

# FRAME ERROR RATE EVALUATION OF A C-ARQ PROTOCOL WITH MAXIMUM-LIKELIHOOD FRAME COMBINING

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**Abstract:** In this paper a *Cooperative-ARQ (C-ARQ)* with a *maximum-likelihood frame combiner (ML-FC)* protocol is studied. C-ARQ is well suited for wireless transmission in either infrastructure or ad-hoc networks, as it exploits some of the unique characteristics of wireless media, such as the natural broadcast of wireless transmission and receiver diversity. The frame combiner can help in the case in which any received frame is correct, exploiting the same characteristics of wireless transmission but at a bit level. The paper studies the Frame Error Rate for this kind of system, showing that significant improvements can be obtained.

## 1 INTRODUCTION

This paper deals with a recently proposed variant of the well known ARQ (Automatic Repeat reQuest) protocol, called *Cooperative-ARQ (C-ARQ)* (Miu, 2005), (Monti, 2005), (Morillo, 2005), (Zhao, 2005). C-ARQ is well suited for wireless transmission in either infrastructure or ad-hoc networks, as it exploits the natural broadcast characteristic of wireless transmission and receiver diversity (Goldsmith, 2005). As in any ARQ protocol, when a node receives a frame with erroneous bits, it will ask for a frame retransmission. In C-ARQ, however, the node has previously designated a subset of nearby nodes as *cooperator nodes*. In case that a cooperator node has correctly received the frame, it will perform the retransmission instead of the transmitter node. In case no cooperator node has the correct copy of the frame, a retransmission from the transmitter node will be asked.

In this document, a variant of this basic operation scheme is studied: when all the cooperator nodes have erroneous copies of the frame, instead of asking a retransmission from the transmitting node, the receiver uses the erroneous frames from them in an attempt to reconstruct the original one. This frame reconstruction will be performed by means of a so-called *frame-combiner (FC)*. The frame-combining technique studied in this paper is called *maximum-likelihood (ML)*.

Assume that a node have M-1 cooperators. The frame-combiner will take bit by bit the M frames (frames from the receiving node plus frames from the M-1 cooperator nodes) and will try to produce a correct copy of the original frame. Although a frame combiner is a Layer 2 module, -it deals with bits and not with signals-, it shares some commonalities with techniques used at physical level, such as the well known Maximal Ratio Combining (MRC) technique, that combines different signals received on different antennas on the same device; see (Goldsmith, 2005).

The paper presents a numerical evaluation of the Frame Error Rate (FER) of a C-ARQ protocol with a maximum-likelihood frame combiner (ML-FC) in Rayleigh channels. The idea behind the frame combiner is give to the system a second opportunity when C-ARQ fails: instead of asking a retransmission from the source we retransmit from neighboring nodes of the destination.

Using C-ARQ with a frame combiner can introduce degradation in throughput, as in some occasions M-1 cooperator nodes must re-transmit their incorrect frames to the receiving node. On the other hand using higher order constellations in the frame retransmissions from the cooperators could alleviate this. The study of this, however, will be left out of the scope of this paper.

The studied scheme does not present major implementation challenges, as it only involves changes in the driver software together with the

inclusion of new signalling packets (for example for establishing the relation of cooperation and for requesting frame copies). The ML-FC operation requires that each node knows the average SNR during frame reception. This information is readily available, with different levels of accuracy, from many of the wireless NICs used in current networks.

Previous studies on C-ARQ (e.g. (Miu, 2005), (Monti, 2005), (Zhao, 2005)) report significant improvements in terms of transmission power, transmission range or throughput. Reference (Morillo, 2005) studies a majority voting frame combiner for AWGN and Rayleigh channels. An ARQ variant (that the authors call *Memory ARQ*) that uses *frame combining* of erroneous frames was previously introduced in (Lau, 1986). This work, however, is not in the framework of cooperative ARQ, as all the copies come from the original transmitter. Moreover, (Lau, 1986) analyzes the performance of such system in an AWGN channel while our work is focused on Rayleigh fading channels. The work reported in (Eaves, 1977), studies the probability of block error (i.e. FER) for slow Rayleigh fading channels. Although it is not a work on cooperative techniques nor on ARQ protocol, it gives to us a good base line scenario to compare with our proposal.

Next, Section II gives a brief description of the C-ARQ variant that is studied in this paper. Section III presents the Maximum-Likelihood (ML) frame combiner (FC) integrated in the studied C-ARQ scheme. Section IV presents the evaluation of the FER for such kind of system.

