

MODELLING OF SAW BIOSENSORS

Marija Hribšek, Slavica Ristić, Zdravko Živković
Goša Institute, Milana Rakića 35, Belgrade, Serbia

Dejan Tošić
Faculty of Electrical Engineering, Bulevar kralja Aleksandra 73, Belgrade, Serbia

Keywords: SAW devices, SAW filters, biosensors.

Abstract: New approach in surface acoustic wave (SAW) biosensor's modelling is presented. Biosensor is modelled as a three port network. The model is general and can be used also in the case of transponder type of sensor. The closed form solutions for transfer function and input admittance at the electrical port of SAW devices with uniform transducers based on complex equivalent circuit are presented. Transfer function and input admittance in two different cases are calculated and compared with the experimental results showing very close agreement.

1 INTRODUCTION

Surface acoustic waves (SAW) were discovered in 1885 by Lord Rayleigh, and are often named after him: Rayleigh waves. A surface acoustic wave is a type of mechanical wave motion which travels along the surface of a solid material. Rayleigh showed that SAWs could explain one component of the seismic signal due to an earthquake, a phenomenon not previously understood. The velocity of acoustic waves is typically 3000 m/s, which is much lower than the velocity of the electromagnetic waves. A basic SAW device consists of two interdigital transducers (IDTs) on a piezoelectric substrate such as quartz. The IDTs consist of interleaved metal electrodes which are used to launch and receive the waves, so that an electrical signal is converted to an acoustic wave and then back to an electrical signal (Morgan, 1985).

The basic application of the SAW device is as delay line. Central frequency and the bandwidth are determined by the IDT's geometry and the substrate type. The IDT geometry is capable of almost endless variation, leading to a wide variety of devices. Starting around 1970, SAW devices were developed for pulse compression radar, oscillators, and bandpass filters for domestic TV and professional radio. In the 1980s the rise of mobile radio, particularly for cellular telephones, caused a

dramatic increase in demand for filters. New high-performance SAW filters emerged and vast numbers are now produced, around 3 billion annually. In the last two decades SAW devices have found numerous different applications outside their conventional fields of application: communications and signal processing. In the last decade considerable work has been done in the development of SAW sensors of different types of high quality. SAW filters are used as temperature, pressure and stress sensors as well as chemical and biosensors (Seifert, 1994, Pohl, 2000). At Imperial College in London are working on implantable and wearable SAW devices for long term clinical monitoring. Saw sensors are also used for wireless monitoring in harsh environment. There are two different types of SAW sensors: transversal and resonant. In liquids usually SH SAW type of sensors are used. In the references only analyses in time domain of the sensors exist. In the frequency domain only resonant type of SAW sensors are modelled (Campbell, 1989).

In this paper modelling of transversal SAW sensors is presented. It is well known that the exact analysis of SAW devices using surface wave theory is very complex (Matthews, 1977). Because of that approximate methods of analysis are developed. The simplest method of analysis is using delta function model. It gives the approximate results relatively

fast, but its use is limited to small loads and substrates with lower coupling constants.

The better approximate methods use equivalent models for IDTs, where the analysis tools known in electrical engineering can be applied. In these methods the accuracy depends on the complexity of the model. The closed form solutions for transfer functions and input admittances mostly only for simple models are given (Matthews, 1977, Morgan, 1985, Smith 1, 1969, Smith 2, 1969, Debnath, 1983, Hribšek, 1983). In this new algorithm SAW biosensor is modelled as three port network. The IDT's equivalent circuit based on Milsom's and Redwood's equivalent model (Milsom and Redwood, 1971) is most complex. Using the symbolic analysis method (Hribšek, 2007) the closed form solutions for transfer function and input admittance of the SAW biosensor are derived. The algorithm is valid for both types of sensors: direct and transponder types (Pohl, 2000). The algorithm is verified by two examples: insertion loss and input admittance are computed and compared with experimental results. It is also shown that in the case of the transponder straightforward dependence of the loss and measured value is obtained.

