

SYNTHESIS OF THE LOW-PASS AND HIGH-PASS WAVE DIGITAL FILTERS

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Abstract: In this paper we propose a very simple procedure for the design and analysis of low-pass and high-pass wave digital filters derived from reference filter given in the lattice configuration. Wave Digital Filters derived from reference filter in lattice configuration can be designed with excellent pass band properties. They can be proposed and implemented without the knowledge of classical filter theory. In this paper we present tables for Butterworth, Chebychev and Elliptic low-pass filter design. In the examples we demonstrate programs in MATLAB that permits analyze the attenuation properties of the designed filters. In the end of our article we realize wave digital filter using Embedded Target for Texas instruments TMS320C6000 DSP Platform. The model of the WDF was created by means of serial and parallel blocks that were added to the window Simulink Library Browser between common Used Blocks.

1 INTRODUCTION

Wave digital lattice filters are derived from LC-filters. The reference filter consists of parallel and serial connections of several elements. Since the load resistance R_L is not arbitrary but dependent on the element or source to which the port belongs, we cannot simply interconnect the elements to a network. The elements of the filters are connected with the assistance of serial and parallel adapters. These adapters in the discrete form are connected in one port by delay elements. The possibility of changing the port resistance can be achieved using parallel and serial adapters. These adapters contain the necessary adders, multipliers and inverters. In this paper we use adapters with three ports. The block of the serial and parallel adapter and their signal-flow diagram are shown in figures 1 and 2. (Fettweis, 1973)

The coefficient of the 3-port reflection-free serial adapter in figure 1A) is calculated from the port resistances R_i $i=1,2$ by equation (1). (Fettweis and Meerkoetter, 1975)

$$B = \frac{R_1}{R_1 + R_2} \quad (1)$$

The coefficient of the reflection-free parallel adapter in figure 1B) can be calculated from the port conductances G_i $i=1,2$ by eq. (2).

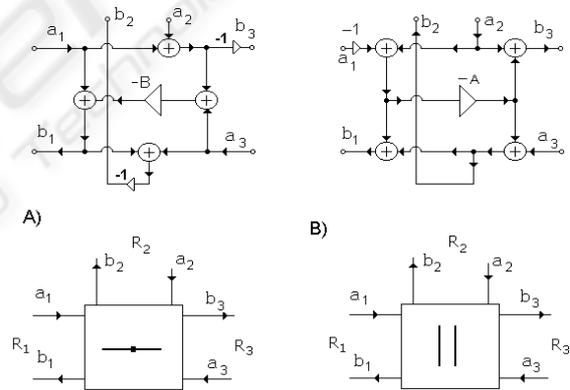


Figure 1: A) Three port serial adapter whose port 3 is reflection-free and its signal flow-graph, B) Three port parallel adapter whose port 3 is reflection-free and its signal flow-graph.

$$A = \frac{G_1}{G_1 + G_2} \quad (2)$$

The coefficients of the dependent parallel adapter in the figure 2B) can be get from the port conductances G_i $i=1,2,3$ by eq. (3)

$$A_1 = \frac{2G_1}{G_1 + G_2 + G_3} \quad A_2 = \frac{2G_2}{G_1 + G_2 + G_3} \quad (3)$$

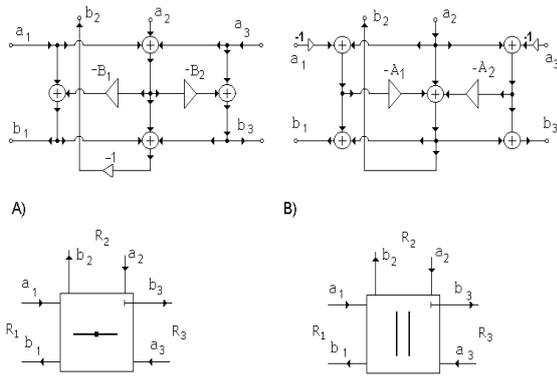


Figure 2: A) Three port serial dependent adapter and its signal flow-graph, B) Three port parallel dependent adapter and its signal flow-graph.

