DECORRELATION TECHNIQUES IN IMAGE RESTORATION

Catalina Cocianu
Dept. of Computer Science, Academy of Economic Studies, Bucharest
Calea Dorobantilor #15-17, Bucuresti –1, Romania

Luminita State
Dept. of Computer Science, University of Pitesti, Pitesti
Caderea Bastliei #45, Bucuresti – 1, Romania

Panayiotis Vlamos
Ionian University, Corfu, Greece

Doru Constantin
Department of Computer Science, University of Pitesti, Pitesti, Romania

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Abstract: The restoration can be viewed as a process that attempts to reconstruct or recover an image that has been degraded by using some a priori knowledge about the degradation phenomenon. The multiresolution support provides a suitable framework for noise filtering and image restoration by noise suppression. We present the algorithms GMNR, a generalization of the MNR algorithm based on the multiresolution support set for noise removal in case of arbitrary mean, and NFPCA. A comparative analysis of the performance of the algorithms GNMR and NFPCA is experimentally performed against the standard AMVR and MMSE.

1 INTRODUCTION

The effectiveness of restoration techniques mainly depends on the accuracy of the image modeling. The image restoration tasks correspond to the process of finding an approximation to the overall degradation process and finding the appropriate inverse process to estimate the original unknown image (Gonzales, 2002).

Noise is any undesired information that contaminates an image and appears in images from a variety of sources. The digital image acquisition process is the primary process by which noise appears in digital images. Typically, the noise can be modeled with either a Gaussian, uniform or salt and pepper distribution.

The image restoration tasks mainly correspond to the process of finding an approximation to the overall degradation process and finding the appropriate inverse process to estimate the original unknown image. The most successful denoising algorithms fulfill at least the two following features. They use translation invariant overcomplete representations with local kernels selected to scale and orientation and apply a multidimensional shrinkage function based on joint observations of the coefficients in the neighborhoods. Some of these methods can be viewed as extensions of the classical Wiener estimate which assumes a global Gaussian behavior of both signal and noise. (Portilla, 2005)

In (Balster, Zheng, Ewing, 2003) a selective wavelet shrinkage algorithm for digital image denoising aiming to improve the performance and computation scheme of a wavelet shrinkage algorithm is proposed, the denoising methodology incorporated in this algorithm involving two-

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threshold validation process for real time selection of wavelet coefficients.

The MNR technique is essentially based on the statistical significance of the wavelet coefficients specifying the support. The statistical significance is established, somehow heuristically in terms of second order statistics. (Stark, 1995)

We present the algorithms GMNR, a generalization of the MNR algorithm based on the multiresolution support set for noise removal in case of arbitrary mean, and NFPCA. A comparative analysis of the performance of the algorithms GNMR and NFPCA is experimentally performed against the standard AMVR and MMSE.

The paper reports the conclusions experimentally derived on the convergence rates and their corresponding efficiency for specific image processing tasks.

2 NOISE DECORRELATION TECHNIQUE

The multiresolution support provides a suitable framework for noise filtering and image restoration by noise suppression. The procedure used is to determine statistically significant wavelet coefficients and from this to specify the multiresolution support, therefore a statistical image model is used as an integral part of the image processing. The support is used subsequently to hand-craft the filtering processing.

The MNR algorithm is (Stark, 1995),

Input: The image X_0, the number of the resolution levels *p* and the heuristic thresold *k* (the value of *k* should be taken close to 3).

Step 1. Compute the sequence of image variants $\{X_j\}_{j=1,p}$ and the wavelet coefficients using the "À Trous" algorithm (Stark, 1995)

$$X_j(r,c)=\sum_l\sum_k h(l,k)X_{j-1}\left(r+2^{j-1}l,c+2^{j-1}k\right)$$

$$\omega_j(r,c)=X_{j-1}(r,c)-X_j(r,c), \tag{1}$$

where *h* is a discrete low-pass filter.

Step 2. Apply the significance test, $\omega_j(r,c)$ is significant if and only if $\left|\omega_j(r,c)\right|\geq k\sigma_j$, for $j=1,...,p$

Step **3.** Compute the restored image,

$$\tilde{X}(r,c)=X_p(r,c)+\sum_{j=1}^{p}g\left(\sigma_j,\omega_j(r,c)\right)\omega_j(r,c), \tag{2}$$

where *g* is defined by,

$$g\left(\sigma_j,\omega_j(r,c)\right)=\begin{cases}1, & \left|\omega_j(r,c)\right|\geq k\sigma_j\\ 0, & \left|\omega_j(r,c)\right|<k\sigma_j\end{cases}$$

Output The restored image $\tilde{X}$.

