SIGNAL-DEPENDENT ANALYSIS OF SIGNALS SAMPLED BY SEND ON DELTA SAMPLING SCHEME

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Keywords: Signal driven A/D conversion, send on delta sampling scheme, spectral analysis, signal dependent transformation.

Abstract: Interest in the application of signal driven sampling schemes is increasing as they offer various advantages over traditional sampling. The paper describes the principles and discusses the properties of sampling, which is based on the send-on-delta concept. In such a way, it is possible to decrease the sampling density, and since the samples are placed non-equidistantly it is possible to suppress the distortion due to frequency aliasing. The non-uniform location of samples requires an advanced processing method. The paper discusses the spectral estimation, which is based on the use of a bank of minimum variance filters. To improve the resolution and accuracy, iterative updating of autocorrelation matrix is used. The results of simulations are presented. The use of an iterative algorithm allows correcting spectral estimation even if the mean sampling density is several times less than the Nyquist criterion. The proposed approach can be of interest for distributed wireless data acquisition in remote sensing systems, because it allows the amount of transmitted data to be decreased considerably.

1 INTRODUCTION

Regarding signal sampling procedures, generally, the signal can be approximated with fewer samples per interval using appropriate non-equidistantly spaced samples than using a uniform sampling procedure, where the sampling frequency is defined taking into account only the highest signal component. The problem of processing non-uniformly sampled signals has quite a long history. However it typically deals with cases, where non-uniformity is introduced in a deliberate or deterministic way. Intuitively speaking, the sampling flow has to reflect the local properties of the signal. For example, it is more efficient to sample the low frequency regions at a lower rate than the high frequency regions. A special class of non-uniform sampling is derived if the sampling process is driven by the signal itself this is so called signal dependent sampling. The popular types of signal-dependent sampling are zero crossing, reference signal crossing, level crossing or send-on-delta concepts.

In the paper, sampling based on the send-on-delta (SoD) concept is employed. The motivation of such a choice is based on three key aspects. Firstly, it is signal dependent sampling, which offers various advantages over traditional uniform sampling (Hauck, 1995). Secondly, this sampling scheme can be simply implemented in hardware, because it guarantees a certain minimal interval between samples. Thirdly, the non-uniformity in sampling provides the possibility of suppressing frequency aliasing (Masry, 1978), and therefore the SoD scheme can be used to reduce the sampling density in comparison with the Nyquist rate. The properties of send-on-delta sampling will be discussed in Section 2.

The reason, why the SoD sampling is not widespread in practice, lies in the fact that such digitized signals can not always be successfully processed using standard algorithms. One of the main processing tasks is the estimation of the signal spectrum. In the case of uniform sampling, the Nyquist criterion determines the minimum sampling rate, which must be fulfilled in order to avoid frequency aliasing. To estimate the spectrum of a non-uniformly sampled signal, an advanced signal processing method is required, especially in the cases where the sampling density can be below the Nyquist criterion. The paper discusses the spectrum estimation method that is based on signal-dependent transform (Greitans, 2005), which uses the minimum variance filter principle and provides spectral estimation with high resolution and accuracy. The algorithm will be described in Section 3, and simulation results will be shown in Section 4.

Greitans M. and Shavelis R. (2008).
SIGNAL-DEPENDENT ANALYSIS OF SIGNALS SAMPLED BY SEND ON DELTA SAMPLING SCHEME.
In *Proceedings of the International Conference on Signal Processing and Multimedia Applications*, pages 125-130
DOI: 10.5220/0001937201250130

2 SEND-ON-DELTA SAMPLING

According to the level-crossing (LC) concept the sampling is triggered if the input signal crosses any of the fixed quantization levels (Allier and Sicard, 2003). The principle of LC sampling is illustrated in Figure 1a. The number of samples captured depends on signal itself (Mark and Todd, 1981), (Greitans, 2006). The minimum distance $\Delta t_{min} = \min(\Delta t_n)$, where $\Delta t_n = t_{n+1} - t_n$, between the sampling points t_n can be very small and thus the analog-to-digital (A/D) converter can not ensure all the samples are captured due to the limited performance of electronic components.

