ON THE INFLUENCE OF LOW FREQUENCY COMPONENTS IN THE WEIGHT BEHAVIOUR OF THE LMS ALGORITHM

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Abstract: The Least Mean Square (LMS) algorithm is a very important tool in the estimation and filtering of biomedical signals. Amongst these signals are the periodic and quasiperiodic. For example, the LMS algorithm was used to estimate the coefficients of the Fourier series at a given frequency or even in a spectral analysis. In this paper we study the behavior of the weights of the LMS algorithm when the signal to be estimated acts at very low frequencies. We prove theoretically that lower frequency noise affects the estimation of the weights at higher frequencies. We carried out simulations and showed that experimental findings are in agreement with the theoretical results. Moreover, we exemplify the problem with electrocardiogram signals (ECG).

1 INTRODUCTION

Quasiperiodic physiological signals such as the electrocardiogram (ECG) are susceptible to various interferences. According to (Friesen et al., 1990), the main sources of noises and artifacts in ECG signal are: power line interference, electrode contact noise, motion artifacts, muscle contraction (electromyographic, EMG), baseline drift and ECG amplitude modulation with respiration, instrumentation noise generated by electronic devices used in signal processing, electrosurgical noise and impedance cardiography signals (ZCG). Some of those interference signals have low frequency (0.04–0.15 Hz) and very low frequency (0.0033–0.04 Hz) components, that influence on the acquisition and analysis of the ECG signal.

In figure 1, we show the power spectrum of an ECG signal under the influence of low and very low frequencies. Analyzing its spectrum it is possible to ascertain the disturbances at low and very low frequencies. These kind of noises may interfere in the analysis of the signal.

Many solutions to filter general disturbances on biological signals have been proposed. Amongst them there are those which use adaptive methods, such as the Fourier Linear Combiner (FLC) proposed by (Vaz and Thakor, 1989), (Vaz et al., 1994). After that, it was suggested the Scaled FLC (SFLC) in (Barros et al., 1995) to eliminate not only non-correlated noises but also body movements (Barros and Ohnishi, 1997). It is important to emphasize that the FLC is used as spectrum analyzer as proposed by Widrow (Widrow et al., 1987). In this method, the reference inputs are sinusoidal and co-sinusoidal functions and the LMS algorithm (Widrow and Hoff Jr., 1960) is used to estimated the coefficients of a Fourier series in (Widrow et al., 1976), (Widrow and Stearns, 1985).

In this work we used the FLC to estimate the spectrum of biomedical signals, specifically ECG signals in the presence of low or very low frequency noise, and studied the behavior of the weights estimated by the LMS algorithm. We show that if there is a constant component or even a low frequency noise added to the desired signal, the behavior of the weights of the LMS algorithm will be changed.

This work is organized as follows. In section 1 we describe the problem present a brief introduction on the topic. In 2 we make the problem demonstration

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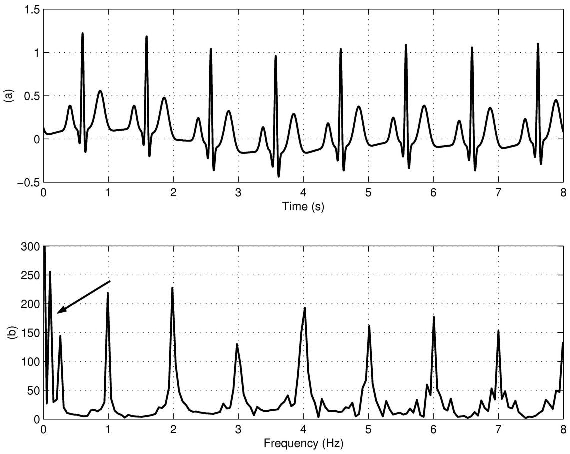

Figure 1: The spectrum response of a signal electrocardiogram (ECG) synthetic. In (a) we have an ECG synthetic signal and (b) its Fast Fourier Transform (FFT).

describe the methods used in our analysis. In section 3 we show the results obtained in simulations and the applications of the method. Finally in sections 4 and 5 we make comments to improve the understanding of the work and the conclusion respectively.