In the following, we present the algorithm GMNR, a generalization of the MNR algorithm based on the multiresolution support set for noise removal in case of arbitrary mean (Cocianu, 2003). Let *g* be the original "clean" image, $\eta\sim N\left(m,\sigma^2\right)$ and the analyzed image $f=g+\eta$. The sampled variants of *f*, *g* and η obtained using the two-dimensional filter φ are given by,

$$c_0(x,y)=\left\langle f(l,c),\varphi(x-l,y-c)\right\rangle,$$

$$I_0(x,y)=\left\langle g(l,c),\varphi(x-l,y-c)\right\rangle,$$

$$E_0(x,y)=\left\langle \eta(l,c),\varphi(x-l,y-c)\right\rangle,$$

$$c_0=I_0+E_0. \tag{3}$$

Consequently, the wavelet coefficients of c_0 computed by the algorithm "À Trous" are,

$$\omega_j^{c_0}(x,y)=\frac{1}{2^j}\left\langle f(l,c),\psi\left(\frac{l-x}{2^j},\frac{c-y}{2^j}\right)\right\rangle=$$

$$=\omega_j^{I_0}(x,y)+\omega_j^{E_0}(x,y), \tag{4}$$

where $\frac{1}{2}\psi\left(\frac{x}{2}\right)=\phi(x)-\frac{1}{2}\phi\left(\frac{x}{2}\right)$.

For any pixel (x,y), we get

$$c_p(x,y)=\frac{1}{2^p}\left\langle f(l,c),\varphi\left(\frac{l-x}{2^p},\frac{c-y}{2^p}\right)\right\rangle=$$

$$=I_p(\mathrm{x},\mathrm{y})+E_p(x,y). \tag{5}$$

The representation of the image c_0 is given by,

$$c_0(x,y)=c_p(x,y)+\sum_{j=1}^{p}\omega_j^{c_0}(x,y)=$$

$$=I_p(x,y)+E_p(x,y)+\sum_{j=1}^{p}\omega_j^{I_0}(x,y)+\sum_{j=1}^{p}\omega_j^{E_0}(x,y). \tag{6}$$

Note that only $E_p(x,y)$ and $\sum_{j=1}^{p}\omega_j^{E_0}(x,y)$ include noise component. The mean of the noise can be decreased using the following algorithm.

Step1. Determine the images $E^{(i)}$, $1\leq i\leq n$, by superimposing noise sampled from $N\left(m,\sigma^2\right)$ on the "white wall" image.

Step2. For all *j*, $1\leq j\leq p$, compute c_j, $E_j^{(i)}$, $1\leq i\leq n$ and the coefficients $\omega_j^{c_0}$, $\omega_j^{E^{(i)}}$ using the "À

Trous" algorithm, where h is given by the filtering mask $\begin{pmatrix} \frac{1}{16} & \frac{1}{8} & \frac{1}{16} \\ \frac{1}{8} & \frac{1}{4} & \frac{1}{8} \\ \frac{1}{16} & \frac{1}{8} & \frac{1}{16} \end{pmatrix}$ according to,

$$c_j(x,y) = \sum_l \sum_c h(l,c) c_{j-1}\left(x + 2^{j-1}l, y + 2^{j-1}c\right)$$
$$E_j^{(i)}(x,y) = \sum_l \sum_c h(l,c) E_{j-1}^{(i)}\left(x + 2^{j-1}l, y + 2^{j-1}c\right)$$
$$\omega_j^{c_0}(x,y) = c_{j-1}(x,y) - c_j(x,y)$$
$$\omega_j^{E^{(i)}}(x,y) = E_{j-1}^{(i)}(x,y) - E_j^{(i)}(x,y) \tag{7}$$

Step 3. Compute the image $\widetilde{I}$ by,

$$\widetilde{I}(x,y) = \frac{1}{n}\sum_{i=1}^{n}\Big[c_p(x,y) - E_p^{(i)}(x,y) + \sum_{j=1}^{p}\left(\omega_j^{c_0}(x,y) - \omega_j^{E^{(i)}}(x,y)\right)\Big]. \tag{8}$$

Step 4. Compute a variant of the original image I_0 using the multiresolution filtering based on the statistically significant wavelet coefficients.

