

# REALISTIC RAIN RENDERING

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**Abstract:** We propose a method for rendering realistic three-dimensional raining scene. The proposed method gives two main contributions to rendering raining scene. The first one is to propose a simple model that deals with temporal-spatial localities such as wind effect, density and intensity of rainfall, and raindrop brightness, so as to represent environmental conditions that differs on a scene-to-scene basis. The other one is to propose a raindrop trajectory computing method whose computation load is immune to the number of raindrops, wind effect and complicated camera movement. Due to the above-mentioned contributions, the proposed method can represent essential aspects in raining scene, such as spatial changes of rain in urban canyon or temporal changes due to the moving wisp of rain.

## 1 INTRODUCTION

Although various factors are concerned with the reality of landscape rendered with Computer Graphics (CG), the representation about weather or meteorological condition is one of the essential factors. This paper proposes a method for realistically rendering raining scenes, which can represent the temporal-spatial localities such as wind effect, density and intensity of rainfall, and raindrop brightness.

For the clarification of background, the previous studies related to rainfall is summarized below, especially focusing on the relationship with our study. Kaneda et al. proposed a representation method of water droplet on the windshield for driving simulator (Kaneda et al., 1999). Sato et al. proposed a faster method by using the capability of graphics hardware (Sato et al., 2002). Many researchers have studied rendering methods of the trajectory of raindrops. In early CG animation, the falling motion of translucent white spheres was synthesized into the background scene on the basis of the particle system (Reeves, 1983; Sims, 1990). Starik et al. proposed a technique for combining raindrop trajectory with video images, where rain intensity and raindrop size can be adjusted by controlling parameters (Starik and Werman, 2003). Feng et al. proposed a fast GPU-based method of rendering the trajectory of windblown raindrops and raindrop splashes (Feng et al., 2005a; Feng et al., 2005b). Garg and Nayar proposed a method

for improving appearance of rain trajectory that utilizes a database of rain streaks under various illumination conditions (Garg and Nayar, 2006). Several other studies have been done for creating real-time rain animation taking into account environmental light condition. For example, handwritten rain strokes were colored based on the environmental light condition (Wang et al., 2006). Multiple layers of rain texture were used particularly for expressing rain drop motion parallax (Tatarchuk, 2006). A texture distorted according to optical properties of raindrops was mapped onto a raindrop (Rousseau et al., 2006).

These previous contributions are very useful for rendering raining scenes with feasible computation requirements. Now it should be noted that particle system was employed in all study except Starik's method. These particle-based methods generated raindrops randomly and uniformly. In addition, the raindrop movement was not influenced by the wind. Even so, these wind conditions were uniform. However it is clear that the density and the falling trajectory of real-life raindrops reflect the temporal-spatial changes of the wind conditions. These characteristic localities in raining scene are considered as essential factors for providing raining reality, and the problem about these factors in the previous studies should be improved for more broad applications. In addition, apparent movements of raindrops caused by the camera movement were not considered or if so, they were considered partially.

This paper proposes a rendering method of the falling trajectory of raindrops that can deal with both the temporal-spatial localities of raining and arbitrary camera movements. At first, section 2 gives a brief overview of the proposed method and section 3 describes a simulation model of rainfall and 3-D wrapping-around of raindrops that was introduced for permitting arbitrary camera movements. Section 4 explains the raindrop-generating model for the temporal-spatial localities of raining, that is, the density and intensity of raining. This section also describes the method for rendering raindrops considering with locality of occlusion of the sky. Section 5 shows usefulness of the method using CG animations rendered with the proposed method. Finally, section 6 describes the conclusion and the future work.

## 2 OVERVIEW

### 2.1 Prior Conditions

In real-life raining, all the raindrops have differences in location, velocity, its size and shape. In addition, there are several factors to be considered in rendering, that is, the changes in direction and velocity of raindrops due to non-steady wind effect, and the changes in light intensity coming through a raindrop depending on illumination condition due to the sky-light. The above-mentioned factors are referred as temporal-spatial locality of raining in our study. It is desired to reflect these localities for improving the rendering reality, but practical real-time applications such as walk-through systems or driving simulators require as simple processing as possible. In order to solve this trade-off, we introduce the following prior-conditions:

- (1) The viewpoint is located near the ground level. Therefore the vertical velocity of every raindrop is assumed to be equivalent of terminal velocity. On the other hand, only horizontal velocity is controlled by temporal-spatial changes in wind.
- (2) Trajectory of a raindrop is drawn with a polyline connecting the positions of the raindrop at neighboring two frames.
- (3) The shape of a raindrop is represented with sphere and its diameter is determined by the raining intensity.

### 2.2 Fundamental Idea

From two viewpoints, the following description illustrates the problems in conventional methods and

the characteristics of the proposed method.

