ONE-SHOT 3D SURFACE RECONSTRUCTION FROM INSTANTANEOUS FREQUENCIES
Solutions to Ambiguity Problems

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Abstract: Phase-measuring profilometry is a well known technique for 3D surface reconstruction based on a sinusoidal pattern that is projected on a scene. If the surface is partly occluded by, for instance, other objects, then the depth shows abrupt transitions at the edges of these occlusions. This causes ambiguities in the phase and, consequently, also in the reconstruction. This paper introduces a reconstruction method that is based on the instantaneous frequency instead of phase. Using these instantaneous frequencies we present a method to recover from ambiguities caused by occlusion. The recovery works under the condition that some surface patches can be found that are planar. This ability is demonstrated in a simple example.

1 INTRODUCTION

We consider the problem of 3D object surface reconstruction based on a sinusoidally modulated illumination pattern. Figure 1 shows an example. Depth information of the surface can be obtained from the phase of the pattern observed by a camera. This is the principle of phase-measuring profilometry. In this paper we study the use of the instantaneous frequency (IF) instead of phase. The IF is defined as the rate at which the phase changes.

The depth of a surface patch of the scene is encoded in the IF of the observed image. For example, the IF at the centre of the cylinder in Figure 1 is smaller than the IF observed at the background. The explanation is simple: in our case, the illumination pattern is almost orthographically projected onto the scene. Due to the perspective projection of the camera the IF is proportional to the depth.

In order to find the depth from the IF, we cannot simply reverse this relation because the inclination of the surface patch also influences the IF. For instance, on the side of the cylinder the IF increases with the depth, but also with the inclination angle of the patch. The dependency of the IF on both depth and inclination angle seems to introduce an ambiguity in the inverse solution. However, under the assumption that the surface is smooth (no abrupt transitions) we are able to bypass this ambiguity as will be shown in the sequel. With that, the solution based on IF is equivalent to the solution provided by phase-measuring profilometry.

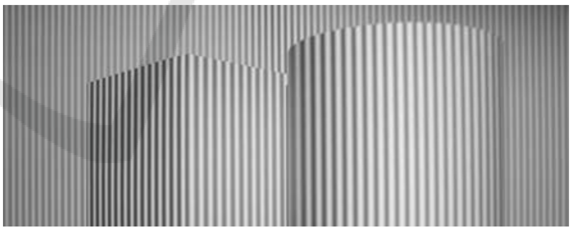

Figure 1: Sinusoidal illumination of a scene.

Possible occlusions in the scene (self-occlusion or occlusion from other objects) do cause discontinuities in the depth. At these discontinuities, the unwrapping of the phase fails, and as a result, the reconstructions will be ambiguous. This holds true especially for phase-measuring profilometry. At first sight, one would expect that IF based methods suffer from the same defect. However, this paper introduces a workaround for these types of ambiguities. The validity of the workaround is limited to piecewise planar surfaces such as the surfaces of the block and the background.

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The outline of the paper is as follows. Section 2 provides a short overview of related work. Section 3 analytical describes the image formation process leading to a forward model. Section 4 introduces an inverse model. Here, the ambiguity problems are discussed, and the workarounds are introduced. Experiments that are conducted are reported in Section 5. The paper finalizes with a conclusion in Section 6.

2 RELATED WORK

The 3D reconstruction technique addressed in this paper belongs to the category of structured lighting. The literature on this topic is numerous. Salvi, Pagès and Battle (2004) give an overview. Most systems rely on the principle of a triangulation set up between a ray of light projected on a surface patch in the scene and the corresponding line of sight of that patch as observed by a camera. To prevent time-consuming scanning of the scene a 2D pattern of light is projected on the scene so that all surface patches are concurrently illuminated. The various approaches of structured lighting differ in the way they uniquely identify a ray of light amongst other rays of the same projected pattern.

One-shot methods encode the position of a given ray in just one single illumination pattern. Usually, the identification of a projected point amongst other points of the pattern is done by using the context of grey levels (or colours) in the spatial neighbourhood of the projected point. A popular method to do so is PMP (phase-measuring profilometry) introduced by Srinivasan, Liu and Halioua (1985). Here, a sinusoid pattern is projected on the scene. PMP exploits the phase of the image of this pattern. For each pixel, the triangulation is set up by means of a difference between the phase derived from a reference plane and the phase derived from the surface under study.