The coefficient of the dependent serial adapter in the figure 2A) can be obtained from the port resistances R_i $i=1,2,3$ by (4)

$$B_1 = \frac{2R_1}{R_1 + R_2 + R_3} \quad B_2 = \frac{2R_2}{R_1 + R_2 + R_3} \quad (4)$$

When connecting adapters, the network must not contain any feedback loops without a delay element in order to guarantee that the structure is realizable. This means that we cannot connect the dependent adapters from figure 2A) and 2B). The three-port dependent parallel adapter is reflection free at port 3 if $G_3 = G_1 + G_2$ and three port dependent serial adapter is reflection free if $R_3 = R_1 + R_2$.

2 EXAMPLES

In this part we shall demonstrate in the examples calculation of the low-pass and high-pass wave digital filter. The most important components for the realization of wave digital filters according to the Fettweis procedure are the ladder LC filters. The tables for wave digital structures was designed for the corner frequency $f_1 = 1/(2 \cdot \pi) = 0.159155$ and sampling frequency $f_s = 0.5$

2.1 Realization of the Low-pass Filter

In the first example we shall realize Butterworth low-pass of the 5th order and $A_{max} = 3dB$. In the figure 3 we show the structure of a 5th order ladder LC reference Butterworth filter and its corresponding block connection in the digital form.

First we must calculate from FIG. 3 wave port resistances R_1, R_2, R_3, R_4 , and finally the coefficients of the

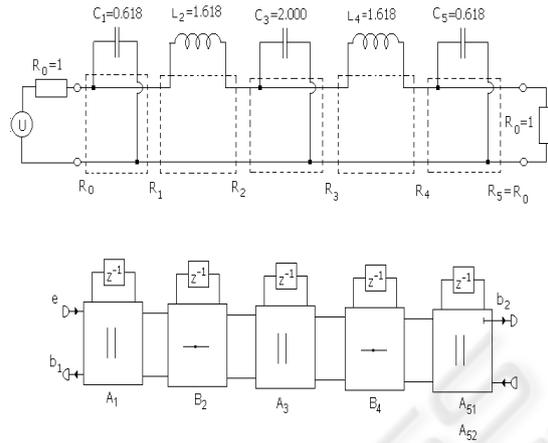


Figure 3: LC reference Butterworth low-pass filter and its corresponding block connection.

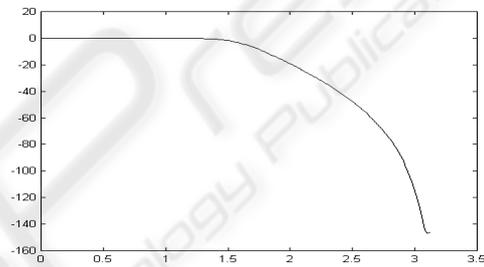


Figure 4: Frequency response of the Butterworth low-pass filter.

parallel and serial adapters A_1, B_2, A_3, B_4 and the coefficients of the dependent parallel adapter A_{51}, A_{52} according to equations (1)-(3).

$$\begin{aligned}
 G_1 &= G_0 + C_1 = 1.618 & G_3 &= G_2 + C_3 = 2.447 \\
 R_1 &= 1/G_1 = 0.618 & R_3 &= 1/G_3 = 0.408 \\
 R_2 &= R_1 + L_2 = 2.236 & R_4 &= R_3 + L_4 = 2.026 \\
 G_2 &= 1/R_2 = 0.447 & G_4 &= 1/R_4 = 0.493 \\
 A_1 &= \frac{G_0}{G_1 + C_1} = 0.618 & B_4 &= \frac{R_3}{R_3 + L_4} = 0.201 \\
 B_2 &= \frac{R_1}{R_1 + L_2} = 0.276 & A_{51} &= \frac{2G_4}{G_4 + C_5 + G_5} = 0.443 \\
 A_3 &= \frac{G_2}{G_2 + C_3} = 0.182 & A_{52} &= \frac{2C_5}{G_4 + C_5 + G_5} = 0.556
 \end{aligned}$$