**(1) Generation and Tracking of Raindrops.** Problems in the previous methods which employ particle system for generating and vanishing raindrops, are summarized as :

- Stationary camera; Some researches discussed the dynamic camera, but they synthesized stationary-camera rendering of raindrops with the dynamic-camera background.
- Temporal-spatial uniform generation of raindrops.
- Due to the consistency of the vertical axis of the camera-coordinate system with that of the world-coordinate system, raindrops are generated above the top face of the view volume and fall down only in the direction of gravitational force.

Conventional methods focused on rendering the raindrop trajectory and some of them were very realistic for off-line processing or very fast for real-time applications. On the contrary, the above-mentioned constraints narrow the range of their applications. To solve these problems, we provide a new raindrop movement model with the following characteristics:

- A raindrop generating model that can deal with localities on density and intensity of raining.
- A raindrop movement model that can express apparent movements of raindrops caused by the rotation and/or translation of camera.
- A raindrop descent tracking model that invokes no rapid fluctuation in the number of raindrops with satisfying two above-mentioned points.

**(2) Rendering of Raindrops.** As suggested by Wang et al. (Wang et al., 2006), the light intensity coming through a raindrop is determined on basis of the reflection of the surrounding environment. Wang et al. utilized a environment mapping for obtaining the reflection-based light intensity. The luminance of the object surface, however, distinctly differs from that of the sky during daytime even in the rain. For instance, in the light-raining scene shown in Figure 1 (captured by a digital camera with automatic exposure mode), the actual measurement of luminance gives  $6,886cd/m^2$ ,  $562cd/m^2$  and  $53cd/m^2$  at point *A*, *B* and *C*, respectively. On the other hand, the pixel value on the digital camera picture gives  $(R,G,B) = (254, 254, 254), (182, 171, 181)$  and  $(51, 50, 46)$ . As you can see, 8 bit-colored image-based method is entirely inadequate for computing the optical energy received from the surrounding environment due to lack of the dynamic range, therefore resulting in inaccurate light transport. In order to solve this problem, we assume that the light intensity coming through a

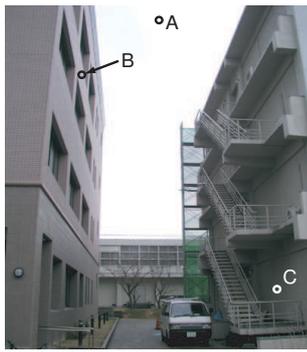


Figure 1: A photograph of a light-rain day.

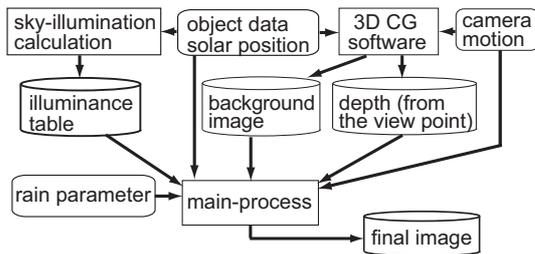


Figure 2: Rain rendering procedure.

raindrop is proportional to the optical energy received from the sky, and determines the light intensity based on the illuminance due to skylight.

### 2.3 Procedure of Generating Raining Scene

As depicted in Figure 2, the proposed method for generating raining scene consists of three steps. As a preprocessing, the illuminance due to skylight is calculated. In the second step, background image and depth information for each animation frame is generated and stored. In the third step, the raindrops are generated based on both the camera movement and the local density of raindrop occurrence. The velocity and direction of wind field are computed and applied to the updating of raindrops to be tracked. Then, the trajectories of raindrop are rendered based on both the illuminance due to skylight at the view point and the raindrop movement in the camera-coordinate system. Finally, the background image and the corresponding raindrop-rendering image are synthesized.

## 3 RAINDROP MOVEMENT

In this section, the computing and drawing method for the trajectory of raindrops is described. The pro-

posed method regards only raindrops in the rectangular parallelepiped (RP, see Figure 3) located in front of the viewpoint as a target for computing and drawing. At first, let us explain how to compute the velocity vector of a raindrop including its apparent movement due to camera movement. Then the effective wrapping-around reuse of raindrops running over the RP is shown.

### 3.1 Calculation of Velocity Vector

(1) **Stationary Camera Model.** The RP is defined as a bounding box that including a part of the view volume, which is illustrated with bold lines in Figure 3. We consider that every raindrop which is tracked its trajectory exists only in this RP as described before. According to the prior condition (1), the factors affecting the raindrops' movement are the gravity and wind. Consequently, the velocity vector of a raindrop ( $V_r'$  in Figure 4) can be computed by synthesizing the following two factors. The first one is the vertical falling velocity which reaches to the terminal velocity,  $V_t$ . The second one is the wind velocity,  $V_w$ , that is calculated by solving Navier-Stokes equation (We discuss more in Section 4.2).

