, with the primary characteristics that it allows to establishing the total number of vertices that the simplified object will have, that is, the level of detail of the resulting mesh, and that it allows to set a maximum running time to obtain the simplified mesh.

In order to check the efficiency of this neural network method, we compare it with the well known Quadric Error Metrics (QEM) by Garland (Garlang & Heckbert, 1997). The reason to choose QEM is that it constitutes perhaps the best balance yet between speed, fidelity, and robustness of the proposed models and algorithms for mesh simplification in the latest years. See Luebke (2001) for a more detailed comparison of classical algorithms for mesh simplification.

2 THE GNG3D MODEL

The GNG3D algorithm has been designed taking as a basis the GNG model, with an outstanding modification consisting on the possibility of removing some nodes or neurons that do not provide relevant information about the original model. Besides, a reconstruction phase has been added in order to construct the faces of the optimized mesh.

Therefore, the GNG3D algorithm consists of two different phases: a Mesh Optimization Phase and a Mesh Reconstruction Phase.

A. Phase 1. Mesh Optimization. The primary objective of this optimization phase is the calculation of the best distribution of vertices that shapes the new simplified mesh. To perform this task an optimization algorithm has been implemented. For a detailed description of the algorithm, see the paper (Alvarez, Noguera, Tortosa & Zamora, 2007).

B. Phase 2. Reconstruction of the 3D Object. In general, phase 1 can be seen as a training process based on neural networks. At the end of this process a set of nodes, which represent the new vertices of the optimized mesh is computed. The edges connecting these nodes show the neighboring

Álvarez R., Tortosa L., Vicent J. and Zamora A. (2009).
COMPARING GNG3D AND QUADRIC ERROR METRICS METHODS TO SIMPLIFY 3D MESHES.
In *Proceedings of the Fourth International Conference on Computer Graphics Theory and Applications*, pages 99-102
DOI: 10.5220/0001791300990102

relations among the nodes generated by the optimization algorithm.

The reconstruction phase constitutes a post-process which uses the information on new nodes provided by the optimization phase and the information on the nodes of the original model. With these sets of nodes, a concordance process can be carried out between the nodes of the original object and the nodes generated by the optimization algorithm. This concordance process allows us to reconstruct the faces of the new optimized mesh.

3 THE QEM MODEL

The QEM algorithm provides a useful characterization of local surface shape, and it has modest computational and storage requirements. Combining quadric error metrics with iterative vertex pair contraction results in a fast algorithm for producing high-quality approximations of polygonal surfaces. The main characteristic of this algorithm is that is based on the iterative contraction of vertex pairs, proceeding by iteratively merging pairs of vertices, which need not be connected by an edge. Candidate vertex pairs include all vertex pairs connected by an edge, plus all vertex pairs separated by less than a user specified distance threshold t. It is introduced the quadric error metric to represent the error introduced by a sequence of vertex merge operations. The error introduced by a vertex-merge operation can be quickly derived from the sum of the quadric error metrics of the vertices being merged and that sum will become the merged vertex quadric error metric. The resulting algorithm is extremely fast. The visual fidelity of the resulting simplifications tends to be quite high, even at drastic levels of simplification. The advantages of this model can be summarized in the *Efficiency* (the algorithm is able to simplify complex models quite rapidly) and the *Quality* (the approximations maintain high fidelity).

4 EVALUATION OF THE 3D MESHES

Garland, Heckbert (1997) developed a surface simplification algorithm based on iterative contraction of vertex pairs to simplify models and maintains surface error approximations using quadric metrics.

They observed that, given a simple plain *(n, d)* one can express the squared distance from the plane to a point x by $error(x) = x^T Ax + 2\, b^T x + c$, where $(A,b,c) = (nn^T, dn, d^2)$ is the fundamental quadric of the plane (n,d).