The program for the analysis of the wave digital filter of the 5th order written in MATLAB follows, and the frequency response obtained by MATLAB is presented in the figure 4. The equation in the program for computing $XN1-XN4, BN4-BN1, N1-N9$ and $YN(i)$ was received from the structure in the Fig. 5

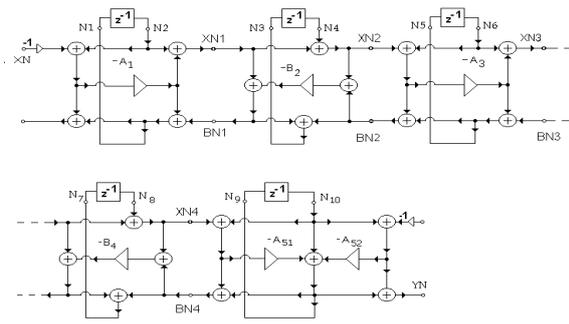


Figure 5: Butterworth wave digital low-pass filter of the 5th order.

```

A1=0.618146; B2=0.276524; A3=0.182858;
B4=0.201756; A51=0.467553; A52=0.947276;
N2=0; N4=0; N6=0; N8=0; N10=0; XN=1;
for i=1:1:50
    XN1=A1*XN-A1*N2+N2;    XN2=XN1+N4;
    XN3=-A3*XN2-A3*N6+N6;  XN4=XN3+N8;
    BN4=XN4-A51*XN4+2*N10-A51*N10-A52*N10;
    BN3=XN3-B4*XN4-B4*BN4;
    BN2=XN2-A3*XN2+BN3+N6-A3*N6;
    BN1=XN1-B2*XN2-B2*BN2;
    N1=XN*A1-A1*N2+BN1;    N3=BN1+BN2;
    N5=-A3*XN2-A3*N6+BN3; N7=BN3+BN4;
    N9=-A51*XN4+N10-A51*N10-A52*N10;
    YN(i)=-A51*XN4+2*N10-A51*N10-A52*N10
    N2=N1; N4=N3; N6=N5; N8=N7; N10=N9; XN=0;
end
[h,w]=freqz(YN,1,50)
plot(w,20*log10(abs(h)))
    
```

2.2 Design of the Low-pass and High-pass Chebychev Filter

In the second example we shall propose low-pass and high-pass Chebychev WDF for $n=5$, $A_{max} = 3$ dB. From the table 4 we get the values of the WDF $A_1 = 0.223$, $B_2 = 0.226$, $A_3 = 0.182$, $B_4 = 0.192$, $A_{5,1} = 0.383$ and $A_{5,2} = 0.360$. Using previous MATLAB program we obtain the attenuation of the Chebychev filter presented in the Fig. 6. High-pass we get by changing in the preceding program $N2=-N1$, $N4=-N3$, $N6=-N5$, $N8=-N7$ and $N10=-N9$.

2.3 Realization of the Low-pass Cauer WDF

In the figure 7 the structure of the 5th order ladder LC reference Cauer low-pass filter is shown. The values of the LC filter was obtained from the table C 0550 for $\Theta = 30^\circ$ for $A_{max} = 1.2494$ dB, $A_{min} = 70.5$ dB and $\Omega_s = 2.0000$ (Saal, 1979). Parallel resonant LC circuits in the low-pass filter Fig. 7 will be realized

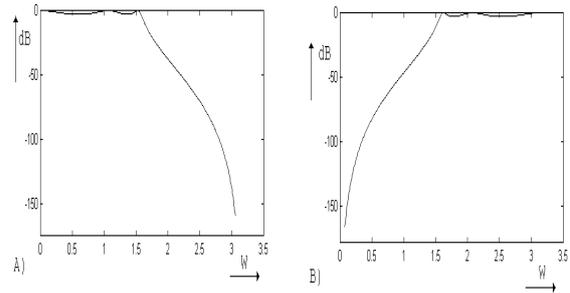


Figure 6: Frequency response of the low-pass and high-pass Chebychev filter.