Note that $\widetilde{I}$ computed at Step 3 is $\widetilde{I} = I_0 + E'$, where $E' \sim N(m', \sigma'^2)$, $m' \approx 0$ and $E(\sigma'^2) \approx \sigma^2$.

An alternative approach in solving image restoration task can be performed by PCA neural network. The idea is to use features extracted from the noise in order to compensate the lost information and improve the quality of images.

The NFPCA algorithm is presented in the following. We consider the additive normal distributed degradation model. Let I^0 be a RxC matrix, where $C = nC_1, 2 \le n < C$ representing the initial image of L gray levels and let I be the distorted variant resulted from I^0 by superimposing random noise $N(0, \Sigma)$,

$\forall i = 1,\dots,R$, $k = n(j-1),\dots,nj$, $j = 1,\dots,C_1$,

$I_{i,j}(k) = I_{i,j}^0(k) + \eta(k)$, where

- $I_{i,j}$ is the sequence of n pixels of the *i-th* row from the $n(j-1)$-th pixel to the *nj*-th of the image I
- $I_{i,j}^0$ is the sequence of n pixels of the *i-th* row from the $n(j-1)$-th pixel to the *nj*-th of the image I^0
- η is a n-dimensional random vector distributed $N(0, \Sigma)$.

The algorithm for removing the noise component proceeds in two stages:

- in the first stage the noise features Φ are computed. The columns of Φ are the eigen vectors of Σ, taken according to the decreasing order of their corresponding eigen- values;
- in the second stage, using Φ, we apply a noise removal method M for cleaning each pixel (i, j) of the de-correlated transformed image.

The restoration process of the image I using the learned features is performed as follows:

Step 1. Compute the image I' by de-correlating the noise component,

$\forall i = 1,\dots,R$, $j = 1,\dots,C_1$,

$I'_{i,j} = \Phi^T I_{i,j} = \Phi^T I_{i,j}^0 + \eta'$, where

$\eta' = \Phi^T \eta \sim N(0, \Sigma')$,

$\Sigma' = \Phi^T \Sigma \Phi = \Lambda$,

$\Lambda = diag\{\lambda_1, \lambda_2, \dots, \lambda_n\}$.

Step 2. The noise component η' is removed for each pixel P of the image I' using the multirezolution support of I' by the labeling method of each wavelet coefficient of P, resulting I''.

$I''_{i,j} = MNR(I'_{i,j}) \cong \Phi^T I_{i,j}^0$,

$\forall i = 1,\dots,R$, $j = 1,\dots,C_1$,

where $MNR(I'_{i,j})$ is produced by applying the above mentioned method to $I'_{i,j}$.

Step 3. An approximation $\widetilde{I} \cong I^0$ of the initial image I^0 is produced by applying the inverse transform of T_{Φ^T} to I'',

$$\widetilde{I}_{i,j} = \Phi I''_{i,j} \cong \Phi\Phi^T I_{i,j}^0 = I_{i,j}^0, \tag{9}$$

$\forall i = 1,\dots,R$, $j = 1,\dots,C_1$

Note that the decorrelation of the noise component is performed by the computation carried out at **Step 1** because the resulted image is

$$I'_{i,j}(k) = \Phi^T I_{i,j}^0(k) + \eta'(k), \tag{10}$$

$k = n(j-1),\dots,nj$, $j = 1,\dots,C_1$,

where for each $k = n(j-1),\dots,nj$, $j = 1,\dots,C_1$, $\eta'(k) \sim N(0, \sigma_{i,k}^2)$, $\sigma_{i,k}^2 = \lambda_{k,k}$.

When the assumption of zero mean noise is not acceptable, the method GMNR to remove the noise resulted by the decorrelation process instead.

In order to evaluate the performance of the proposed noise removal algorithms, a series of experiments were performed on different 256 gray level images. We compared the performance of our algorithm NFPCA against MMSE (Umbaugh, 1998), AMVR (Umbaugh, 1998), and GMNR.