2.1 Sampling Scheme

The situation changes in case of send-on-delta sampling scheme, according to which the sampling is triggered if the signal deviates by defined value $\Delta l > 0$, called the threshold, or delta (Miskowicz, 2006). The principle of SoD sampling is shown in Figure 1b. There is a constant difference equal to Δl between consecutive signal values

$$|s(t_{n+1}) - s(t_n)| = \Delta l, \quad (1)$$

where $s(t_n)$ is a signal sample at the time instant t_n. The threshold Δl determines the resolution of signal observations. The smaller Δl, the higher resolution of input signal tracking.

2.2 Minimum Sampling Interval

Obviously, $N_{LC} \geq N_{SoD}$, where N_{LC} and N_{SoD} are the numbers of samples captured by using respectively LC and SoD sampling schemes. The minimum distance Δt_{min} in case of SoD depends on Δl and on the spectral properties of the signal. For example, if a single sinusoid

$$s(t) = A\sin(2\pi f t + \varphi) \quad (2)$$

is sampled, then the minimum distance, if $\omega = 2\pi f$, is

$$\Delta t_{min} \geq \frac{\Delta l}{2A\pi f} = \frac{\Delta l}{A\omega} \quad (3)$$

as the maximum value of derivative is $A\omega$.

Another example is the signal with constant power spectral density

$$P(f) = \begin{cases} A^2 & \text{for} \quad |f| \leq f_{max} \\ 0 & \text{elsewhere} \end{cases}. \quad (4)$$

To achieve the maximum slope of such a signal the phase has to be the same at all frequencies. In this case the iverse Fourier transform is described by *sinc* function:

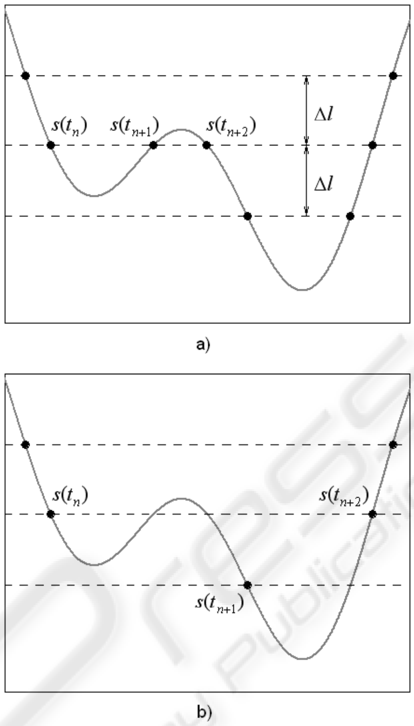

Figure 1: Signal dependent sampling based on a) level-crossing and b) send-on-delta concepts.

$$s(t) = 4A\pi f_{max}\text{sinc}(2\pi f_{max} t) \quad (5)$$

To estimate the maximum value of derivative of $s(t)$, lets first look at signal $u(t) = \text{sinc}(t) = \frac{\sin(t)}{t}$. The first and second order derivatives of $u(t)$ are:

$$u'(t) = \frac{t\cos(t) - \sin(t)}{t^2} \quad (6)$$

and

$$u''(t) = \frac{-t^2\sin(t) - 2t\cos(t) + 2\sin(t)}{t^3} \quad (7)$$

To estimate the maximum value of (6), we solve the equation

$$u''(t) = 0 \quad (8)$$

From (8) it follows, that $t \neq 0$ and

$$t\cos(t) = \sin(t) - 0.5t^2\sin(t) \quad (9)$$

Lets say we have the solution of (9) if $t = t_0$, then inserting (9) into (6) gives:

$$u'(t_0) = -0.5\sin t_0 \quad (10)$$

It means, that the maximum value of $u'(t)$

$$u'(t)_{max} \leq \frac{1}{2} \quad (11)$$

In turn, the derivative of $s(t)$ can be expressed as

$$s'(t) = 8A\pi^2 f_{max}^2 (\text{sinc}(t))' = 8A\pi^2 f_{max}^2 u'(t). \quad (12)$$