2 THE MODEL

The transversal SAW biosensor can be schematically presented as in Figure 1: between the interdigital transducers (IDT) on the top of the piezoelectric substrate the chemical or bio sensitive layer is placed.

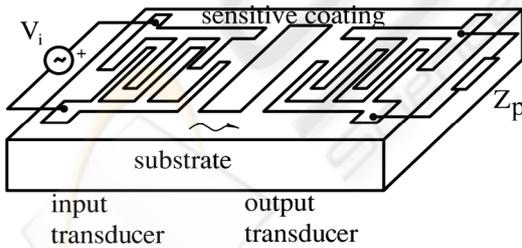


Figure 1: The basic configuration of SAW biosensor.

The surface wave is induced by electrical signal applied to the input IDT. The output signal (voltage) is taken from the second IDT. The interdigital transducers are wideband with uniformly spaced electrodes. The configuration presented in Figure 1 can be modelled by equivalent electrical scheme given in Figure 2 where IDT's are three port

networks and the middle sensing part is a two port network.

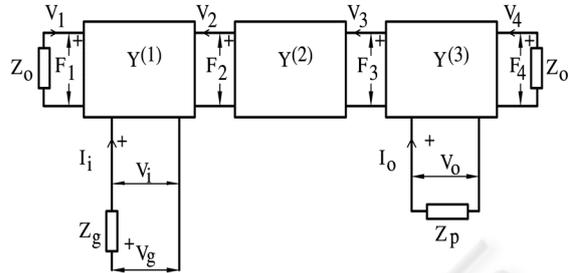


Figure 2: Equivalent electrical model of a saw sensor.

The characteristic acoustic impedance of the unloaded substrate is Z_o , and the electrical impedances of the generator and load, are Z_g and Z_p , respectively. Each part is defined by its corresponding admittance matrix $Y^{(1)}$, $Y^{(2)}$ and $Y^{(3)}$.

These matrices are calculated following the procedure presented in (Hribšek, Tošić). In general case matrices $Y^{(1)}$ and $Y^{(3)}$ are different, but usually in sensors they are equal with small number of electrodes, thus simplifying the calculations. Since the circuit is passive for all matrices the passivity condition: $Y_{ij} = Y_{ji}$, for $i \neq j$, is valid. F_i 's and v_i 's denote the electrical equivalents of mechanical forces and velocities. The elements of $Y^{(2)}$ of the sensing part is given by:

$$y_{11}^{(2)} = y_{22}^{(2)} = \frac{\cot g \theta}{jZ_s} \tag{1}$$

$$y_{12}^{(2)} = \frac{j \operatorname{cosec} \theta}{Z_s} \tag{2}$$

where $\theta = \pi f / f_o$, f_o is the central frequency, Z_s is the acoustic impedance of the sensing part, and f is the frequency of the input signal.

Now the whole sensor can be represented as an equivalent two port where one port is the electrical input of the input IDT and the second port is the electrical port of the output transducer. The transfer function of the two port defined as:

$$T = \frac{V_o}{V_g} \tag{3}$$

can be expressed in the following form:

$$T = \frac{-Y_{21}}{(Y_{22} + \frac{1}{Z_p})(1 + Y_{11}Z_g) - Y_{12}Y_{21}Z_g} \quad (4)$$

where Y_{ij} are the admittance parameters of the equivalent two port. Therefore, for the transfer function determination y parameters should be found. They are found in several steps. In each step one partial Y matrix is derived. In the first step matrix Y' , which connects input voltage and current with force F_2 and velocity v_2 , is found. Then the matrix Y'' which gives the relationship between the input signals and the F_3 and v_3 is determined. Finally, the Y matrix is derived in terms of parameters of the matrices $Y^{(1)}$, $Y^{(2)}$ and $Y^{(3)}$. The expressions are in closed form, but very bulky and that is the reason why they are omitted in this text. The symbolic circuit analysis method is used. The obtained relations are general. In the case of SAW sensors they can be less complex if input and output transducers are equal. Also if the simpler models of IDTs are used, the calculation will be easier, but in any case computer must be used.

