A MATHEMATICAL FORMULATION OF A MODEL FOR LANDFORM ATTRIBUTES REPRESENTATION FOR APPLICATION IN DISTRIBUTED SYSTEMS

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Abstract: This study presents a methodology based on nonlinear regression for landform attributes representation. The equations to estimate the parameters of a two-dimensional polynomial are shown and, for testing the methodology, it was used the data from landform attributes of the state of Minas Gerais (Brazil) obtained by the Digital Elevation Model (DEM) in GTOPO30 project. They form a regular grid, with spacing of approximately 900m. The presented methodology can be used to minimize time of sending landform attributes information through network, to minimize space by storing the parameters of the estimated polynomial, and to make possible the process distribution of the polynomial coefficients calculations to different CPUs over internet network.

1 INTRODUCTION

The increased efficiency and reliability of computer networks (Fowler, 2005), as well as the popularization of the Internet, have generated new opportunities for applications development. Now, with the growing number of connected computers in the Internet (Zakon, 2006), it is possible to distribute applications, using communication infrastructure, without great efficiency problems. An application that has access to the resources of the Internet can use the services as HTTP, SMTP, POP3, FTP, for updating through remote servers (Tanenbaum, 2002).

In Geo-informatics, there is an area called Numerical Model of Surface that treats the mathematical representation of landforms (INPE, 2001). One of the numerical models most common used is the Regular Grid Model whose function is to generate a grid starting from a group of elevation, longitude and latitude points as in Figure 1.

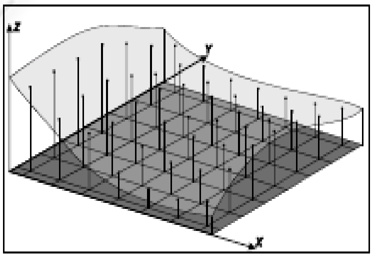

Figure 1: Regular Grid Model.

This grid can be generated by point interpolation for a polynomial regression model that adjusts a two-dimensional polynomial which best represents the landform attributes of an area (Burrough, 1986) (Ervin and Hasbrouck, 2001). The regression analysis is a statistical instrument often used in science. Its common use is to make possible the description of a phenomenon by means of a mathematical model (equation), based on a data sample. Graphically, it is equivalent to identify the curve or mathematical surface that best adjusts to the points in a dispersion diagram. The mathematical models of regression are based in three statistical assumptions: a) the relationship among the

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dependent and independent variables is deterministic instead of stochastic; b) the error measurements are random, following the normal distribution, average zero and constant variance; and c) the explanatory variables don't show correlation among themselves (Seber and Wild, 2005). This technique of landform attributes representation has some advantages over other techniques, basically by representing the landform attributes by mathematical equations instead of images, latitude, longitude and elevation coordinates, or map of elevation levels. One of the advantages is the significant reduction of the amount of necessary information to represent the landform attributes of a certain area, since, with the regression technique, the landform attributes of an entire area can just be represented through the coefficients of two-dimensional polynomial. By the polynomial representation it is possible to generate images with different resolution levels because, with a polynomial function, we may generate as many points as needed. Besides these advantages, there is a possibility to apply, on the polynomial, several mathematical operations, such as finding the maximal and minimal point, derivation, etc. However, the technique of polynomial regression has some disadvantages too, since the complexity and the computational power demanded in obtaining such polynomial is very high and sometimes impractical.

For that reason the objective of this article is to present a methodology designed to make possible the distribution of the necessary processing to compute a two-dimensional polynomial that represents the landform attributes of an area and, as an example, we use the area of the state of *Minas Gerais*, in Brazil, located in the Southeastern area of the country. Some estimative calculations, presented in this article, show that the necessary time of centralized processing to estimate such a polynomial is prohibitive, being in order of dozens of uninterrupted years of processing.

2 MATHEMATICAL METHOD

In this work we present a methodology for landform attributes representation using the method of nonlinear regression to adjust a two-dimensional polynomial. The regression analysis is a statistical instrument very used in science. Its frequent use is due to the fact of making possible the description of phenomena through mathematical models from a data sample. Graphically, it is equal to identify the curve or mathematical surface that best adjusts to the points in a dispersion diagram. The mathematical models of regression are based on three statistical facts: a) the relationship among the dependent and independent variables is deterministic instead of stochastic; b) the errors are random with normal distribution, average zero and constant variance; and c) the explanatory variables don't present correlation among themselves (Seber and Wild, 2005).