From (12) and (11) follows, that the maximum value of $s'(t)$

$$s'(t)_{max} \leq 4A\pi^2 f_{max}^2 = A\omega_{max}^2 \quad (13)$$

and the minimum distance

$$\Delta t_{min} \geq \frac{\Delta l}{A\omega_{max}^2}. \quad (14)$$

2.3 Sampling Density

Due to the signal-dependent nature of SoD, the sampling density depends on the statistical characteristics of the signal. If l_k are the values of quantization levels crossed by the signal, then in case of only one quantization level l_1 the number of SoD samples captured will be $N_{l_1} = 1$. In case of two levels l_1 and l_2 the numbers of samples $N_{l_1} = N_{l_2}$ as every next sampling is triggered by crossing the quantization level, which differs from the previous one. If we start the numbering of levels from the lower one, then in case of three levels l_1, l_2 and l_3 the numbers of samples $N_{l_1} = N_{l_2} - N_{l_3}$ since SoD captures at the second level is also caused by the third level. In general, if there are K quantization levels, then

$$N_{l_1} = \sum_{k=2}^{K} (-1)^k \cdot N_{l_k} \quad (15)$$

Now let us first discuss the case, when the signal is sampled by level crossing. For a single sinusoid $A\sin(2\pi ft + \varphi)$ each quantization level during one period is crossed twice and the sampling density can be expressed as

$$\sigma_{LC} = 2Kf. \quad (16)$$

For a zero mean Gaussian process with power spectral density given by expression (4) the expected value of the sampling density is (Mark and Todd, 1981):

$$\mathrm{E}[\sigma_{LC}] = \frac{2 f_{max}}{\sqrt{3}} \sum_{k=1}^{K} e^{-\frac{l_k^2}{4A^2 f_{max}}}, \quad (17)$$

Now let us analyse the SoD case for both previously chosen signals. For a single sinusoid each quantization level gives two SoD samples during one period. The exception is with the upper and lower levels each giving only one SoD sample per period. Thus, the sampling density can be expressed as

$$\sigma_{SoD} = 2(K-1)f \quad (18)$$

for $K \geq 2$. If $K = 1$, then only one SoD sample will be captured during the whole observation time of the signal. For the Gaussian process the estimation of sampling density is not so obvious. To simplify the analysis, we assume the signal, whose consecutive local extremes are with different signs varying around zero level as shown in Figure 2.

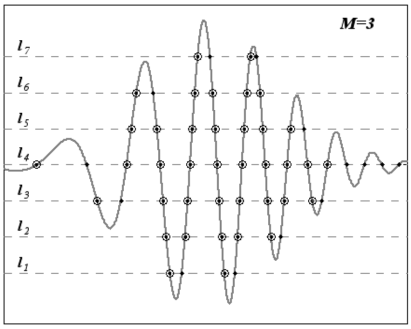

Figure 2: Signal for simplified analysis of SoD sampling density. The black points are LC samples, while the small circles are SoD samples.

The number of quantization levels is $K = 2M+1$ with $2M$ levels placed symmetrically around the zero level. The number of SoD samples captured can be estimated from the number of LC samples for each quantization level. Obviously for the first level we get

$$N_{SoD_{l_1}} = \frac{N_{LC_{l_1}}}{2}. \quad (19)$$

For the second level we get

$$N_{SoD_{l_2}} = \frac{N_{LC_{l_2}} - N_{LC_{l_1}}}{2} + N_{LC_{l_1}} = \frac{N_{LC_{l_2}} + N_{LC_{l_1}}}{2}. \quad (20)$$

And similarly for the Mth level we get

$$N_{SoD_{l_M}} = \frac{N_{LC_{l_M}} + N_{LC_{l_{M-1}}}}{2}. \quad (21)$$

The number of SoD samples captured on the zero level equals the number of LC samples on the Mth level

$$N_{SoD_{l_{M+1}}} = N_{LC_{l_M}}. \quad (22)$$

The total number of SoD samples captured, assuming that $N_{SoD_{l_m}} = N_{SoD_{l_{K+1-m}}}$, is

$$N_{SoD} = 2\left(N_{SoD_{l_1}} + \ldots + N_{SoD_{l_M}}\right) + N_{SoD_{l_{M+1}}}. \quad (23)$$

From (21), (22) and (23) follows, that

$$N_{SoD} = 2\left(N_{LC_{l_1}} + N_{LC_{l_2}} \ldots + N_{LC_{l_M}}\right) \quad (24)$$

and considering (17) the sampling density is

$$\mathrm{E}[\sigma_{SoD}] = \frac{4 f_{max}}{\sqrt{3}} \sum_{k=1}^{\frac{K-1}{2}} e^{-\frac{l_k^2}{4A^2 f_{max}}}. \quad (25)$$

3 SIGNAL DEPENDENT SPECTRUM ANALYSIS

Typically the digital spectral analysis is based on the transformation of the signal samples set $\{x(t_n)\}$ from time to frequency domain by a set of transformation functions $\{W(f,t_n)\}$

$$X(f) = \sum_n x(t_n)W(f,t_n), \tag{26}$$

For example, the discrete Fourier transform is based on the set of exponent functions $\{e^{-j2\pi f t_n}\}$ that are unrelated to the spectral nature of the signal.