For matching purposes, the input admittance must be determined. It can be expressed in terms of the y parameters of the input transducer as follows:

$$Y_i = y_{33} - y_{13}^2 \frac{2y_{12} + 2y_{11} + Y_o + Y_s}{(y_{11} + Y_o)(y_{11} + Y_s) - y_{12}^2} \quad (5)$$

Using the algorithm presented, the computer program which calculates frequency dependence of input conductance and susceptance and insertion loss of SAW device with two uniform transducers was made. The program was verified by two examples taken from (Smith et al.1., Smith et al.2). In these cases the substrate between transducers was unloaded, e.g. $Z_s = Z_o$. The computed results are presented in Figures 3. and 4., solid lines. To allow comparison between the computed and measured data, the experimental results are also presented in the same figures, dotted lines.

From Figures 3 and 4 is obvious that experimental and calculated data are in excellent agreement, even better than in (Smith et al.1., Smith et al.2).

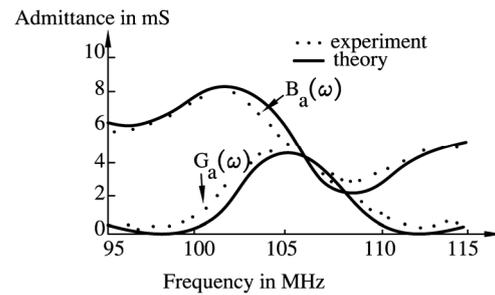


Figure 3: Real (G_a) and imaginary (B_a) part of the input admittance.

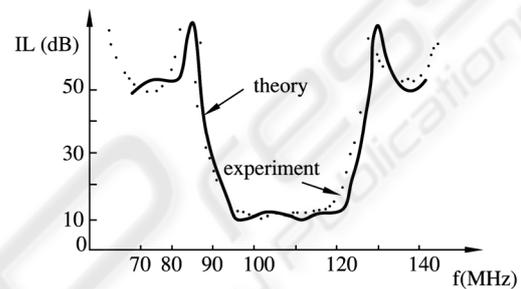


Figure 4: Calculated and measured insertion loss.

Since the algorithm and computer programs are verified than can be successfully used in the analysis or prediction in any particular case in SAW biosensors. In that case due to the loading of the sensitive film the acoustic impedance will change accordingly, and therefore Y_i and T have to be changed.

The algorithm is general. It can be also very efficiently used in the frequency domain analysis of SAW transponders (Pohl, 2000). SAW transponders are SAW devices which do not have sensing film between the transducers. They get the signal obtained from the actual physical sensor on the impedance Z_p . In that case from relation (4) is obvious that for one device all admittances are constants and the only variable is Z_p . Then, since $Z_g = Z_o$, $Z_s = Z_o$, and $Z_o = R_o$, transfer function can be expressed as:

$$T(Z_p) = T(\infty) \frac{1}{1 + \frac{R_o}{Z_p}} \quad (6)$$

where $T(\infty)$ denotes the transfer function when Z_p is infinite. Now two cases can be discussed: when Z_p is real and when it is of capacitive or inductive type.

If Z_p is real e.g. $Z_p = R_p = 1/G_p$, ratio of $T(Z_p)$ and $T(\infty)$ can be represented as in Figure 5.