When a mathematical model of regression is used, the most used method of estimating the parameters is the Least Squares Method which consists of estimating a function to represent a group of points, minimizing the square of the deviations (Nobel and Daniel, 1986). Considering a group of geographic coordinates (x, y, z), representing longitude, latitude, and elevation of each point, respectively, we may take an estimate elevation function $\hat{z} = f(x, y)$ of these points. A polynomial of degree r in x and degree s in y can be given, according to Equation 1, and the estimated error ε_{ij} is given by Equation 2 where $0 \le i \le m$ and $0 \le j \le n$.

$$\hat{z} = f(x_i, y_j) = \sum_{k=0}^{r}\sum_{l=0}^{s} a_{kl} x_i^k y_j^l \qquad (1)$$

$$\varepsilon_{ij} = z_{ij} - \hat{z}_{ij} \qquad (2)$$

The coefficients a_{kl} ($k = 0, 1, ..., r$, $l = 0, 1, ..., s$) that minimize the errors of the estimated function $f(x, y)$ can be obtained by solving Equation 3 for $c = 0, 1, ..., r$ and $d = 0, 1, ..., s$.

$$\frac{\partial \xi}{\partial a_{cd}} = 0 \qquad (3)$$

where

$$\xi = \sum_{i=1}^{m}\sum_{j=1}^{n} \varepsilon_{ij}^2 = \sum_{i=1}^{m}\sum_{j=1}^{n} (z_{ij} - \hat{z}_{ji})^2 \qquad (4)$$

and

x_i	longitude i of DEM column, for $1 \le i \le k$
y_j	latitude j of DEM line, for $1 \le j \le l$,
zij	elevation of point (xi, yi)
r	polynomial degree in x,
s	polynomial degree in y,
akl	coefficients which minimize the error of the estimated function f (x, y)

$$\hat{z}_{ij} = f(x_i, y_j) = \sum_{k=0}^{r}\sum_{l=0}^{s} a_{kl} x_i^k y_j^l \tag{5}$$

$$\varepsilon_{ij}^2 = \left(z_{ij} - \sum_{k=0}^{r}\sum_{l=0}^{s} a_{kl} x_i^k y_j^l \right)^2 \tag{6}$$

$$z_{ij} = \sum_{k=0}^{r}\sum_{l=0}^{s} a_{kl} x_i^k y_j^l + \varepsilon_{ij} \tag{7}$$

$$\xi = \sum_{i=0}^{m}\sum_{j=0}^{n} \left(Z_{ij} - \sum_{k=0}^{r}\sum_{l=0}^{s} a_{kl} x_i^k y_j^l \right)^2 \tag{8}$$

$$\frac{\partial \xi}{\partial a_{cd}} = 2\sum_{i=0}^{m}\sum_{j=0}^{n} \left[\left(z_{ij} - \sum_{k=0}^{r}\sum_{l=0}^{s} a_{kl} x_i^k y_j^l \right) x_i^c y_j^d \right] \tag{9}$$

$$\sum_{i=0}^{m}\sum_{j=0}^{n} \left[\left(z_{ij} - \sum_{k=0}^{r}\sum_{l=0}^{s} a_{kl} x_i^k y_j^l \right) x_i^c y_j^d \right] = 0 \tag{10}$$

$$\sum_{i=0}^{m}\sum_{j=0}^{n}\sum_{k=0}^{r}\sum_{l=0}^{s} a_{kl} x_i^{k+c} y_j^{l+d} = \sum_{i=0}^{m}\sum_{j=0}^{n} z_{ij} x_i^c y_j^d \tag{11}$$

$$\sum_{i=0}^{m}\sum_{j=0}^{n} \left[z_{ij} x_i^c y_j^d - \left(\sum_{k=0}^{r}\sum_{l=0}^{s} a_{kl} x_i^{k+c} y_j^{l+d} \right) \right] = 0 \tag{12}$$

$$\hat{Z}_{ij}(x_i, y_j) = a_{00}x^0y^0 + a_{01}x^0y^1 + a_{02}x^0y^2 + a_{10}x^1y^0 + a_{11}x^1y^1 + a_{12}x^1y^2 + a_{20}x^2y^0 + a_{21}x^2y^1 + a_{22}x^2y^2 \tag{13}$$