In order to construct signal dependent transformation functions an approach using Minimum variance (MV) filter is suggested. The basic idea of the MV filter is to minimize the variance of the signal on the output of a selective filter. The frequency response of such a filter adapts to the spectral components of the input signal on each frequency of interest (Marple, 1987).

Given the filter coefficients $a_1, a_2, \ldots, a_p$, the output of the filter at time n is

$$y_n = \sum_{n=1}^{p} a_n x_{n-k} = \mathbf{x}^T(n)\mathbf{a}, \tag{27}$$

where $\mathbf{x}^T(n) = [x_n, x_{n-1}, \ldots, x_{n-p+1}]$ and $\mathbf{a} = [a_1, a_2, \ldots, a_p]^T$. The variance of the output signal is determined as

$$\rho = E\left\{|y_n|^2\right\} = \mathbf{a}^H\mathbf{R}\mathbf{a}, \tag{28}$$

where $\mathbf{R} = E\left\{\mathbf{x}^*(n)\mathbf{x}^T(n)\right\}$ is the autocorrelation matrix and $E\{(\cdot)\}$ denotes the expectation operator. The filter coefficients are found to ensure the sinusoidal signal with frequency f_0 passes through the filter designed for this frequency without distortion and the variance (28) for spectral components differing from f_0 is minimal. The first condition can be written as

$$\sum_{n=1}^{p} a_n e^{-j2\pi f_0 t_n} = \mathbf{e}^H(f_0)\mathbf{a} = 1, \tag{29}$$

where $\mathbf{e}(f_0) = \left[e^{j2\pi f_0 t_1}, e^{j2\pi f_0 t_2}, \ldots, e^{j2\pi f_0 t_p}\right]^T$. It means that the gain of the filter response on frequency f_0 is one. In order to satisfy the second requirement under condition (29) the coefficients of the MV filter for the frequency f_0 are determined as (McDonough, 1983):

$$\mathbf{a}_{MV}(f_0) = \frac{\mathbf{R}^{-1}\mathbf{e}(f_0)}{\mathbf{e}^H(f_0)\mathbf{R}^{-1}\mathbf{e}(f_0)}. \tag{30}$$

Inserting (30) into (28) gives the minimum variance:

$$\rho_{MV} = \frac{1}{\mathbf{e}^H(f_0)\mathbf{R}^{-1}\mathbf{e}(f_0)}. \tag{31}$$

The value (31) indicates the power of spectral components of the input signal at the frequency f_0.

The proposed approach assumes that frequency band of spectral analysis is covered by a set of such MV filters. In general, the frequencies of these filters can be chosen arbitrarily. The particular case is if the filter frequencies are located equidistantly and the frequency step is selected equal to the frequency step $\Delta f = \frac{1}{\Theta}$ of the Discrete Fourier transform (DFT), where Θ is observation time of the signal being analyzed. To obtain a high resolution spectral estimation, it is reasonable to select the frequency step several times smaller than the Fourier frequency step.

The expression (31) requires the knowledge of the signal autocorrelation. The Wiener-Khinchin theorem relates it to the power spectral density $P(f)$ via the Fourier transform:

$$R(\tau) = \int_{-\infty}^{\infty} P(f)e^{j2\pi f\tau}df \tag{32}$$

In order to obtain $P(f)$ estimate at the frequencies $\mathbf{f} = [f_1, f_2, \ldots, f_M]$ from the non-uniformly spaced signal samples $\hat{\mathbf{x}} = [x_1, x_2, \ldots, x_N]$ the DFT can be used

$$\hat{\mathbf{P}}^{(DFT)} = \left|\frac{\hat{\mathbf{x}}\mathbf{B}^T}{N}\right|^2, \tag{33}$$