Our method belongs to the one-shot category using neighbourhoods, but differs from all other techniques in the sense that it does not set up an explicit triangulation. Furthermore, we do not use phase, but instead, use the rate at which the phase in the image changes. As such our method is a variation on PMP.

The literature on PMP is wealthy. Most papers deal with the way in which the phase is measured. Srinivasan et al. (1985) used a method called phase shifting. Before that, Takeda & Mutoh (1983) used a non-sinusoidal pattern and exploited Fourier analysis to find the phase. See the review of Su & Chen (2001). Cuevas et al. (1999) estimated the phase using a PLL method. Tang & Hung (1990) used synchronous detection. Tay et al. (2004) used a simple interpolation technique. In fact, phase shifting is not a one-shot technique since it requires multiple patterns. The phase shifting technique is elaborated by Guan et al (2003) and Sansoni & Redaelli (2005) to a true one-shot technique. They describe modulation/demodulation techniques that combine the multiple patterns to one. The method of Hu et al. (2007) has the same goal but they use colour.

We did not find much literature about the usage of IF. Neither did we find literature about the recovery from ambiguities due to occlusions. Sansoni & Redaelli (2005) use the IF to find an expression for the maximum slope that can be recovered.

3 IMAGE FORMATION

Figure 2 shows the geometric set-up of the camera. A profile of the object surface, taken along the x-direction and at a fixed value of y, is parametrically represented by $\left(x(\xi), z(\xi)\right)$ where ξ is the running variable. We choose ξ to be the pinhole mapping of $\left(x(\xi), z(\xi)\right)$ on the image plane. So, if D is the focal distance, then:

$$\xi = \frac{x(\xi)D}{z(\xi)} \tag{1}$$

Eq. (1) establishes a constraint on ξ, $z(\xi)$ and $x(\xi)$.

Occlusions are parts of the surface that are not observable from the position of the focal point. They bring intervals of the x-axis for which no corresponding values of ξ exist. An example is the interval S in Figure 2. Due to this occlusion, the mapping $x(\xi)$ shows a discontinuity, i.e. an abrupt transition, at $\xi = a$. The occurrence of a number of such occlusions splits the ξ-axis into a number of disjoint intervals in which the mappings $x(\xi)$ are continuous and piecewise differentiable. In the sequel, we will refer to these intervals as the 'continuity intervals'.

For the moment, we assume that the pattern is parallel projected on the object surface along the z-direction. Such an orthographic projection makes additional requirements on the optical arrangement

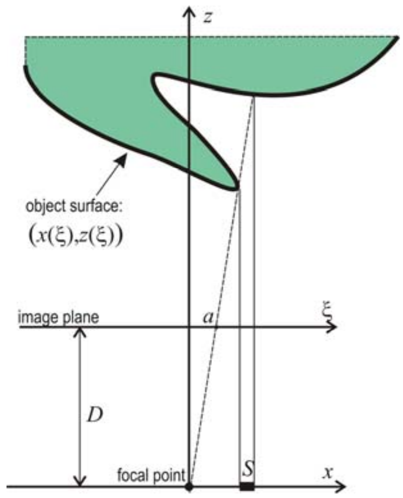

Figure 2: Camera and Scene geometry.

but it greatly simplifies the mathematical analysis. With this arrangement, the illumination pattern is described by $A\cos(2\pi ux+\phi)$ where u is the spatial frequency measured along the x-axis, and ϕ is a phase constant. The result of the orthographic projection is that the image of the pattern can be described by $x(\xi)$ without a reference to $z(\xi)$: the observed pattern is simply: $B(\xi)\cos(2\pi ux(\xi)+\phi)$. With that, the observed phase in the image becomes $\varphi(\xi)=2\pi ux(\xi)+\phi$. The instantaneous frequency of the observed signal is defined as follows:

$$IF(\xi) \overset{def}{=} \frac{1}{2\pi}\frac{d}{d\xi}\varphi(\xi) = u\frac{d}{d\xi}x(\xi) \qquad (2)$$

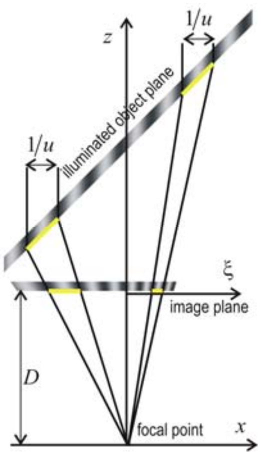

Figure 3: Image formation model.