If the loading is purely inductive or capacitive ratio of $T(Z_p)$ and $T(\infty)$ can be expressed as follows:

$$T(B_p) = T(\infty) \frac{1}{1 + jB_p R_o} \quad (7)$$

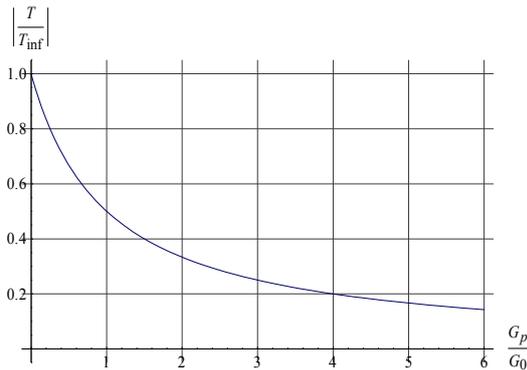


Figure 5: Relative amplitude versus resistive load.

In that case relative insertion loss can be represented as in Figure 6.

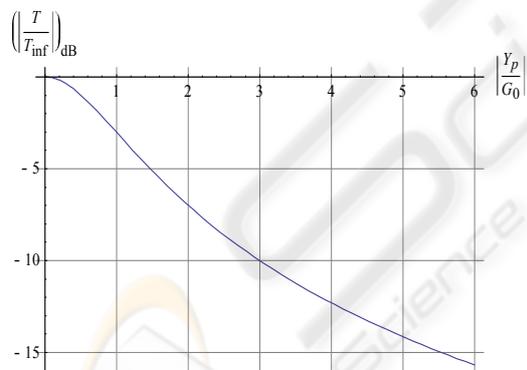


Figure 6: Relative insertion loss versus load susceptance.

3 CONCLUSIONS

The new developed algorithm is general. It can be used in frequency analysis of SAW based biosensors, as well as of SAW transponders. The efficiency of the presented algorithm is demonstrated with calculations of frequency dependence of input conductance and susceptance and insertion loss of SAW devices with uniform transducers. The results are compared with

corresponding experimental data showing very close agreement.

ACKNOWLEDGEMENTS

The authors thank the Ministry of Science and Technological Development of Serbia for financial support under the project number 11026.

REFERENCES

- Morgan D. P., 1985, *Surface Wave Devices for Signal Processing*, Elsevier, London.
- Seifert F., Bulst W.E., Ruppel C., 1994, Mechanical sensors based on surface acoustic waves, in *Sensors and Actuators*, A44, pp. 231-239.
- Pohl A., 2000, A Review of Wireless SAW Sensors, in *IEEE Transactions on ultrasonics, ferroelectrics, and frequency control*, vol. 47, no. 2, pp. 317-332, (Invited Paper).
- Campbell C., 1989, *Surface Acoustic Wave Devices and Their Signal Processing Applications*, Academic Press, Boston.
- Matthews, ed. 1977, *Surface Wave Filters*, John Wiley, New York.
- Smith W.R., et al, 1, 1969., Analysis of interdigital surface wave transducer by use of equivalent circuit model, in *IEEE Trans. on MTT*, vol.17, No 11, pp.856-864.
- Smith W.R., et al, 2, 1969., Design of surface acoustic delay lines with interdigital transducers, in *IEEE Trans. on MTT*, vol.17, No 11, pp. 865-873.
- Debnath N., Ajmera J.C., Hribšek M. F., Newcomb R.W., 1983. Scattering and Admittance Matrices of SAW Transducers, in *Circuits, Systems and Signal Processing*, vol.2., No.2, pp. 161-178.
- Hribšek M., Tošić D., 1983, An Improved Algorithm for Analysis of Uniform SAW Transducers, in *Proc. 26th Midwest Symposium On Circuits and Systems*, INAOE, Mexico.
- Milsom R. F., Reddwood M., 1971, Interdigital Piezoelectric Rayleigh Wave Transducer: an improved Equivalent Circuit, in *El. Letters*, vol.7, p. 217.
- Hribšek M., Tošić D., 2007, Symbolic analysis and design of current-differencing-amplifier filters, in *Scientific Technical Review*, vol. LVII, No. 2.