where $\mathbf{B}$ is $M \times N$ matrix whose element in row m and column n is $b_{mn} = e^{-j2\pi f_m t_n}$. The elements of autocorrelation matrix from the spectral estimation can be calculated on the bases of inverse DFT

$$\hat{r}_{lk} = \sum_{m=1}^{M} \hat{p}_m^{(DFT)} b_{ml}^* b_{mk}. \tag{34}$$

As the signal autocorrelation matrix (34) is rather rough estimate then the resulting PSD function values $\hat{\mathbf{P}} = [\hat{p}_1, \hat{p}_2, \ldots, \hat{p}_M]$, where $\hat{p}_m = \rho_{MV}(f_m)$, obtained by (31)

$$\hat{\mathbf{P}} = \frac{1}{\mathrm{diag}(\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^H)} \tag{35}$$

will not provide precise results.

To increase the precision a special iterative algorithm is used (Liepinsh, 1996), (Greitans, 1997) according to which the $(i+1)$-th order estimate of signal autocorrelation matrix is updated from i-th order $\hat{\mathbf{P}}^{(i)}$ estimate in the following way

$$\hat{r}_{lk}^{(i+1)} = \sum_{m=1}^{M} \hat{p}_m^{(i)} b_{ml}^* b_{mk}. \tag{36}$$

The values $\hat{\mathbf{P}}^{(i)}$ are obtained as

$$\hat{\mathbf{P}}^{(i)} = \left|\frac{\hat{\mathbf{x}}\mathbf{R}^{(i)^{-1}}\mathbf{B}^H}{\mathrm{diag}(\mathbf{B}\mathbf{R}^{(i)^{-1}}\mathbf{B}^H)}\right|^2 \tag{37}$$

considering that the power of the output of the designed MV filter

$$p(f_0) = |\hat{\mathbf{x}}\mathbf{a}_{MV}(f_0)|^2 \tag{38}$$

is interpretable similarly to the power of the output of selective Fourier filter

$$p_{DFT}(f_0) = |\hat{\mathbf{x}}\mathbf{e}^*(f_0)|^2 \tag{39}$$

at the frequency f_0.

The iteration process begins with the initial PSD values $\hat{\mathbf{P}}^{(0)} = \hat{\mathbf{P}}^{(DFT)}$. The iteration process can be stopped when the difference $\|\hat{\mathbf{P}}^{(i+1)} - \hat{\mathbf{P}}^{(i)}\|$ becomes small.

4 SIMULATION RESULTS

As a test-signal an autoregressive moving-average (ARMA) process with AR coefficients $c(1) = -2.76$, $c(2) = 3.809$, $c(3) = -2.654$, $c(4) = 0.924$ and MA coefficients $d(1) = -0.9$, $d(2) = 0.81$ was used. The PSD function of such signal is given as (Marple, 1987):

$$P_{ARMA}(f) = \frac{1}{2 f_{max}} \left| \frac{1+\sum_{v=1}^{V} d(v) e^{-j\pi v f / f_{max}}}{1+\sum_{u=1}^{U} c(u) e^{-j\pi u f / f_{max}}} \right|^2, \tag{40}$$

and is shown as a dashed line in Figure 3.

To show the potential of the send-on-delta sampling and performance of proposed signal-dependent method, the results were compared with uniformly sampled signal case. The test-signal with $f_{max} = 0.5$ was sampled during 256 seconds and processed with iterative algorithm described in Section 3. The solid line in Figure 3a illustrates the PSD estimate if signal is sampled uniformly at Nyquist rate (256 samples), while the dotted line shows spectrum estimated by the standard minimum variance filter (Capon filter) approach. The plots are the averages of 100 independent realizations similarly as in (Li and Stoica, 1998). If the sampling rate is reduced two times (128 samples), the frequency aliases appear as shown in Figure 3b.