By solving Equation 3, we get Equation 5 through Equation 12. By solving Equation 1 for a particular case of $r = s = 2$, we get Equation 13. By solving Equation 12 for the particular case of $r = s = 2$, we get the matrix system of equations in Figure 2, represented by AX=B where the matrix A is formed by x_{lc} terms, matrix X is formed by a_{kl}, estimated coefficients as the system solution, and matrix B as the independents terms b_l. These terms, x_{lc} and b_l, are shown as Equation 14 and Equation 15, respectively, as a general solution for any r and s of Equation 12**.** The estimated time to calculate the coefficients of Equation 14 and Equation 15 varies with degree of the polynomial, and may range from 28 sec for $r = s = 2$ to 45.9 years for $r = s = 500$ as show in Figure 3. The estimated error decreases with the increasing of the polynomial degree, as shown in Figure 4. Equation 14 and Equation 15 can be processed in an independent way, fundamental characteristic of distributed processing in different CPUs over a network system.

3 RESULTS

To validate the present methodology and derived equations, they will be applied to represent the landform attributes of an area of the state of *Minas Gerais* (Brazil). The data source of the chosen area, comes from a Digital Elevation Model (DEM) called GTOPO30 project (GTOPO30, 2006), in the form of a regular matrix with 1,043 lines and 1,343 columns, with spacing approximately 900m in the geographical coordinates.

Using the data source form the GTOPO30 (GTOPO30, 2006) project, the statistical analyses of the elevations of the state of *Minas Gerais* indicate a dispersion from 1m (meter) to 2,863m (meter). The distribution of these points, in 200m by 200m intervals, is presented in Table 1.