## 2 COOPERATIVE ARQ

In this section we give a brief description of the C-ARQ variant studied in this paper: Let us assume a wireless ad-hoc network. For a given node  $x$  of the network, we define  $R_x$  as the set of nodes which receive the signal from  $x$  with some minimal quality parameters. Let  $y$  be a node of  $R_x$ , to which node  $x$  wants to send a frame, and let  $d$  be the distance between  $x$  and  $y$ . From the set  $R_y$ , we form a subset of cooperating nodes, that we call  $C_y$ , which includes all nodes from that  $y$  receives signals with an excellent quality (including  $y$  itself), and that are willing to cooperate with  $y$ . We assume a perfect channel between  $y$  and nodes in  $C_y$ , due to this excellent signal quality. Although this is a strong assumption, it can be justified by the proximity between  $y$  and nodes in  $C_y$ . In the rest of the paper we assume  $|C_y| = M$ . Usually, we will have that

distance between  $y$  and nodes of  $C_y$  will be bounded to  $d'$ , with  $d' \ll d$ . We can thus approximate the distance between  $x$  and nodes of  $C_y$  to the value  $d$ .

We assume that when  $x$  transmits a frame addressed to  $y$ , nodes of  $C_y$  observe different values of SNR, following a Rayleigh distribution:

$$f(\gamma) = \frac{1}{\gamma^*} e^{-\frac{\gamma}{\gamma^*}} \quad \gamma \geq 0$$

where the average SNR,  $\gamma^*$ , is the same for all nodes and constant with time. We also assume that for each node, the SNR value is constant during a frame reception, but it can be different for each frame.

Let  $x$  transmit a frame to  $y$ , which is received by nodes of  $C_y$ . We assume that these nodes can identify that the final destination of the frame is  $y$  even in presence of transmission errors (e.g. using a strong error correction code for the corresponding header fields). After receiving the frame, every node checks for its correctness using for instance a CRC. For simplicity, it is assumed throughout this paper that the error detection code used will detect all errors introduced by the channel. In practice, this is a reasonable assumption since the probability of an undetected error can be made very small.

In a C-ARQ system without frame combining, nodes in  $C_y$  that correctly receive the frame will keep a copy of it. If node  $y$  detects that its reception is erroneous, it will ask one of these nodes for a retransmission of a correct copy. Only in the case that any node in  $C_y$  has correctly received the frame,  $y$  would ask for a retransmission to node  $x$ .

In a C-ARQ with FC, in contrast, even if the frame was received with errors, nodes in  $C_y$  will keep a temporary copy of it. In case node  $y$  finds that the frame has suffered errors, it will request their cooperators for a correct copy of the frame. In case there is no correct frame received by any cooperator, it will send a signaling packet to its cooperators requesting a retransmission of their erroneous frame copies (Figure 1). Cooperators of  $y$  will send, in turns, their copies of the frame, attaching the measured value of SNR during the frame reception,  $\gamma_i$ , until  $y$  is able to correctly decode the frame by performing the *frame combining*. Recall that we assume a slow fading channel and, therefore, that  $\gamma_i$  is constant during frame reception, but in general different for each cooperator. At each reception of the information sent by each cooperator - and assuming that this received copy of the frame has not a correct CRC-, terminal  $y$  uses a Maximum-Likelihood (ML) decision rule for constructing a "hypothetically correct" received frame (Figure 1). That is,  $y$  uses a statistically optimal fusion rule in

terms of minimum detection error probability. Only in the case that this reconstructed frame is still incorrect in the last step ( $\gamma$  has received all copies from all its cooperators), a retransmission is required from node  $x$  (Figure 1) if the maximum number of such retransmissions has not been exceeded (this number is normally set to 11 in 802.11 but could be set below this value for this system with greater resilience).

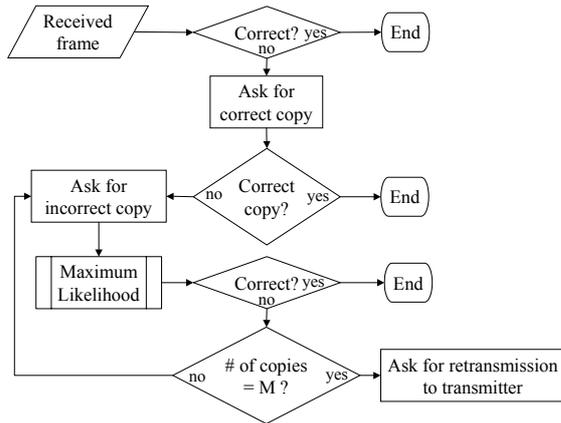


Figure 1: Cooperative ARQ scheme with maximum-likelihood frame combiner.