The *IF* can be estimated indirectly by numerical differentiation of the phase. For the estimation of the phase, many techniques are available (Section 2). Another possibility is to directly estimate the *IF* from the image. Modulation theory has produced several algorithms for that (Boashash, 1992a & 1992b).

4 INVERSE MODELLING

Eq. (2) is the forward model of the image formation. It predicts $IF(\xi)$ of the observed image if the geometry $(x(\xi),z(\xi))$ of the object is given. The sequel of this report focuses at the inverse problem. How to reconstruct the geometry $(x(\xi),z(\xi))$ of the surface if the instantaneous frequency $IF(\xi)$ of the observed image is given?

Figure 4 illustrates the fact that this question is not easy to answer. The figure shows the image observed from the profile of a planar surface as presented in Figure 3. The interval a spans exactly one period of the associated *IF* observed in a. The line segments b, c and d are three different solutions. Each of them complies with the observed *IF*. That is, each solution is mapped to a, and the projection of each solution on the x-axis has a length that matches the period $1/u$ of the projected pattern. In other words, the solution at $\xi\in a$ is ambiguous. The observation of the *IF* at a merely establishes a relation between the depth of a surface patch and its slope.

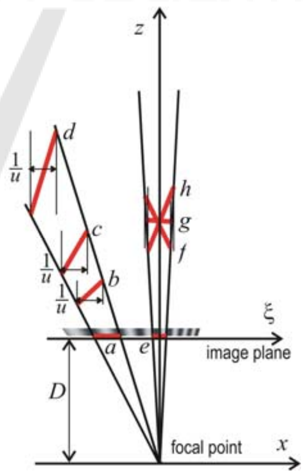

Figure 4: Ambiguous solutions.

Surprisingly, the ambiguity does not occur at $\xi=0$. In Figure 4, the interval e spans one period of the *IF* observed near $\xi=0$. The line segments f, g and h are different solutions that maps to e. However, the solutions all intersect at a unique, common point $(0, z(0))$. This point can be retrieved unambiguously from the *IF*.

4.1 The Phase based Solution

If the surface slice is not occluded, then an unambiguous, full reconstruction is possible. The reconstruction starts at $\xi = 0$ where the depth can be recovered unambiguously. Next, using the continuity of the surface the full solution is obtained by integration eq. (2) along ξ:

$$x(\xi) = \frac{1}{u} \int_{\alpha=\xi_0}^{\xi} IF(\alpha) d\alpha + x(\xi_0) \tag{3}$$

The integral is valid for any interval $[\xi_0, \xi]$ in which no occlusion occurs.

If $x(\xi)$ is known to be continuous everywhere (no occlusion), then eq. (3) provides the full solution since according to eq. (1), we have $x(0) = 0$. Thus, for continuous surfaces the integration starts at $\xi_0 = 0$. In fact, we are then just reconstructing the phase $\varphi(\xi) - \phi$, and due to our orthographic projection of the pattern this gives us directly $x(\xi)$. The actual value of the phase constant ϕ is irrelevant, and there is no need to calibrate this parameter.

Suppose that $x(\xi)$ has been resolved by numerical integration yielding an estimate:

$$\hat{x}(\xi) = \frac{1}{u} \int_{\alpha=0}^{\xi} IF(\alpha) d\alpha \tag{4}$$

Then the depth can be recovered by eq. (1):

$$\hat{z}(\xi) = \frac{\hat{x}(\xi) D}{\xi} \tag{5}$$

which provides the full solution $(\hat{x}(\xi), \hat{z}(\xi))$. The only point that remains unsolved is $\hat{z}(0)$ because $\hat{x}(\xi)D/\xi$ is undetermined for $\xi = 0$. However, it can be found by $\hat{z}(0) = D \cdot IF(0)/u$.

4.2 An IF based Solution

If the surface is occluded, then $x(\xi)$ is piecewise continuous. The solution of eq. (4) and (5) is then only valid within the continuity interval that contains $(0, z(0))$. Each of the other intervals holds an integration constant that is unknown yet. In order to find the full solution one needs to identify the intervals, and, for each interval, estimate the corresponding integration constant. These integration constants corresponds to the jumps that are made at the discontinuities in $x(\xi)$. A clue for finding the positions of the discontinuities in $x(\xi)$ is that at these positions $IF(\xi)$, and also $B(\xi)$, are likely to be discontinuous. Edge detection applied to $IF(\xi)$ and $B(\xi)$ may therefore recover these discontinuity points.