2 METHODS

Our method was developed by implementing the Fourier linear combiner (FLC) according to the block diagram on figure 2. The LMS algorithm is used to estimate the coefficients of the Fourier series. This method is known as LMS spectrum analyzer. The reference inputs are pairs of sines and co-sines. We propose the LMS spectrum estimation for a given frequency band and suppose that these low frequency noises affects the spectrum estimation of ECG signals.

2.1 Periodic and Quasiperiodic Signals

A periodic or quasiperiodic signal can be expressed as a combination of Fourier series and therefore can be reconstructed as:

$$s_k = \sum_{n=1}^{l} A_{nk} sin(c_n \omega_0 k) + \sum_{n=1}^{l} B_{nk} cos(c_n \omega_0 k), \quad (1)$$

where ω_0 is the fundamental frequency and $c_n = [c_1, \cdots, c_l]$ is a vector defining the frequency band. $c_1\omega_0$ is lowest estimated frequency components and $c_l\omega_0$ the highest. k is the discrete time index and A_{nk} and B_{nk} time variant Fourier series coefficients. The variations on these coefficients occur due to the quasiperiodic nature of the signal, which means that the fundamental frequency of the signal, together with the harmonics, have little variation along time.

To estimate the coefficients A_{nk} and B_{nk} we used adaptive linear combiner (ALC), developed by (Vaz et al., 1994). The reference input vectors, $\mathbf{X}_k$, is defined as,

$$\mathbf{X}_k = \frac{1}{\sqrt{N}} \left[sin(c_n \omega_0 k) \ \cdots \ cos(c_n \omega_0 k) \ \cdots\right]^H, \quad (2)$$

where N correspond to the number of harmonic and H is the Hermitian operator.

2.2 LMS Algorithm

We used the LMS algorithm as a spectrum analyzer as employed by Widrow (Widrow et al., 1987), in which he discussed the relationship between the DFT (Discrete Fourier Transform) and the vector weight $\mathbf{W}_k$ estimated by the algorithm. This algorithm calculates the possible variations in time which result in an instantaneous output, y_k, which is given by the internal product between the reference inputs $\mathbf{X}_k$ and the vector weight $\mathbf{W}_k$. Mathematically this is presented as follows,

$$y_k = \mathbf{X}_k^H \mathbf{W}_k = \mathbf{W}_k^H \mathbf{X}_k, \quad (3)$$

where $\mathbf{W}_k$ is composed of updated weights $[w_{1k}, w_{2k}, \cdots, w_{lk}]$. The initial weight vector was set to zero, as used by Widrow et al. in (Widrow et al., 1987). Consider a noise ν_k, inherent to the acquisition process of the desired signal, which is non-correlated to ECG signal.

The weights of the adaptive system are adjusted or updated by steepest descent method, as described by Widrow in (Widrow and Stearns, 1985). The output of the LMS algorithm y_k, is subtracted from the signal d_k, which is corrupted by noises generating an error ε_k, which we want to minimize to obtain the LMS spectrum.

2.3 Weight Behavior

In this section we demonstrate that low frequency disturbances present on ECG signal interfere on the estimation of LMS spectrum. We consider low and very low noises as being those disturbances below the fundamental frequency of the signal.