Note that we do not focus on the order in which cooperators send their respective frames to  $\gamma$ , and we assume some pre-established order. How this order is set up is left for future work and is out of the scope of this document.

This paper studies the impact of C-ARQ and ML Frame-Combining on the FER.

### 3 THE ML FRAME COMBINING TECHNIQUE

The ML decision rule for obtaining a possible correct copy of the frame is the following: Let  $BER_i(\gamma_i)$  be the  $i$ -th node BER (directly derived from its SNR,  $\gamma_i$ ). Let  $S_1$  and  $S_0$  be the sets of cooperator nodes of  $\gamma$  that have detected a given bit as 1 or 0 respectively. Assuming that the a-priori probabilities of “1” or “0” are identical, the ML decision rule for this bit would be:

Decide “1” if

$$\prod_{i \in S_1} BER_i(\gamma_i) \prod_{i \in S_0} (1 - BER_i(\gamma_i)) < \prod_{i \in S_1} (1 - BER_i(\gamma_i)) \prod_{i \in S_0} BER_i(\gamma_i) \quad (1)$$

and decide “0” otherwise

Note that when SNR is the same for all cooperating nodes, then the proposed decision rule is equivalent to the Majority Voting (MV) scheme proposed in (Morillo, 2005), due to the fact that all frames have the same weight. Each cooperator node coherently detects its own information with a given BER depending on the modulation scheme used and the channel model considered. In the case of Rayleigh channels and for some modulation schemes, this ML decision rule can be simplified.

### 4 FER PERFORMANCE EVALUATION

The exact evaluation of the FER for C-ARQ+FC is involved, even for simple modulation techniques. Even the evaluation of FER for C-ARQ leads to non-closed expressions; see (Eaves, 1977). On the other hand, FC mechanism only enters into play when all frames received by a node and its cooperators are incorrect, meaning that we should introduce this condition into the probability expressions.

We take thus the next approach: Firstly, we study the ML-FC in isolation. Secondly, an analysis of the performance of C-ARQ without FC is done. It is easy to show that each of these mechanisms in isolation would lead always to worse cases than the C-ARQ+FC mechanism in conjunction. Finally, we study FER for the combined C-ARQ-FC scheme using Monte-Carlo simulations. We assume that the modulation method is BPSK, without loss of generality.

The expression for the FER for a Rayleigh fading channel in the case in which no cooperation is exploited is given by:

$$FER(\gamma) = \int_0^{\infty} \frac{1}{\gamma^*} e^{-\gamma/\gamma^*} [1 - (1 - BER(\gamma))^L] d\gamma \quad (2)$$

where  $\gamma^*$  is the average SNR, and  $L$  is the frame length; see (Eaves, 1977). In the case of BPSK modulation,  $BER(\gamma) = Q(\sqrt{2\gamma})$ . Figure 2 presents the calculated FER versus frame length  $L$  for an average SNR of 24 dB, which corresponds to a  $10^{-3}$  BER.

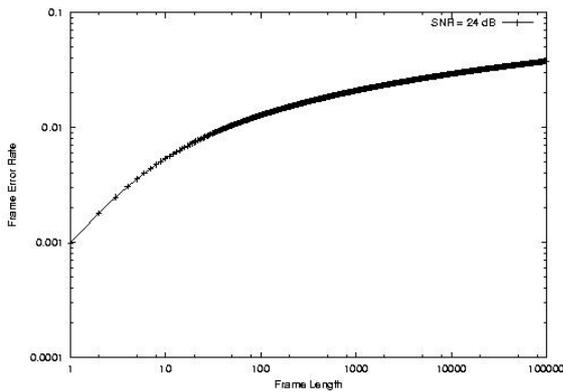


Figure 2: FER vs Frame Length in Rayleigh fading channels. SNR=24dB. BPSK modulation.

### 4.1 FER of ML-FC

We focus first on the maximum-likelihood FC mechanism and study it in isolation. We have performed a Monte-Carlo simulation calculating the FER that can be obtained for different values of average SNR, and for different values of M, the number of cooperators. Simulations have been done using OCTAVE for frame lengths (L) equal to 1,000 and 10,000 bits. The results for L=1,000 bits are presented in Figure 3.