Suppose that a single point $(\hat{x}(\xi_0), \hat{z}(\xi_0))$ has been found within a continuity interval. Then, eq. (3) provides the solution for the full continuity interval. The question is: how to find such a solution? A general answer is hard to find. However, in the special case of having a surface patch that is locally flat, this section provides an answer. For such a surface the profile is locally of the form $z = ax + z_0$.

Our solution is based on the derivative of $IF(\xi)$. We analyse the local behaviour of the geometry around a fixed ξ. Thus, we examine the properties of $(x(\xi+h), z(\xi+h))$ and the associated instantaneous frequencies $IF(\xi+h)$ and its derivatives for $h \to 0$. If the surface slice around ξ is of the form $z = ax + z_0$, then the parametric representation is:

$$\begin{aligned} z(\xi+h) &= z(\xi) + a g(\xi,h) h \\ x(\xi+h) &= x(\xi) + g(\xi,h) h \end{aligned} \tag{6}$$

$g(\xi,h)$ is a scale factor that is needed to fulfil the constraint on $\xi + h$, $x(\xi+h)$ and $z(\xi+h)$ expressed by eq. (1):

$$\xi + h = D \frac{x(\xi+h)}{z(\xi+h)} \tag{7}$$

Substitution of eq. (6) in eq. (7), and solving for $g(\xi,h)$ yields:

$$g(\xi,h) = \frac{z(\xi)}{D - a\xi - ah} \tag{8}$$

Next, substitution of $g(\xi,h)$ in the expression for $x(\xi+h)$ in eq. (6) gives:

$$x(\xi+h) = x(\xi) + \frac{z(\xi)}{D - a\xi - ah} h \tag{9}$$

From this expression we can derive the derivatives with respect to h, and evaluate these at $h=0$. This finally enables us to find $IF(\xi)$ and its first derivative $IF_\xi(\xi)$:

$$IF(\xi) = u\left.\frac{dx(\xi+h)}{dh}\right|_{h=0} = \frac{z(\xi)u}{D-a\xi}$$
$$IF_\xi(\xi) = \frac{2auz(\xi)}{(D-a\xi)^2} \qquad (10)$$

In a practical situation, $IF_\xi(\xi)$ can be estimated from the measured $IF(\xi)$. For each fixed ξ, we have two equations and two unknown, i.e. $z(\xi)$ and a. Solving eq. (10) yields:

$$\hat{a}_\ell(\xi) = \frac{D\,IF_\xi(\xi)}{\xi\,IF_\xi(\xi)+2\,IF(\xi)}$$
$$\hat{z}_\ell(\xi) = \frac{2D\,IF^2(\xi)}{u\left(\xi\,IF_\xi(\xi)+2\,IF(\xi)\right)} \qquad (11)$$

Finally, eq. (1) gives the estimate of $x(\xi)$:

$$\hat{x}_\ell(\xi) = \frac{\xi\hat{z}_\ell(\xi)}{D} \qquad (12)$$

The subscript ℓ has been introduced to emphasis the fact that these estimators are based on a linearity assumption of the surface.

Together, eq. (11) and (12) present a local solution based on the instantaneous frequency $IF(\xi)$ and its first derivative $IF_\xi(\xi)$. Since the solution is local, and there is no need for integration, the solution bypasses the problem of having continuity intervals and unknown integration constants.

The assumption of having a locally linear profile is essential. Suppose, that the neighbourhood of $(x(\xi), z(\xi))$ is locally approximated by a quadratic curve, i.e.:

$$z(\xi+h) = z(\xi) + a\left(x(\xi+h)-x(\xi)\right) + b\left(x(\xi+h)-x(\xi)\right)^2 \qquad (13)$$

The constraint of eq. (7) causes the second order constant b to enter the expression for $IF_\xi(\xi)$ given in eq. (10). Consequently, the estimator $\hat{z}_\ell(\xi)$ is only valid if $b=0$. It is not applicable to curved parts of the surface.

If within a finite neighbourhood of ξ the linearity assumption holds, then the estimated coefficient $\hat{a}_\ell(\xi)$ should be constant within this neighbourhood. Thus, if within a given interval $\hat{a}_\ell(\xi)$ fails to be constant, then the linearity assumption falls down there. In that case, $g(\xi, h)$ does not fulfil the constraint (7), and the solution given by eq. (11) is invalid.