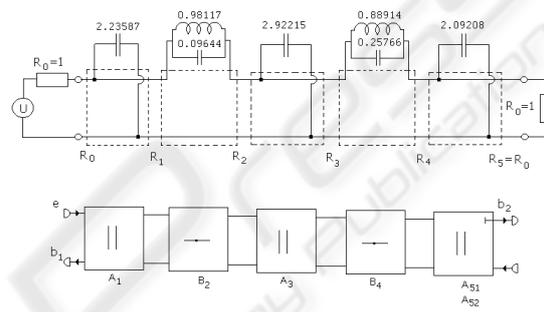


Figure 7: LC reference Cauer low-pass filter.

by digital structure according the Fig. 8.

With the assistance of the following MATLAB program we can calculate coefficients of the WDF A_1 , B_2 , A_3 , B_4 , $A_{5,1}$ and $A_{5,2}$. The attenuation of the low-pass Cauer filter is presented in Fig. 9. The program was obtained from the structure in the Fig. 10. The input data of the LC filter was obtained from the catalog of the Cauer filter (Saal, 1979).

```

C(1)=2.235878; C(3)=2.922148; C(5)=2.092084;
L(2)=0.981174; L(4)=0.889139; C(2)=0.096443;
    
```

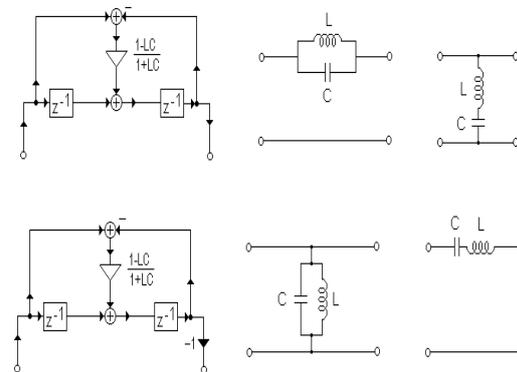


Figure 8: Discrete realization of parallel and serial LC circuits.

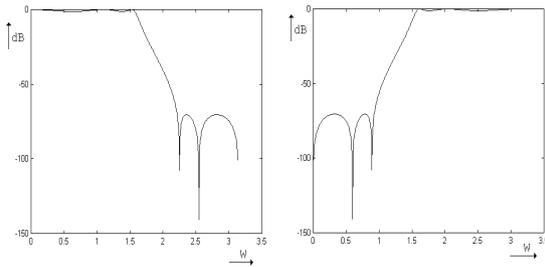


Figure 9: Frequency response of the Cauchy low-pass and high-pass filter.

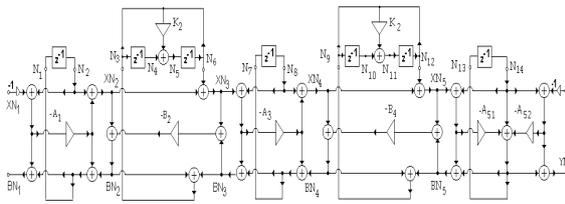


Figure 10: Structure of the Cauchy low-pass filter n=5.