In order to obtain SoD samples, the uniformly sampled signal (at Nyquist rate) was interpolated by *sinc* functions. To decrease the interpolation error the test-signal with duration of 512 seconds was interpolated and thereafter sampled by SoD from 128 to 384 seconds. Figure 3c demonstrates the case, where the sampling was done using 11 quantization levels. In total 120 samples (the average number for 100 independent realizations) were captured. The dotted line shows PSD estimate obtained by the expression (35)

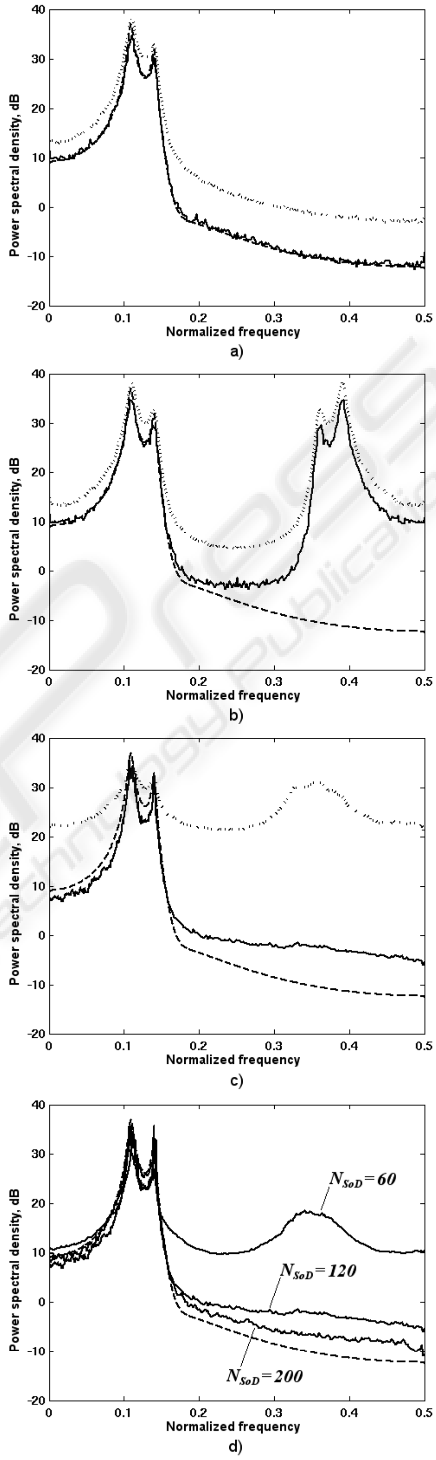

Figure 3: PSD estimates using different sampling schemes: a) uniform at Nyquist rate, b) uniform below Nyquist rate, c) SoD with 11 levels d) SoD with 7, 11 and 15 levels (dashed line - true PSD, dotted line - estimate without iterations and solid line - estimate after the 10th iteration.

without iterative updating of autocorrelation matrix **R**. The frequency aliasing is well observable. The iterative procedure improves the result and suppresses aliasing. It is shown in figure by solid line, which illustrates PSD obtained after the 10th iteration. Although the estimate is close to true PSD values, it does not reach the lower power level of -10dB. However, the estimate is better than the result obtained in uniform sampling case at Nyquist rate using the standard Capon filter approch (see dotted line in Figure 3a).

If the number of SoD quantization levels was increased to fifteen then average 200 samples were captured. The results after the 10th iteration are illustrated in Figure 3d. In this case the PSD estimate gets closer to the lower power level of true PSD since we have more data about the signal. In contrast, if the number of SoD quantization levels was decreased to seven then average 60 samples were captured and the precision of PSD estimate got worse.

5 CONCLUSIONS

The use of send-on-delta sampling provides several interesting features – the local sampling density reflects the local properties of the signal, samples are without quantization errors in amplitude, nonuniform location of sampling instants allows suppression of frequency aliasing that leads to the possibility of processing signals with a reduced sampling density. As was shown in the paper, in contrast to the level-crossing sampling, the SoD scheme guarantees a certain minimal interval between samples, which is a principal factor for practical implementations. However it is done at the expense of decreasing the number of samples.

To deal with non-uniformity and the reduced density of sampling flow, it was proposed to use a processing method, which is based on signal dependent transformation. The shortage of samples can be compensated by the iterative update of the estimation of the autocorrelation matrix. Simulation results show correct spectral estimation even if the sampling density is decreased several times in relation to the Nyquist rate. However it is done at the expense of an increased computation burden, because a linear system of equations has to be solved in each iteration.

In fact, the complexity of the data acquisition phase is transferred to the processing phase. Such a strategy offers possibilities for distributed wireless data acquisition in remote sensing systems. The signal dependent nature of sampling, the decreased number of samples and the possibility to code data with one bit using position on time axis allow considerably diminish the amount of transmitted data.

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