We have chosen two methods of error evaluation. For the first one, we use a metric that measures the squared distance between the approximation and the original model. We define the distance $d(v,A) = min_{p \in A} \, ||v - p||$ as the minimum distance from v to the closest vertex p in the optimized mesh. This metric provides two error measurements that permit us to evaluate the approximations we are generating. These error measurements are:

- Mean error value of the minimum squared distance, given by

$$E_{avg} = \frac{1}{|M|} \sum_{v \in K} d^2(v, A)$$

- Maximum error value of the minimum squared distance, given by

$$E_{\max} = \max_{v \in K} \left\{ d^2(v, A) \right\}.$$

Remember that K is the set of vertices of the original model, $|M|$ is the number of elements of K, and A is the set of vertices of the simplified object. The second error measurement method computes the difference between the area comprised by the faces of the original object and the area corresponding to the faces of the simplified object (Cignoni, Rocchini, & Scopigno, 1998). Taking that the faces of the three-dimensional models considered here are triangular, this metric can be computed by the expression $E_{sur} = S_K - S_A$, with

$$S_X = \frac{1}{2} \sum_{f \in X} v_a \cdot v_b \cdot sen\alpha = \frac{1}{2} \sum_{f \in X} \left| \vec{v}_a \otimes \vec{v}_b \right|.$$

X is the set of faces of the original mesh and v_a, v_b the vectors joining the vertices belonging to face f.

The quality of the mesh being generated can be known at any time employing the metric of the distance to the vertices on any iteration during the training of the neural network in phase 1. The area difference metric can only be computed after applying phase 2 because there are no faces in the mesh being optimized during phase 1.

Using the three error measurements exposed in this section, E_{avg}, E_{max}, E_{sur}, we have performed numerical experiments for a variety of models with different geometric characteristics.

5 NUMERICAL RESULTS

To perform the comparisons between QEM and GNG3D models, we have tested more than twenty 3d models with different topologies and levels of complexity. Besides, we have registered measurements for the average error, the maximum error and the surface error for both models.

Because of the characteristics of the GNG3D model, we are able to obtain error values for different iterations during the training process; the error values in QEM model are obtained once the simplification process has been performed. The example we show in Table 1 is a 3D model named gargoyle, which has *21379* vertices and *40348* faces. In the table we summarize the error values E_{avg}, E_{max}, and E_{sur} obtained for the GNG3D and QEM methods when the number of iterations in the GNG3D algorithm is increasing. The error values for the QEM algorithm have been determined once the mesh has been simplified.

The second and third columns show us the number of vertices and faces of the simplified mesh for the corresponding number of iterations in the first column. Therefore, we can see that when the optimization algorithm in the GNG3D method performs *149653* iterations, and we run the reconstruction phase, we obtain a simplified mesh with *4009* vertices and *7234* faces. For that mesh, the error values are: $E_{avg} = 0.050$, $E_{max} = 0.690$, and $E_{sur} = 3.64$. The constant error values for the QEM algorithm are $E_{avg} = 0.010$, $E_{max} = 0.443$, and $E_{sur} = 27.321$. From Table 1, we remark:

- comparing the simplified meshes obtained from GNG3D and QEM models, it can be concluded that the approximations generated by the GNG3D model are better than those generated by the QEM model, since the error values are lower for GNG3D model, comparing the corresponding columns.
- it is observed that the surface error for the GNG3D model is always lower than the corresponding value for QEM algorithm, even when the number of iterations is extremelly small. In the case of the E_{avg} and E_{max} measurements, it is observed that from a number of iterations (*256548* for E_{avg} and *128274* for E_{max}), the error values for the GNG3D model are much better than the measured for the QEM model.
- for the gargoyle 3d object in Table 1, the differences in the error measurements are very important, with the GNG3D model being always better. This is particularly evident in the case of the surface error, meaning that the mesh obtained by the neural network algorithm is of higher quality than the generated by the classical algorithm.

More than thirty 3d objects have been used to test both models. In most cases, the GNG3D model produces higher quality approximations than the QEM algorithm.

6 CONCLUSIONS

We have performed comparisons between two different models to simplify meshes representing 3D objects. To compare the resulting simplified meshes from both models we implement three error measurements, E_{avg}, E_{max}, and E_{sur}, which allow us to evaluate the quality of the approximations generated. The numerical experiments with more than thirty 3D objects with different topologies and geometric characteristics shows that we can afirm that the quality of the approximations generated by the GNG3D model are better than the generated by the QEM model. For the particular objects detailed in the paper we observe significant differences in the error measurements in favor of the GNG3D model, especially when the number of iterations is high.

ACKNOWLEDGEMENTS

The Generalitat Valenciana (GVPRE/2008/363) has supported this work.