We start this analysis from the adaptation rule of the LMS algorithm (Widrow and Hoff Jr., 1960):

$$\mathbf{W}_{k+1} = \mathbf{W}_k + 2\mu\varepsilon_k \mathbf{X}_k, \quad (4)$$

where μ is a real positive number which represents the size of the step or learning rate and controls the

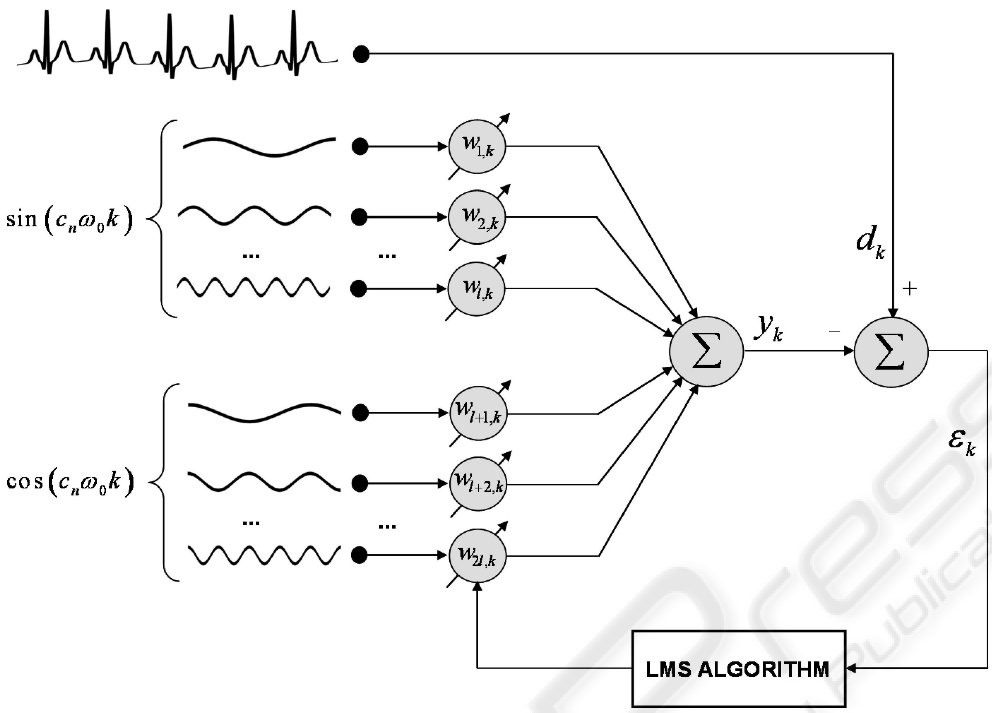

Figure 2: Block diagram of the Fourier Linear Combiner and of the LMS algorithm implemented. d_k is a synthesized ECG signal add ν_k a very low frequency noise inherent the acquisition. $\left[x_{1,k}, x_{2,k}, \cdots x_{l,K}\right]^T$ is the vector of reference inputs, composed by pairs of sines and cosines and ε_k is the error given by $\varepsilon_k = d_k - y_k$.

system stability. Let us define the error $\varepsilon_k = d_k - y_k$ and the signal $d_k = s_k + \nu_k$ in which s_k is the desired signal and ν_k is the additive low frequency noise.

Substituting ε_k in equation (4), the updated weights become,

$$\mathbf{W}_{k+1} = \mathbf{W}_k + 2\mu\left(s_k\mathbf{X}_k + \nu_k\mathbf{X}_k - \mathbf{X}_k^H\mathbf{W}_k\mathbf{X}_k\right), \quad (5)$$

Applying the expected to the value updated weights in equation (5) and doing appropriate considerations (appendix) we obtain the following result,

$$E\left[\mathbf{W}_k\right] = 2\mu D\mathbf{x}_0 E\left[D^{-k}\right] E\left[\nu_{k-1}\right]\frac{1-\tilde{\mathbf{H}}^k}{1-\tilde{\mathbf{H}}} + \mathbf{W}_*, \quad (6)$$

where $\mathbf{W}_*$ is the optimal weight.

The term $E\left[D^{-k}\right]E\left[\nu_{k-1}\right]$ is sinusoidally time variant for it is non-stationary and its expected value changes through time, therefore we call sinusoidal perturbation factor (SPF).