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C(4)=0.257662; XN=1; N2=0; N4=0; N6=0; N8=0;
N10=0; N12=0; N14=0; G(1)=1;
G(2)=G(1)+C(1); R(2)=1/G(2);
R(3)=R(2)+1/(C(2)+1/L(2));
G(3)=1/R(3); G(4)=G(3)+C(3); R(4)=1/G(4);
R(5)=R(4)+1/(C(4)+1/L(4)); G(5)=1/R(5);
K(2)=(L(2)*C(2)-1)/(L(2)*C(2)+1);
K(4)=(L(4)*C(4)-1)/(L(4)*C(4)+1);
A(1)=G(1)/G(2); B(2)=R(2)/R(3);
A(3)=G(3)/(G(3)+C(3));
B(4)=R(4)/R(5); A(5,1)=2*G(5)/(G(5)+C(5)+1);
A(5,2)=2/(G(5)+C(5)+1);
for i=1:1:500
    XN1=A(1)*XN+N2-A(1)*N2; XN2=XN1+N6;
    XN3=-A(3)*XN2+N8-A(3)*N8; XN4=XN3+N12;
    BN4=XN4-XN4*A(5,1)+2*N14-N14*A(5,1)-A(5,2)*N14;
    BN3=-BN4*B(4)+XN3-B(4)*XN4;
    BN2=XN2-A(3)*XN2+BN3+N8-A(3)*N8;
    BN1=XN1-XN2*B(2)-BN2*B(2);
    N1=XN*A(1)-N2*A(1)+BN1; N3=BN1+BN2;
    N5=-K(2)*N3+K(2)*N6+N4; N7=-XN2*A(3)+BN3-N8*A(3);
    N9=BN3+BN4; N11=N10-N9*K(4)+N12*K(4);
    N13=-A(5,1)*XN4+N14-A(5,1)*N14-A(5,2)*N14;
    YN(i)=-A(5,1)*XN4+2*N14-A(5,2)*N14-A(5,1)*N14;
    N2=N1; N4=N3; N6=N5; N8=N7; N10=N9; N12=N11;
    N14=N13; XN=0;
end
[h,w]=freqz(YN,1,500); plot(w,20*log10(abs(h)))
    