$$\begin{bmatrix}
\underset{\{0,0\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{0} y_j^{0}} & \underset{\{0,1\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{0} y_j^{1}} & \underset{\{0,2\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{0} y_j^{2}} & \underset{\{0,3\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{1} y_j^{0}} & \underset{\{0,4\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{1} y_j^{1}} & \underset{\{0,5\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{1} y_j^{2}} & \underset{\{0,6\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{2} y_j^{0}} & \underset{\{0,7\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{2} y_j^{1}} & \underset{\{0,8\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{2} y_j^{2}} \\
\underset{\{1,0\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{0} y_j^{1}} & \underset{\{1,1\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{0} y_j^{2}} & \underset{\{1,2\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{0} y_j^{3}} & \underset{\{1,3\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{1} y_j^{1}} & \underset{\{1,4\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{1} y_j^{2}} & \underset{\{1,5\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{1} y_j^{3}} & \underset{\{1,6\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{2} y_j^{1}} & \underset{\{1,7\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{2} y_j^{2}} & \underset{\{1,8\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{2} y_j^{3}} \\
\underset{\{2,0\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{0} y_j^{2}} & \underset{\{2,1\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{0} y_j^{3}} & \underset{\{2,2\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{0} y_j^{4}} & \underset{\{2,3\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{1} y_j^{2}} & \underset{\{2,4\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{1} y_j^{3}} & \underset{\{2,5\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{1} y_j^{4}} & \underset{\{2,6\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{2} y_j^{2}} & \underset{\{2,7\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{2} y_j^{3}} & \underset{\{2,8\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{2} y_j^{4}} \\
\underset{\{3,0\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{1} y_j^{0}} & \underset{\{3,1\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{1} y_j^{1}} & \underset{\{3,2\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{1} y_j^{2}} & \underset{\{3,3\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{2} y_j^{0}} & \underset{\{3,4\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{2} y_j^{1}} & \underset{\{3,5\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{2} y_j^{2}} & \underset{\{3,6\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{3} y_j^{0}} & \underset{\{3,7\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{3} y_j^{1}} & \underset{\{3,8\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{3} y_j^{2}} \\
\underset{\{4,0\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{1} y_j^{1}} & \underset{\{4,1\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{1} y_j^{2}} & \underset{\{4,2\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{1} y_j^{3}} & \underset{\{4,3\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{2} y_j^{1}} & \underset{\{4,4\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{2} y_j^{2}} & \underset{\{4,5\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{2} y_j^{3}} & \underset{\{4,6\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{3} y_j^{1}} & \underset{\{4,7\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{3} y_j^{2}} & \underset{\{4,8\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{3} y_j^{3}} \\
\underset{\{5,0\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{1} y_j^{2}} & \underset{\{5,1\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{1} y_j^{3}} & \underset{\{5,2\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{1} y_j^{4}} & \underset{\{5,3\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{2} y_j^{2}} & \underset{\{5,4\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{2} y_j^{3}} & \underset{\{5,5\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{2} y_j^{4}} & \underset{\{5,6\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{3} y_j^{2}} & \underset{\{5,7\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{3} y_j^{3}} & \underset{\{5,8\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{3} y_j^{4}} \\
\underset{\{6,0\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{2} y_j^{0}} & \underset{\{6,1\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{2} y_j^{1}} & \underset{\{6,2\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{2} y_j^{2}} & \underset{\{6,3\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{3} y_j^{0}} & \underset{\{6,4\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{3} y_j^{1}} & \underset{\{6,5\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{3} y_j^{2}} & \underset{\{6,6\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{4} y_j^{0}} & \underset{\{6,7\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{4} y_j^{1}} & \underset{\{6,8\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{4} y_j^{2}} \\
\underset{\{7,0\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{2} y_j^{1}} & \underset{\{7,1\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{2} y_j^{2}} & \underset{\{7,2\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{2} y_j^{3}} & \underset{\{7,3\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{3} y_j^{1}} & \underset{\{7,4\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{3} y_j^{2}} & \underset{\{7,5\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{3} y_j^{3}} & \underset{\{7,6\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{4} y_j^{1}} & \underset{\{7,7\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{4} y_j^{2}} & \underset{\{7,8\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{4} y_j^{3}} \\
\underset{\{8,0\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{2} y_j^{2}} & \underset{\{8,1\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{2} y_j^{3}} & \underset{\{8,2\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{2} y_j^{4}} & \underset{\{8,3\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{3} y_j^{2}} & \underset{\{8,4\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{3} y_j^{3}} & \underset{\{8,5\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{3} y_j^{4}} & \underset{\{8,6\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{4} y_j^{2}} & \underset{\{8,7\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{4} y_j^{3}} & \underset{\{8,8\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{4} y_j^{4}}
\end{bmatrix}
\begin{bmatrix} a_{00} \\ a_{01} \\ a_{02} \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{20} \\ a_{21} \\ a_{22} \end{bmatrix}
=
\begin{bmatrix}
\underset{\{0,0\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} Z_{ij} x_i^{0} y_j^{0}} \\
\underset{\{1,0\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} Z_{ij} x_i^{0} y_j^{1}} \\
\underset{\{2,0\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} Z_{ij} x_i^{0} y_j^{2}} \\
\underset{\{3,0\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} Z_{ij} x_i^{1} y_j^{0}} \\
\underset{\{4,0\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} Z_{ij} x_i^{1} y_j^{1}} \\
\underset{\{5,0\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} Z_{ij} x_i^{1} y_j^{2}} \\
\underset{\{6,0\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} Z_{ij} x_i^{2} y_j^{0}} \\
\underset{\{7,0\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} Z_{ij} x_i^{2} y_j^{1}} \\
\underset{\{8,0\}}{\sum_{i=0}^{m}\sum_{j=0}^{n} Z_{ij} x_i^{2} y_j^{2}}
\end{bmatrix}$$

Figure 2: Matrix system generated from solving Equation 12 considering a simple case of $r = s = 2$.

$$x_{lc} = \sum_{i=0}^{m}\sum_{j=0}^{n} x_i^{\,l\,div\,(s+1)+c\,div\,(s+1)}\, y_j^{\,l \bmod (r+1)+c \bmod (r+1)} \tag{14}$$

$$b_l = \sum_{i=0}^{m}\sum_{j=0}^{n} z_{ij}\, x_i^{\,l\,div\,(s+1)}\, y_j^{\,l \bmod (r+1)} \tag{15}$$

Table 1: Classes of elevations and relative percentage of the points, in each class, considering the DEM data of the State of *Minas Gerais*. The classes are intervals of 200m, from 0 to 2,900m.