As proposed by Widrow (Widrow et al., 1987) a learning rate equal to 0.5 estimates exactly the DFT of the signal, but lower learning rates can be used to estimate the signal (Barros et al., 1995), and therefore sufficient to make an approximation of the LMS spectrum.

For periodic signals, the coefficients of the series are not time variant. However, in the case of the quasi periodic signals, the behavior of the coefficients of the Fourier series is time variant, since the period of the signal cycles is not constant.

The magnitude of each frequency component can be obtained by

$$\mathbf{C}_{n,k} = \sqrt{\mathbf{A}_{n,k}^2 + \mathbf{B}_{n,k}^2}, \quad (7)$$

where $\mathbf{A}_{n,k} = \mathbf{W}_{n,k}$ and $\mathbf{B}_{n,k} = \mathbf{W}_{n+N,k}$. With those weights we estimate a spectrogram or LMS spectrum. The spectrogram is a graph that represents the signal power, simultaneously in the time and frequency domains in each instant.

3 RESULTS

The adaptive system was implemented as illustrated in figure 2. In the first experiments we used a 1.2 Hz sinusoidal signal as d_k and 0.01 Hz additive sinusoidal noise. The frequency band of the reference inputs was in the range 1.0 Hz to 1.5 Hz, in steps of 0.05 Hz, giving a number of 11 harmonics.

To verify the effect of the low frequency disturbance, as observed on equation (6), we eliminated the low frequency noise using a high-pass, 4^{th} order, butterworth digital filter with 0.50

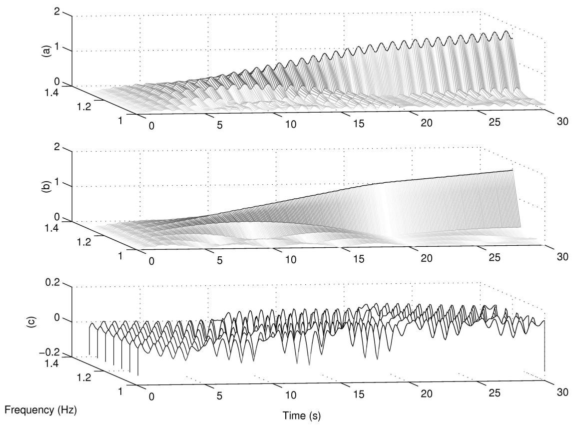

Figure 3: LMS spectrum for a theoretical simulated signal. In (a) we show the LMS spectrum of a sinusoidal signal with frequency of 1.2 Hz with a low frequency additive sinusoidal noise of 0.01 Hz. In (b) the spectrum of the filtered sinusoidal signal and in (c) the error between spectra.

Hz cutoff frequency. In figure 3 we present the LMS spectrum for the simulated theoretical noise sinusoidal signal and the filtered version signal. We also accomplished some experiments with real and synthesized biomedical signals. The synthesized ECG signal was created with an inherent low frequency noise, and it is supposed to approximate an ECG signal from a resting subject generated by ECGSYN program of the PhysioNet (Goldberger et al., e 13). For the ECG signal the frequency range of the reference inputs was 0.5 Hz to 1.5 Hz, which is in general sufficient to estimate the fundamental frequency.

In figure 4 we show the LMS spectrum obtained and the spectral error by the estimation of the synthesized ECG signal and its filtered version using the same filtering process of the previous test. To validate this result, we also accomplished simulation with real biomedical signals, specifically an ECG signal of normal patients. In figure 5 we show the LMS spectrum of the noise signals high-pass filtered.

4 DISCUSSIONS

Observing figure 3, we can say that low frequency noises affect the estimation of the spectrum signal when we use the LMS algorithm as a spectrum analyzer. In figure 3a, we verify, starting from the LMS spectrum, the sinusoidal influence along time as a ripple effect in the fundamental frequency, as well as on the other harmonic frequency components. These results corroborate the theoretical calculations that culminate in the equation (6).