```

High-pass can be obtained by changing in program $N2=-N1$, $N4=-N3$, $N6=-N5$, $N8=-N7$, $N10=-N9$, $N12=-N11$, $N14=-N13$. In the Fig. 9 the attenuation of low-pass and high-pass filter are presented.

2.4 Realization of Wave Digital Filters in DSP C6711 by Simulink

The simulink model showed in Figure 11 corresponds to the realization of a wave digital filter application on TMS320C6711 DSK using Embedded Target for Texas Instruments TMS320C6000 DSP Platform. The model of the WDF was created by means of serial and parallel block that were added to the window Simulink Library Browser between Commonly Used Blocks. In the input and output of the WDF were added ADC and DAC convertes of the TMS320C6711 that are in the window Simulink Library Browser, Embedded Target for TI C6000 DSP and C6711DSK Board Support. This model created in Code Composer Studio project can be see in Figure 13 and can run on the DSP C6711.

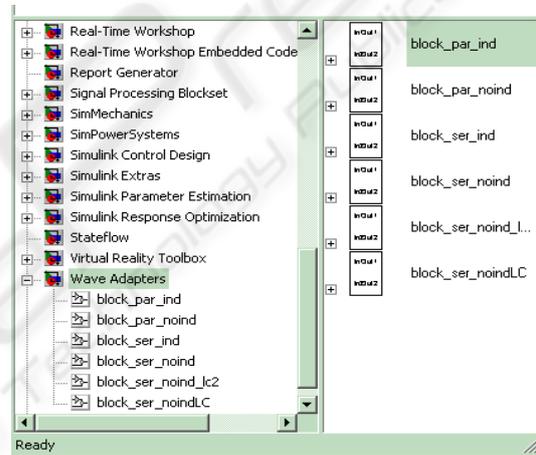


Figure 11: Realization of Wave Digital Filter in TMS320C6711 by Simulink.

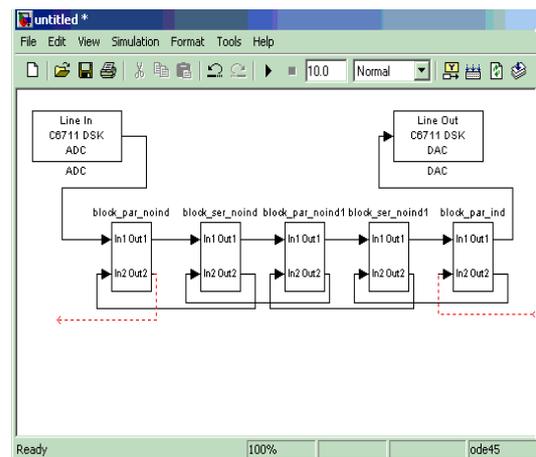


Figure 12: Realization of Wave Digital Filter in TMS320C6711 by Simulink.

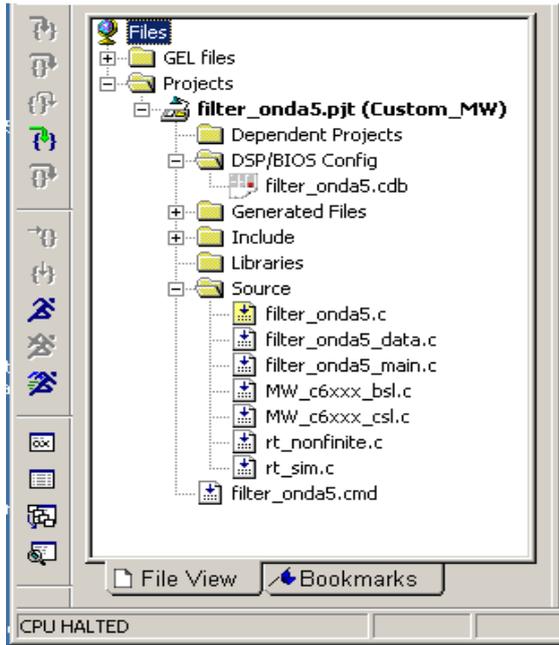


Figure 13: Realization of Wave Digital Filter in TMS320C6711 by Simulink.

3 CONCLUSIONS

Though the structure of the wave digital filter is more complicated than other structures, the algorithm for implementation on the DSP is very simple and it is very easy to propose the general algorithm for the arbitrary order of the filter. These structures are not so sensitive to the error of quantization as other types of the filters. With small modification of the presented programs can be created another tables of the WDF. The parts of the programs can be utilized for implementing of the wave digital filters in digital signal processors DSP.

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APPENDIX

Tables of the Wave Digital Filters

Tables of the Butterworth Wave Digital Filters

In the Fig. 14 is the tolerance scheme of the Butterworth filter and in the Fig. 15 is the structure of the Butterworth Wave Digital Filter (WDF).

The structure is created by cascade connection of the parallel and serial adapters. If at the begin is parallel reflection free adapter, at the end of the structure in case n odd must be connected parallel dependent adapter in order to realize load resistance R_L . In case n even at the end we have to connect serial dependent adapter.