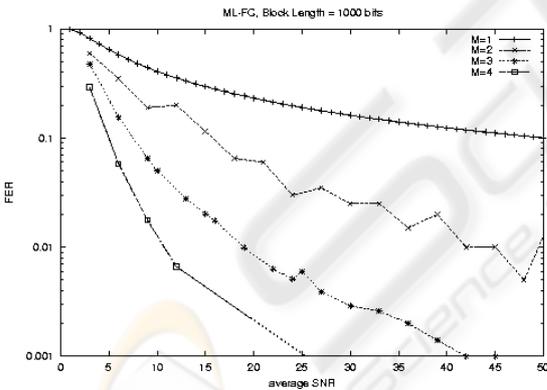


Figure 3: FER of ML-FC vs SNR in Rayleigh fading channels for different values of M and for L=1000 bits. BPSK Modulation.

From Figure 3 we can see the great improvements that can be obtained with the ML-FC technique studied in this paper. For example, for an SNR=33 (~15dB) the FER decreases from 0.15 for the case M=1 -no cooperation- to 0.025 for the case of just combining two frames, meaning that we have a reduction of FER of an order of magnitude. In fact, note that for the case M=2, the ML-FC does not perform any *frame combining* and simply chooses

the frame with higher SNR. For M=3 the FER for the same SNR is about 0.0026. Increasing M, on the other hand, increases the number of frame retransmissions.

Figure 4 presents similar results for the case L=10,000 bits.

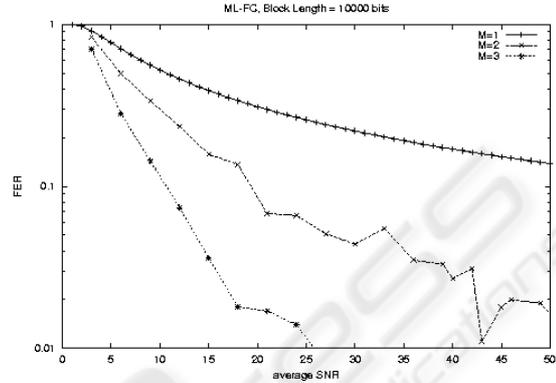


Figure 4: FER of ML-FC vs SNR in Rayleigh fading channels for different values of M and for L=10000 bits. BPSK Modulation.

### 4.2 FER of C-ARQ Protocol

Let us focus now on the C-ARQ protocol and study it in isolation, i.e. without ML-FC, as we have done for the FC.

An analytical expression can be easily derived from (2) for the case of C-ARQ. If we have M cooperating nodes (including the destination node itself) and considering that the  $\gamma_i$  are independent and identically distributed random variables, the FER of C-ARQ will be:

$$FER_{C-ARQ} = \prod_{i=1}^M \int_0^{\infty} \frac{1}{\gamma^*} e^{-\gamma/\gamma^*} \left[ 1 - (1 - BER(\gamma_i))^L \right] d\gamma_i$$

$$FER_{C-ARQ} = [FER(\gamma)]^M \tag{3}$$

This expression represents the FER from the transmitter point of view, i.e., the probability that a retransmission from x in the model presented in Section II would be necessary.

The goodness of the C-ARQ protocol can be observed in Figure 5, together with the impact of the number of cooperators in the FER. It can be seen how FER decays orders of magnitude with just 2 or 3 cooperators.

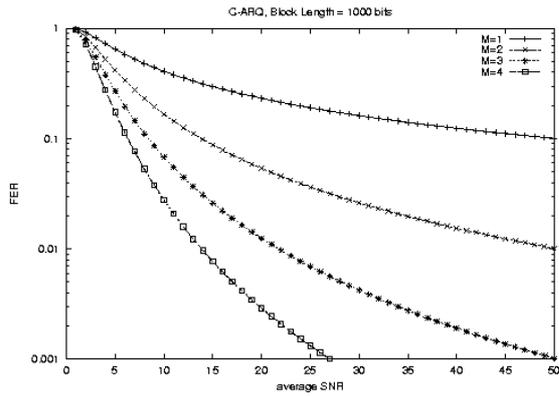


Figure 5: FER of C-ARQ vs SNR in Rayleigh fading channels for different values of M and for L=1000 bits. BPSK Modulation.

### 4.3 FER of Combined C-ARQ + ML-FC

Once presented the results for each component of the system in isolation, let us focus on the performance of such system as a whole.