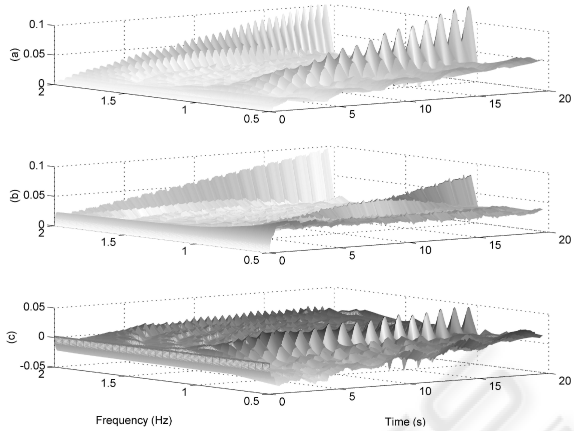

Figure 4: LMS spectrum of an ECG synthesized signal simulating resting conditions. In (a) we show that the spectrum is influenced by the low frequency noises originated of the movement artifacts, breathing etc. In (b) we show the LMS spectrum of the high-pass filtered ECG signal and in (c) the error between spectra.

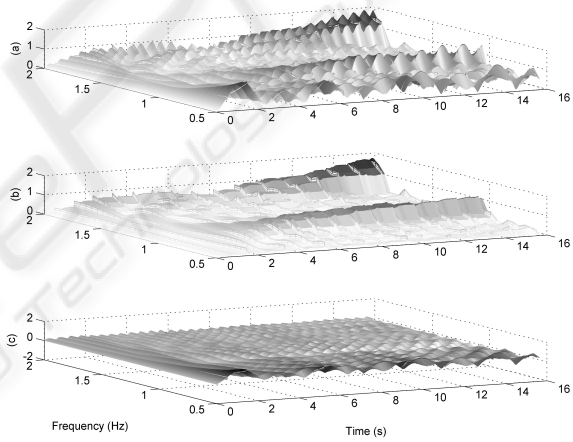

Figure 5: LMS spectrum of real ECG signal from a patient at rest. In (a) we show that the LMS spectrum is influenced by the low frequency noise. In (b) we show the LMS spectrum filtered by high-pass butterworth filter and in (c) the error between spectra.

Examining the equation (6), it can be observed that the expected value of the weights is composed of two terms. The term on the left is the expected value of the weight while the first term on the right side varies sinusoidally in time as a function of the term $E[\nu_{k-1}]$. When the expected value of the noise term is equal to zero, the term for the whole equation will also be zero, since it is multiplied by all of the other variables. The expected value of the weight being equal to the optimum weight is a consequence of the absence of any non-stationary noise into the signal, specifically a noise whose expected value changes in time (ideal condition); or yet when the noise ν_k is in a higher frequency band; then its expected value is also zero. This way, LMS spectrum estimation of ECG signal becomes susceptible to movement artifacts, breathing noise and electromyography signal which are low and very low frequency noises. In fact for the theoretical signal test the fundamental frequency of the desired signal is 1.2 Hz, while the noise was 0.01 Hz. The relationship is only 120 times higher.

In figure 3b, we present an estimation of the filtered sinusoidal signal to eliminate the low frequency noises. Observing the LMS spectrum we can see that there is no sinusoidal influence on the spectrum, as expected, since the low frequency noise was eliminated from the signal. Then we have a most precise spectral estimation being constant through time. In figure 3c we represent the error between LMS spectrum of the noise signal and the filtered signal. Since the influence of the noise in the spectrum is sinusoidal, which can be eliminated by filtering the signal, the error is also sinusoidal.

In figure 4 and 5, we presented the LMS spectrum of biomedical synthesized ECG signal and a real ECG, respectively. Observing figure 4a and 5a, we note a strong sinusoidal influence in the fundamental frequency and along the first harmonic. In figure 4b and 5b, respectively, we have the LMS spectrum of the filtered signal. Therefore we observe the absence of the low frequencies interference in the LMS spectrum. It is practically constant in the fundamental frequency of the signal. In figure 4c and 5c we observe the some pattern of error present in the figure 3c.