Classes of elevation (m)	Number of points	Percentage of point in a class (%)	Accumulated percentage of points at classes (%)
0001 – 200	5,055	0,69986	0.6999
0201 – 400	48,866	6.76538	7.4652
0401 – 600	127,776	17.6903	25.1555
0601 – 800	249,762	34.57899	59.7345
0801 – 1000	226,481	31.3558	91.0903
1001 – 1200	36,246	5.01818	96.1085
1201 – 1400	22,334	3.09209	99.2006
1401 – 1600	4,482	0.62053	99.8211
1601 – 1800	811	0.11228	99.9334
1801 –2000	331	0.04582	99.9792
2001 – 2200	99	0.0137	99.9929
2201 – 2400	24	0.00333	99.9963
2401 – 2600	16	0.00221	99.9985
2601 – 2800	10	0.00139	99.9999
2801 – 3000	1	0.00014	100.0000
Σ	722,294	100.00000	–

The DEM with 1,400,749 points has only 722,294 points into the state of *Minas Gerais*, the others points are outside the state area. Using all the points representing the landform attributes of the state and by using Equations 14 and 15 we estimate the polynomial coefficients for representing the landform attributes of the state of *Minas Gerais*. Since the time to estimate such a polynomial in high degree needs great computer power and a long time of processing, we compute only a small sample, for $r = s = 2$, 3, 10 and 20. The processing was done by a usual PC machine with Intel Pentium 4 processor, running Windows XP.

The processing times in seconds $t(s)$ and the regression coefficient R^2 of the adjusted polynomial of degrees r in x and s in y, with $r = s$, are presented in Table 2.

Table 2: Processing times in seconds $t(s)$ and R2 for the adjusted polynomial of degree r in x and degree s in y, with $r = s$.

$r = s$	$t(s)$	R^2
2	28	0.41547
3	86	0.47168
10	4,746	0.60450
20	62,675	0.64767

With the values from Table 2 and by using polynomial regression, we get Equation 16 and Equation 17 that estimate the time $t(s)$ and R^2, respectively, with $r = s$.

$$t(s) = 11.764s^3 - 87.05s^2 + 168.93s, \quad R^2 = 1.0 \tag{16}$$

$$R^2(s) = 0.102\ln(s) + 0.354, R^2 = 0.9983 \tag{17}$$

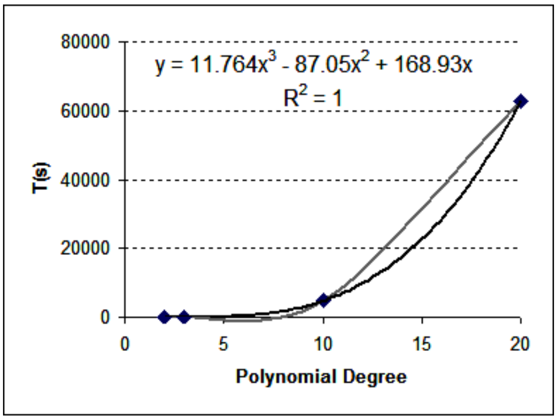

Figure 3: Processing time estimative.

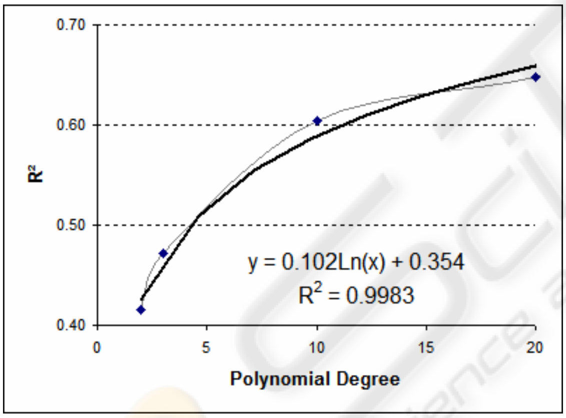

Figure 4: Estimated error.

From Equation 16 and Equation 17 we simulated the time processing in second t(s), and the coefficient of regression R^2, to adjusted a polynomial to represent the landform attributes for r = s, varying from 30 to 500 degree, as shown on Table 3. As an example, for r = s = 500, using the PC machine previously described, the polynomial representing the state of Minas Gerais, will be adjusted with R^2=0.988 and will need 45.9 years to generate the coefficients.

Table 3: Processing time in seconds $t(s)$ and R^2 for the polynomial of degree r in x and s in y, $r = s$, from 30 to 500 degrees.

r=s	$t(s)$	R^2	r=s	$t(s)$	R^2
30	244,351	0.70102	225	129,630,416	0.9065
40	620,373	0.73037	250	178,414,108	0.9173
60	2,237,780	0.77172	300	309,844,179	0.9359
80	5,479,562	0.80107	350	493,777,001	0.9516
90	7,886,055	0.81308	375	608,189,130	0.9587
100	10,910,393	0.82383	400	739,035,572	0.9652
150	37,770,215	0.86518	450	1,054,442,894	0.9772
200	90,663,786	0.89453	500	1,448,821,965	0.988

Using the DEM data source, an elevation map was generated in gray tones, Figure 5-a, where can be observed that the state of *Minas Gerais* has a heavily uneven topography. With the coefficients of the adjusted polynomial of degrees $r = s = 20$, the elevation map was also generated, Figure 5-b. In Figure 5-c, we show the relative errors computed using the source data (DEM) and the estimated data (polynomial).