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Table 1: Comparing GNG3D with QEM for the model gargoyle.

Iteration	Vertices	Faces	E_{avg}	E_{max}	E_{sur}	QEM E_{avg}	QEM E_{max}	QEM E_{sur}
21379	563	0	2,6744	5,3053	25,63	0,01003	0,443243	27,321686
42758	1118	1252	0,7069	1,7517	16,86	0,01003	0,443243	27,321686
64137	1660	2458	0,3242	1,1269	9,25	0,01003	0,443243	27,321686
85516	2228	3670	0,1813	0,6825	2,52	0,01003	0,443243	27,321686
106895	2868	4885	0,1026	0,4889	3,58	0,01003	0,443243	27,321686
128274	3462	6128	0,0685	0,2093	3,45	0,01003	0,443243	27,321686
149653	4009	7234	0,0499	0,6902	3,64	0,01003	0,443243	27,321686
171032	4722	8308	0,0341	0,1586	2,31	0,01003	0,443243	27,321686
192411	5378	9613	0,0248	0,1205	2,09	0,01003	0,443243	27,321686
213790	5961	10844	0,0188	0,1068	0,64	0,01003	0,443243	27,321686
235169	6584	12041	0,0147	0,1058	1,80	0,01003	0,443243	27,321686
256548	7297	13137	0,0113	0,0760	1,96	0,01003	0,443243	27,321686
277927	7833	14318	0,0089	0,0754	2,14	0,01003	0,443243	27,321686
299306	8545	15399	0,0070	0,0554	0,85	0,01003	0,443243	27,321686
320685	9074	16601	0,0057	0,0437	1,44	0,01003	0,443243	27,321686
342064	9787	17761	0,0045	0,0390	0,99	0,01003	0,443243	27,321686
363443	10499	18895	0,0036	0,0390	1,50	0,01003	0,443243	27,321686
384822	10690	19952	0,0032	0,0390	1,20	0,01003	0,443243	27,321686
406201	10690	20667	0,0031	0,0390	1,12	0,01003	0,443243	27,321686
427580	10401	20801	0,0035	0,1432	1,02	0,01003	0,443243	27,321686
448959	10690	20875	0,0029	0,0390	1,25	0,01003	0,443243	27,321686
470338	10690	21216	0,0029	0,0368	1,36	0,01003	0,443243	27,321686
491717	10690	21327	0,0028	0,0292	1,24	0,01003	0,443243	27,321686
513096	10690	21408	0,0028	0,0331	1,12	0,01003	0,443243	27,321686
534475	10674	21412	0,0032	0,1431	1,38	0,01003	0,443243	27,321686
555854	10690	21449	0,0028	0,1054	1,12	0,01003	0,443243	27,321686
577233	10690	21506	0,0028	0,1054	0,97	0,01003	0,443243	27,321686
598612	10690	21471	0,0029	0,0327	1,12	0,01003	0,443243	27,321686

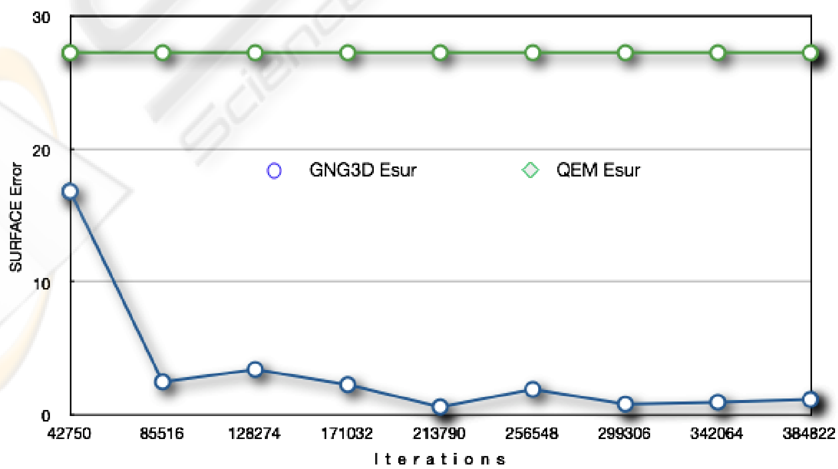

Figure 1: Error surface measurements for the gargoyle 3D object using GNG3D and QEM.