5 CONCLUSIONS

We have demonstrated theoretically and through simulations that, using the LMS algorithm as a spectrum analyzer of biomedical signals, the estimated weights are sinusoidally affected by low frequency noises. In general, biomedical signals, such as ECG, EMG and ZCG, are acquired along with low frequency noises, due to body movements, respiration and other less significant noise sources. This implies that we should be concerned with this type of estimation.

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APPENDIX

We begin the demonstration starting from the expression that represents the updated weight vector given by,

$$\mathbf{W}_{k+1} = \mathbf{W}_k + 2\mu\varepsilon_k\mathbf{X}_k, \tag{A1}$$

where μ is a positive real number representing the step size or the learning rate, while determines and controls the system's stability, and ε_k is the error defined by $\varepsilon_k = d_k - y_k$, with d_k defined by $d_k = s_k + \nu_k$, where s_k is a desired signal and ν_k is the added undesirable signal as it can be observed in figure 2.

Substituting ε_k in equation (A1), the updated weights become,

$$\begin{aligned}\mathbf{W}_{k+1} &= \mathbf{W}_k + 2\mu\varepsilon_k\mathbf{X}_k \\ \mathbf{W}_{k+1} &= \mathbf{W}_k + 2\mu\left(d_k - \mathbf{X}_k^H\mathbf{W}_k\mathbf{X}_k\right) \\ \mathbf{W}_{k+1} &= \mathbf{W}_k + 2\mu(s_k\mathbf{X}_k + \nu_k\mathbf{X}_k - \mathbf{X}_k^H\mathbf{W}_k\mathbf{X}_k),\end{aligned} \tag{A2}$$

By expressing ε_k in terms of statistical error estimation parameters, equation (A2) becomes,

$$\begin{aligned}\mathbf{W}_{k+1} &= \mathbf{W}_k + 2\mu\mathbf{X}_k\left(\mathbf{X}_k^H\mathbf{W}_* + \nu_k - \mathbf{X}_k^H\mathbf{W}_k\right) \\ \mathbf{W}_{k+1} - \mathbf{W}_* &= \mathbf{W}_k - \mathbf{W}_* + 2\mu\mathbf{X}_k\nu_k \\ &\quad -2\mu\mathbf{X}_k\mathbf{X}_k^H\left(\mathbf{W}_k - \mathbf{W}_*\right),\end{aligned} \tag{A3}$$

Replacing $\mathbf{W}_{k+1} - \mathbf{W}_*$ by $\tilde{\mathbf{W}}_{k+1}$ and $\mathbf{W}_k - \mathbf{W}_*$ by $\tilde{\mathbf{W}}_k$ we get an equation for the system weight, given by,

$$\tilde{\mathbf{W}}_{k+1} = \left(\mathbf{I} - 2\mu\mathbf{X}_k\mathbf{X}_k^H\right)\tilde{\mathbf{W}}_k + 2\mu\mathbf{X}_k\nu_k, \tag{A4}$$

Let us define a 2N x 2N diagonal matrix D with diagonal elements, $D(q,q) = \mathbf{D}_q = e^{j\omega_0 q}$, $q = -N,\ -(N-1),\ \ldots,\ -1,\ 1,\ \ldots,\ N$ and $\mathbf{X}_0 = [\overbrace{1,\ 1,\ \ldots,\ 1}^{2N \text{ times}}]^T$. The new inputs $\mathbf{X}_k$ will be expressed by $\mathbf{X}_k = D^{-k}\mathbf{X}_0$ and its conjugate $\mathbf{X}_k^* = D^k\mathbf{X}_0$.