Since we used only points that represent the landform attributes of the state of *Minas Gerais*, the bordering areas suffered heavy alterations, generating non existent elevations. This anomaly can be corrected by taking points that cross the limits of the real area, for estimation of the polynomial parameters.

(a) (b) (c)

Figure 5: Elevation map of Minas Gerais state generated by: (a) DEM of GTOPO project, (b) estimated polynomial with r = s = 20 that represents the DEM and (c) the relative errors form the data source and the estimated data.

The elevations of DEM and the elevations estimated by the polynomial of degrees $r = s = 20$ are displayed in classes in Table 4. By analysis of those data we may verify that the classes are comparable, although the adjusted polynomial generates some values above the maximum and below the minimum real elevations. It is believed that with larger r and s, we can get better values of R^2 and that will improve the accuracy of the polynomial parameters.

4 CONCLUSIONS

Table 4: Elevation classes from the DEM source and estimated polynomial for $r = s = 20$.

Elevation Classes(m)	Points from DEM (source data)	Points from estimated polynomial
0 - 200	5,055	78,725
0201 - 400	48,866	42,757
0401 - 600	127,776	119,401
0601 - 800	249,762	222,355
0801 - 1000	226,481	158,256
1001 – 1200	51,055	47,817
1201 – 1400	22,334	16,348
1401 – 1600	4,482	9,823
1601 – 1800	811	6,443
1801 -2000	331	4,687
2001 – 2200	99	3,548
2201 – 2400	24	2,683
> 2400	27	9,451
Minimum Elevation	1	-4,374
Maximum Elevation	2,863	5,606

The adjusted polynomial of degree 20 in x and y has 441 coefficients. If we use the type float to store them, it will be necessary 1,764 KB of storage. The DEM, on the other hand, with 722,294 points, requires at least 2 bytes to indicate the elevation of each point, being necessary the total of 1,444 MB to store it. The adjusted polynomial needs about 0.1221% of space used by the DEM. From the results, it is verified that is possible to represent, satisfactorily, the landform attributes of an area by a high degree polynomial and the representation has the advantage of smaller space. Additionally, the functional representation of the landform attributes allows larger efficiency, in time and space when sending this information through networks. Efficiency in time is real since, instead of sending an image with millions of points, the coefficients of a mathematical function are sent. Efficiency in space is also obtained since, instead of storing the DEM, we may store the coefficients that represent it. In this methodology, larger polynomial degree, better is the solution. For this work, a polynomial of degree 20 was used, however with a polynomial of degree 200 or larger, the results, statistically, would be better. The presented methodology also has the advantage of being easily adaptable for distributed processing.

REFERENCES

Burrough, P. A., 1986. *Principles of Geographical Information Systems for Land Resources Assessment*. Clarendon Press, Oxford, England.

Ervin, S. M., Hasbrouck, H. H., 2001. *Landscape Modeling: Digital Techniques for Landscape Visualization.* McGraw-Hill Professional Publishing.

Fowler, D., 2005. A Supercomputer in Every Home?. NetNews.

INPE, 2001. *Introdução à Ciência da GeoInformação*. INPE. São José dos Campos, SP, Brazil.

Noble, B., Daniel, J. W., 1986. *Álgebra Linear Aplicada*. Prentice/Hall do Brasil, Rio de Janeiro, Brazil.

Seber, G. A. F., Wild C. J., 2005. *Nonlinear Regression* John Wiley, New York, N.Y IEEE.

Tanenbaum, A. S., 2002. *Computer Networks*. 4th Edition, Prentice Hall, Upper Saddle River, N.J..

U.S. Geological Survey GTOPO30, 2006. [Internet]. Available from: http://edc.usgs.gov/products/elevation/gtopo30/gtopo30.html.

Zakon, R. H., 2006. *Hobbes' Internet Timeline*. [Internet]. Available from: http://www.zakon.org/robert/internet/timeline/#Growth.