The values of the variances to model the noise in images processed by NFPCA represent the maximum of the variances per pixel resulted from the decorrelation process. The implementation of the GMNR algorithm used the masks

$$h_1 = \begin{pmatrix} \frac{1}{256} & \frac{1}{64} & \frac{3}{128} & \frac{1}{64} & \frac{1}{256} \\ \frac{1}{64} & \frac{1}{16} & \frac{3}{32} & \frac{1}{16} & \frac{1}{64} \\ \frac{3}{128} & \frac{3}{32} & \frac{9}{64} & \frac{3}{32} & \frac{3}{128} \\ \frac{1}{64} & \frac{1}{16} & \frac{3}{32} & \frac{1}{16} & \frac{1}{64} \\ \frac{1}{256} & \frac{1}{64} & \frac{3}{128} & \frac{1}{64} & \frac{1}{256} \end{pmatrix} \text{ and}$$

$$h_2 = \begin{pmatrix} \frac{1}{20} & \frac{1}{10} & \frac{1}{20} \\ \frac{1}{10} & \frac{2}{5} & \frac{1}{10} \\ \frac{1}{20} & \frac{1}{10} & \frac{1}{20} \end{pmatrix}$$

A synthesis of the comparative analysis on the quality and efficiency corresponding to the restoration algorithms presented in the paper is supplied in Table 1.

Table 1.

Restoration algorithm	Type of noise	Mean error/pixel
MMSE	U(30,80)	52.08
AMVR		10.94
MMSE	U(40,70)	50.58
AMVR		8,07
MMSE	N(40,200)	37.51
AMVR		11.54
GMNR		14.65
NFPCA		12.65
MMSE	N(50,100)	46.58
AMVR		9.39
GMNR		12.23
NFPCA		10.67

REFERENCES

Bacchelli, S., Papi S. 2006, Image denoising using principal component analysis in the wavelet domain. In *Journal of Computational and Applied Mathematics*, Volume 189, Issues 1-2, 1 May 2006, Pages 606-621

Balster, E. J., Zheng, Y. F., Ewing, R. L. 2003, Fast, Feature-Based Wavelet Shrinkage Algorithm for Image Denoising. In *International Conference on Integration of Knowledge IntensiveMulti-Agent Systems. KIMAS '03: Modeling, Exploration, and Engineering Held in Cambridge,* MA on 30 September-October 4, 2003

Cocianu, C., State, L., Stefanescu, V., Vlamos, P., 2006, PCA-Based Data Mining Probabilistic and Fuzzy Approaches with Applications in Pattern Recognition, *Proceedings of ICSOFT 2006*, Portugal, pp. 55-60, 2006

Cocianu, C., State, L., Vlamos, P.,2002, On a Certain Class of Algorithms for Noise Removal in Image Processing:A Comparative Study, In *Third IEEE Conference on Information Technology ITCC-2002, Las Vegas,* Nevada, USA, April 8-10, 2002

Cocianu, C., State, L., Stefanescu, V., Vlamos, P., 2004, On the Efficiency of a Certain Class of Noise Removal Algorithms in Solving Image Processing Tasks, In: *Proceedings of the ICINCO 2004*, Setubal, Portugal

Diamantaras, K.I., Kung, S.Y., *Principal Component Neural Networks: theory and applications*, John Wiley &Sons, 1996

Gonzales, R., Woods, R., *Digital Image Processing*, Prentice Hall, 2002

Haykin, S., Neural Networks A Comprehensive Foundation, Prentice Hall,Inc. 1999

Hyvarinen, A., Karhunen, J., Oja,E., *Independent Component Analysis*, John Wiley &Sons, 2001

Hyvarinen, A., Hoyer, P., Oja, P., 1999. Image Denoising by Sparse Code Shrinkage, www.cis.hut.fi/projects/ica,

Pitas, I., 1993, *Digital Image Processing Algorithms*, Prentice Hall

Portilla, J. 2005, Image Restoration using Gaussian Scale Mixtures in Overcomplete Oriented Pyramids. In *SPIE's International Symposium on Optical Science and Technology, SPIE's 50th Annual Meeting, Proc. of the SPIE*, vol. 5914, pp. 468-82, San Diego, CA, Aug 2005

Sonka, M., Hlavac, V., 1997, *Image Processing, Analyses and Machine Vision,* Chapman & Hall Computing

Stark, J.L., Murtagh, F., Bijaoui, A., 1995, *Multiresolution Support Applied to Image Filtering and Restoration*, Technical Report

State, L, Cocianu, C, Vlamos, P.., 2001, Attempts in Using Statistical Tools for Image Restoration Purposes, In *Proceedings of SCI2001*, Orlando, USA, July 22-25, 2001

Umbaugh, S., 1998, *Computer Vision and Image Processing*, Prentice Hall