In the table 2 are the elements of the Butterworth WDF for various attenuation A_{max} in the passband. A_i and B_i are the coefficients of the parallel and serial adapters respectively.

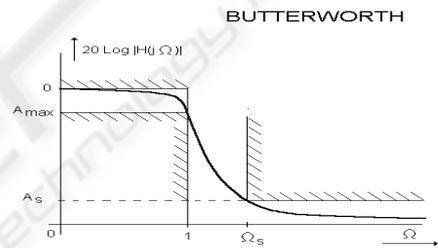


Figure 14: Attenuation of Butterworth WDF.

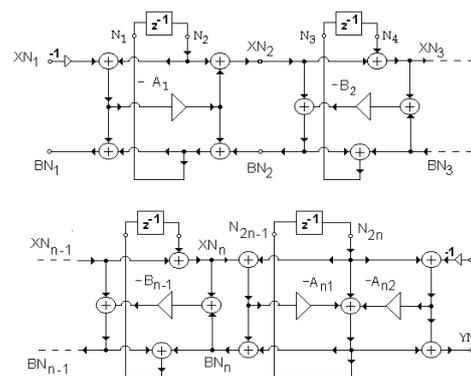


Figure 15: Digital structure of Butterworth WDF.

Tables of the Chebyshev Wave Digital Filter

In Fig. 16 is the tolerance scheme and the attenuation of the the Chebyshev filter. The structure of the Chebyshev wave digital filters is demonstrated in the Fig. 17. This structure is created by connection of

Table 1: Elements of Butterworth WDF n=3, A_{max} in [dB].

A_{max}	A_1	B_2	A_{31}	A_{32}
0.1	0.6519	0.3790	0.5497	0.9454
0.2	0.6248	0.3422	0.5099	0.9310
0.3	0.6083	0.3209	0.4858	0.9211
0.4	0.5964	0.3059	0.4685	0.9134
0.5	0.5864	0.2943	0.4548	0.9069
0.6	0.5791	0.2849	0.4434	0.9014
0.7	0.5723	0.2769	0.4337	0.8964
0.8	0.5664	0.2700	0.4252	0.8920
0.9	0.5611	0.2639	0.4176	0.8878
1.0	0.5562	0.2585	0.4208	0.8840

Table 2: Elements of Butterworth WDF n=5, A_{max} in [dB].

A_{max}	A_1	B_2	A_3	B_4	A_{51}	A_{52}
0.1	0.702	0.387	0.286	0.318	0.602	0.981
0.2	0.687	0.365	0.265	0.295	0.578	0.972
0.3	0.678	0.353	0.253	0.281	0.563	0.974
0.4	0.671	0.344	0.244	0.271	0.553	0.971
0.5	0.666	0.337	0.238	0.264	0.544	0.969
0.6	0.662	0.331	0.232	0.258	0.537	0.968
0.7	0.658	0.326	0.228	0.252	0.531	0.966
0.8	0.655	0.322	0.224	0.248	0.526	0.965
0.9	0.652	0.318	0.220	0.244	0.521	0.964
1.0	0.649	0.314	0.217	0.240	0.517	0.962

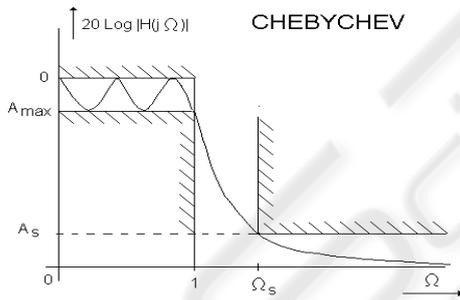


Figure 16: Attenuation of Chebyshev WDF.

the parallel and serial adapters terminated at the port 3 with delay element. At the end of the structure must be connected parallel dependent adapter to realize for n odd load resistance $R_L = 1$.

In table 4 are the elements of the Chebyshev WDF for various attenuation A_{max} in the passband and order filter n=5. The tables was designed for sampling frequency $f_s = 0.5$.

Tables of the Cauer Wave Digital Filters

In the Fig 18 is the attenuation of the Cauer WDF. The structure of the Cauer WDF is presented in the Fig 19. LC parallel resonant circuits in longitudinal branch are realized by serial adapters connected in the 3rd port with two delay elements. The filter with n odd