We have performed simulations for L=1,000 and 10,000 bits, and for different number of cooperators (M). Figure 6 shows the results for M=2 and M=1. It is clear that the use of C-ARQ-FC considerably reduces the overall FER values. Figure 6 is also interesting for a particularity of the M=2 case: normally ML-FC in isolation lead to a lower FER than the C-ARQ in isolation. For M=2, however, this is not true, as in this case ML-FC in fact does not perform any *frame combining*: it will simply choose the frame with higher SNR (that could be erroneous). C-ARQ, on the other hand, will choose a correct frame if it exists. Sometimes it can happen that the correct frame is the one with lower SNR. In this case ML-FC will fail while C-ARQ will not.

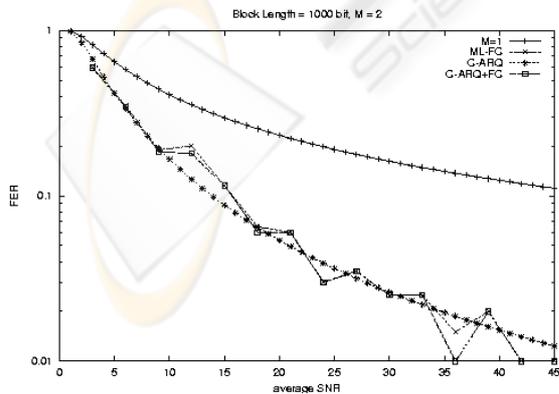


Figure 6: FER of the whole system vs SNR in Rayleigh fading channels for M=2 and L=1000. BPSK Modulation.

Figure 7 presents the case M=3 and L=1,000. As stated previously, ML-FC in isolation performs better than C-ARQ in isolation, while the combination of both mechanisms performs only slightly better than ML-FC.

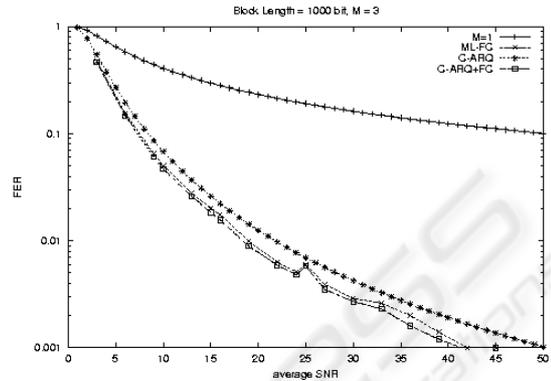


Figure 7: FER of the whole system vs SNR in Rayleigh fading channels for M=3 and L=1000. BPSK Modulation.

Figure 8 shows the obtained values for the case M=4, L=1,000. As expected, similar conclusions can be drawn, although now the difference between the C-ARQ in isolation and the combined C-ARQ-FC scheme is larger.

The question of whether C-ARQ in isolation or the combined C-ARQ-FC schemes are feasible alternatives, is very much system dependent, and is left out of the scope of this paper. The evaluation of this question depends on factors like the distance between source and receiver, between receiver and cooperators, etc. The idea of the FC is to give a second opportunity in trying to avoid retransmissions from a source (possibly far away) by substituting it with transmissions of nearby nodes that could spend less power. Another open issue could be the system performance in the case of using Hybrid-ARQ techniques integrated in the system.

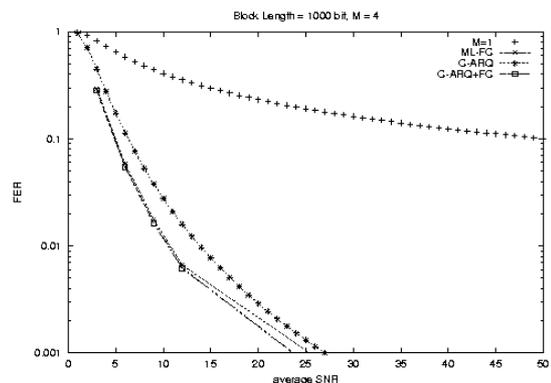


Figure 8: FER of the whole system vs SNR in Rayleigh fading channels for M=4 and L=1000. BPSK Modulation.

## 5 CONCLUSIONS

In this paper we study and evaluate the FER for a cooperative ARQ scheme with a *maximum-likelihood frame combiner* integrated in it that exploits space diversity and cooperation between neighboring nodes. This paper shows how the benefits of space diversity and node cooperation have also a great impact on the FER performance (maybe more interesting at the Link Layer than the BER). The hardware complexity of the system is clearly reduced with respect to MRC or to cooperative transmission techniques, although new protocol signaling is needed.

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