Multiplying both sides of equation (A4) by D^{k+1}, we have

$$\begin{aligned}D^{k+1}\tilde{\mathbf{W}}_{k+1} &= D^{k+1}\tilde{\mathbf{W}}_k \\ &\quad -2\mu D^{k+1}D^{-k}\mathbf{X}_0\mathbf{X}_0^{T*}D^k\tilde{\mathbf{W}}_k \\ &\quad +2\mu D^{k+1}D^{-k}\mathbf{X}_0\nu_k \\ D^{k+1}\tilde{\mathbf{W}}_{k+1} &= D\left(\mathbf{I} - 2\mu\mathbf{X}_0\mathbf{X}_0^T\right)D^k\tilde{\mathbf{W}}_k \\ &\quad +2\mu D\mathbf{X}_0\nu_k,\end{aligned} \tag{A5}$$

Making $\tilde{\mathbf{V}}_k = D^k\tilde{\mathbf{W}}_k$, $\tilde{\mathbf{H}} = D\left(\mathbf{I} - 2\mu\mathbf{X}_0\mathbf{X}_0^T\right)$, $\tilde{\mathbf{V}}_{k+1} = D^{k+1}\tilde{\mathbf{W}}_{k+1}$ and replacing them in equation (A5), we can re-written as,

$$\tilde{\mathbf{V}}_{k+1} = \tilde{\mathbf{H}}\tilde{\mathbf{V}}_k + 2\mu D\mathbf{X}_0\nu_k, \tag{A6}$$

Assuming that D^k and $\mathbf{W}_k$ are statistically independent, thus the value expected of equation (A6) can be re-written as,

$$E\left[\tilde{\mathbf{V}}_{k+1}\right] = E\left[\tilde{\mathbf{H}}\tilde{\mathbf{V}}_k\right] + 2\mu E\left[D\mathbf{X}_0\nu_k\right], \tag{A7}$$

Once D and $\mathbf{X}_0$ are invariant in time, equation (A7) will become,

$$\begin{aligned}E\left[\tilde{\mathbf{V}}_{k+1}\right] &= 2\mu D\mathbf{X}_0E\left[\nu_k\right]\sum_{i=1}^{k}\tilde{\mathbf{H}}^i \\ &\quad + \left(2\mu D\mathbf{X}_0E\left[\nu_k\right]\right)^n,\end{aligned} \tag{A8}$$

Making $n >> 1$, then $\mu^n \to 0$, thus the second term of the right side of the equation (A8) is null, once $0 < \mu < 1$. Expanding the very same equation (A8) into a geometric series, then we attain,

$$E\left[D^k\right]E\left[\tilde{\mathbf{W}}_k\right] = 2\mu D\mathbf{X}_0E\left[\nu_{k-1}\right]\frac{1-\tilde{\mathbf{H}}^k}{1-\tilde{\mathbf{H}}}, \tag{A9}$$

Returning back to former variables and multiplying equation (A9) by $E[D^{-k}]$, making $\tilde{\mathbf{W}}_k = \mathbf{W}_k - \mathbf{W}_*$ we get

$$E\left[\mathbf{W}_k - \mathbf{W}_*\right] = 2\mu D\mathbf{X}_0E\left[D^{-k}\right]E\left[\nu_{k-1}\right]\frac{1-\tilde{\mathbf{H}}^k}{1-\tilde{\mathbf{H}}}, \tag{A10}$$

As $\mathbf{W}_*$ is the optimal weight which is an arbitrary constant, then we can express this result, by

$$E\left[\mathbf{W}_k\right] = 2\mu D\mathbf{X}_0E\left[D^{-k}\right]E\left[\nu_{k-1}\right]\frac{1-\tilde{\mathbf{H}}^k}{1-\tilde{\mathbf{H}}} + \mathbf{W}_*. \tag{A11}$$

where $E\left[D^{-k}\right]E\left[\nu_{k-1}\right]$ is a time variant term that we call sinusoidal perturbation factor (SPF).