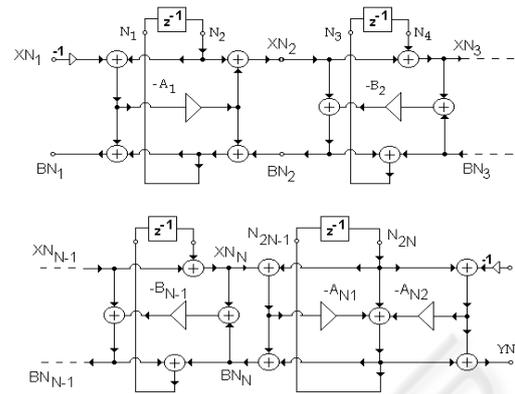


Figure 17: Structure of Chebyshev WDF filter.

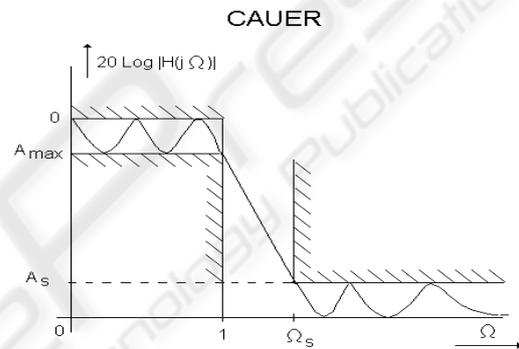


Figure 18: Attenuation of Cauer WDF.

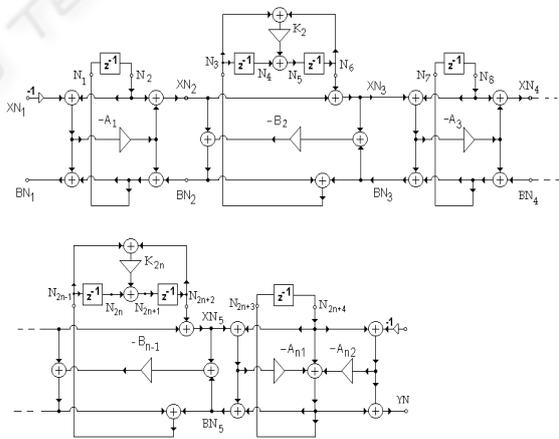


Figure 19: Digital structure of Cauer WDF.

must be terminated with parallel dependent adapter.

In the tables 5 and 6 are the elements of the Cauer WDF for various attenuation A_{max} in passband and sampling frequency $f_s = 0.5$. In the next section we shall demonstrate in the examples calculation of the Wave digital low-pass and high-pas filters.

Table 3: Elements of Chebychev WDF $n=3$, A_{max} in [dB].

A_{max}	A_1	B_2	A_{31}	A_{32}
0.10	0.4922	0.3022	0.4620	0.7574
0.25	0.4341	0.2774	0.4310	0.6812
0.50	0.3852	0.2599	0.4126	0.6114
1.00	0.3307	0.2496	0.3995	0.5293
2.00	0.2695	0.2445	0.3929	0.4331
3.00	0.2300	0.2442	0.3925	0.3696

Table 4: Elements of Chebychev WDF $n=5$, A_{max} in [dB].

A_{max}	A_1	B_2	A_3	B_4	A_{51}	A_{52}
0.10	0.465	0.253	0.216	0.224	0.417	0.737
0.25	0.419	0.240	0.205	0.213	0.398	0.672
0.50	0.369	0.231	0.197	0.204	0.386	0.596
1.00	0.319	0.226	0.191	0.198	0.379	0.516
2.00	0.261	0.225	0.185	0.193	0.379	0.422
3.00	0.223	0.226	0.182	0.192	0.383	0.360

Table 5: Elements of Cauer WDF $n=3$, $\Omega_s = 4.8097$, $K_2 = -0.93686$.

A_m	A_s	A_1	B_2	A_{31}	A_{32}
0.0004	24.7	0.7504	0.5766	0.7314	0.9629
0.0017	30.7	0.7209	0.4907	0.6664	0.9374
0.0039	34.3	0.6673	0.4569	0.6272	0.9160
0.0109	38.7	0.6190	0.4063	0.5778	0.8803
0.0279	42.8	0.5706	0.3612	0.5339	0.8365
0.0436	44.8	0.5461	0.3459	0.5140	0.8114
0.0988	48.3	0.4983	0.3161	0.4804	0.7573
0.1773	50.9	0.4614	0.2979	0.4590	0.7110
0.2803	53	0.4306	0.2855	0.4442	0.6699
1.2494	60	0.3147	0.2593	0.4118	0.4999

Table 6: Elements of Cauer WDF $n=5$, $\Omega_s = 2.0000$ $K_2 = -0.827$ $K_4 = -0.627$.

A_{max}	A_s	A_1	B_2	A_3	B_4	A_{51}	A_{52}
0.0017	41.3	0.657	0.399	0.330	0.436	0.734	0.915
0.0039	44.8	0.627	0.373	0.311	0.404	0.690	0.893
0.0109	49.2	0.585	0.343	0.290	0.369	0.639	0.856
0.0279	53.3	0.543	0.318	0.272	0.341	0.596	0.813
0.0510	55.3	0.522	0.306	0.264	0.323	0.577	0.788
0.0988	58.8	0.479	0.288	0.251	0.310	0.546	0.736
0.1773	61.4	0.446	0.277	0.243	0.298	0.527	0.691
0.2803	63.5	0.418	0.270	0.237	0.289	0.514	0.652
1.2494	70.5	0.309	0.256	0.221	0.269	0.492	0.487