5 EXPERIMENTS

A preliminary experiment is conducted to demonstrate the ability of instantaneous frequencies to recover from occlusion ambiguities. For that purpose, the scene shown in Figure 1 was selected. The scene consists of a cylinder, a block, and a planar background. The cylinder partly occludes the block. Both objects occlude the background. Figure 5 shows a top view map of the geometry.

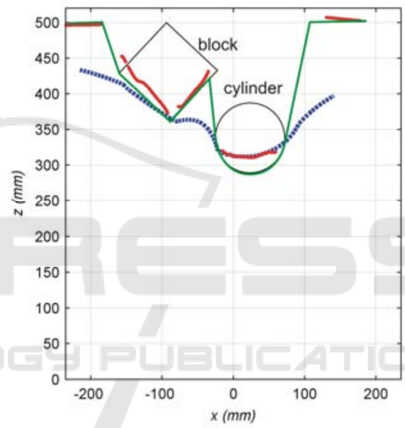

Figure 5: Geometrical set-up and reconstruction results.

The pattern was created by a DLP projector at a distance of 90 cm from the background. The depth range of the scene is about 20 cm. The distance from the camera to the background is 51 cm. The optical axis of the camera (and of the projector) is orthogonal to the background plane, and intersects the cylinder left from its centre.

In this preliminary experiment we used on off-the-shelf FM demodulation technique for the estimation of the *IF*. It uses the analytic signal together with Gabor quadrature filtering. The estimated *IF* of a row extracted from the centre of the image is shown in Figure 6.

Based on the analysis in Section 4, the expectations are as follows:

- The phase-based method can resolve the cylinder since the optical axis intersects this object. Due to the discontinuities, other surfaces cannot be resolved.

- The IF-based solution of eq. (11) and (12) can resolve the background and the two sides of the block.

The phase-based estimate is shown as the blue dashed line in Figure 5. The green thin line is the ground truth. It can be seen that the phase-based estimate corresponds well to the expectation. The estimator finds the surface of the cylinder but it looses track near the edges of this object. At the centre of the cylinder, the error of the estimated depth is about 6 cm. This can be contributed to the illumination which is only approximately parallel.

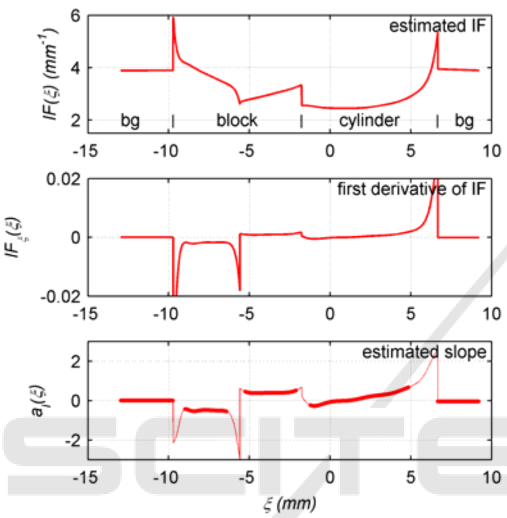

Figure 6: Estimated IF, and its derivative together with the estimated slope of the profile.

The *IF*-based estimates are shown as the red thick lines in Figure 5. The estimated slopes (eq. (11)) are shown in Figure 6. For the background, and the two sides of the block, these slopes corresponds well with the ground truth, i.e. $a_\ell = 0, -1$, and $+1$, respectively. We used the derivative of a_ℓ to decide whether the corresponding surface patch is planar or not. Here too, the estimates correspond well to our expectation, albeit that the accuracy could be improved. Clearly the *IF*-method, being dependant on derivatives, is sensitive to errors in the *IF*.

6 CONCLUSIONS

We have introduced and demonstrated a new method for retrieving depth from images of sinusoidally illuminated scenes. The method is based on the *IF* rather than phase. It has the ability to resolve the ambiguity caused by occlusions in the scene. Phase-based methods cannot resolve these ambiguities. The *IF* method can, but only works for planar surface patches. We are currently working on extensions to relieve this condition by, for instance, allowing quadratic surfaces.

We have assumed an orthographic projection of the illumination pattern. Currently, we are also working on a method that uses a perspective projection model for both the projector and the camera.

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