(2) **Expansion to Dynamic-camera Model.** In applications such as walk-through, the camera movement can be classified into dolly, pan and tilt. The camera movement causes an apparent movement of raindrops on the screen. The proposed method translates this apparent movement into the velocity. Let us explain with the case of dolly as shown in Figure 4. The dolly movement  $V_c$  results in an apparent movement with the opposite-direction velocity  $V_a$  to all raindrops. In the case of pan and tilt movements that arise from the rotation of camera axis, they give apparent movement with the opposite-rotation to all raindrops. The proposed method regards the apparent movement of raindrop caused by pan and tilt as a set of linear movement on each frame. Consequently, the apparent movement caused by camera movement can be represented as a velocity vector. Then, final raindrop's movement,  $V_r$ , is determined by synthesizing  $V_r'$  calculated by physics and  $V_a$  given by apparent movement as shown in Figure 4.

### 3.2 Three-dimensional Wrapping-around of Raindrops

(1) **Basic Idea.** In the proposed method, only raindrops that exist in the RP are regarded as a target for computing and drawing. In a windless scene, as shown in Figure 5(a), all raindrops arising from

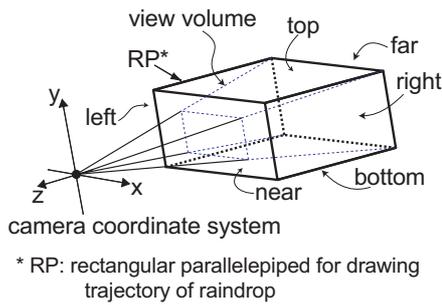


Figure 3: Geometry of the rectangular parallelepiped (RP) for drawing a trajectory of a raindrop.

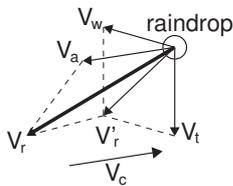


Figure 4: Determination of velocity vector of a raindrop.

the top face of the RP,  $T$ , fall down and reach the bottom face,  $B$ . However, in case of windy scene as depicted in Figure 5(b), some of raindrops run over the side face before arriving at  $B$  and run into region  $P$ , while region  $Q$  has no raindrop arising from the top face. One of the solutions for this problem is to extend the top face horizontally as a function of wind speed. However it will bring redundant computation of raindrops. The proposed method employs more efficient method. Since we may consider that the amount of raindrops in region  $P$  and  $Q$  are almost the same with the geometrical property of the RP, the raindrops running into region  $P$  can be reused as raindrops going into region  $Q$ , which is a basic idea of 3-D wrapping-around of raindrops. The details will be described below.

**(2) Face Attributes of RP.** As illustrated in Figure 5(a), the top face,  $T$ , is usually given as an attribute of raindrop-generating face,  $G$ , and the bottom face,  $B$ , is given an attribute of raindrop-vanishing face,  $V$ . Other four faces have the same attribute of raindrop wrapping-around face. However, in the case of large camera tilt and strong wind as shown in Figure 5(c), there will be a problem that all raindrops may fly out from the RP just after being generated at the top face,  $T$ . Therefore, the proposed method selects the face whose inward-looking normal vector gives maximum inner product with raindrop's velocity vector ( $V_r'$  in Fig. 4) as the raindrop-generating face,  $G$ , and its opposite face as

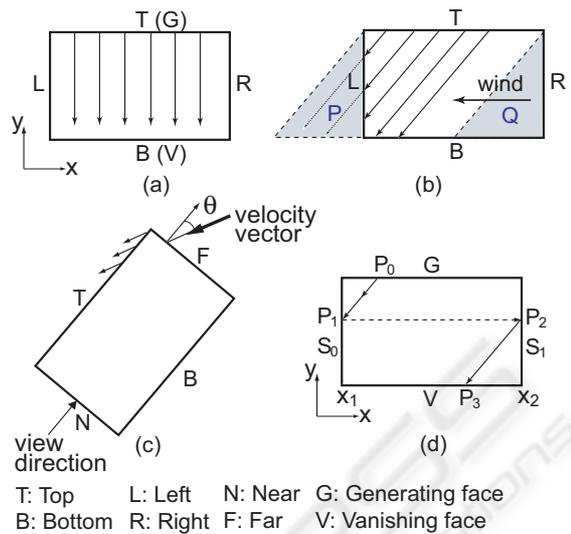


Figure 5: 3-D Wrapping-around of the falling trajectory of the raindrop.

the raindrop-vanishing face,  $V$ .

**(3) 3-D Wrapping-around of Raindrops.** Figure 5(d) illustrates 3-D wrapping-around for tracking the raindrop trajectory. If a raindrop hits a wrapping-around face (in this case  $S_0$ ), the raindrop is automatically moved to the same position on the opposite wrapping-around face (in this case  $S_1$ ) and continues the same movement. In this example, only  $x$  coordinate of the raindrop is changed from  $x_1$  to  $x_2$  as the result of the 3-D wrapping-around processing. Raindrops can go under this processing more than once until reaching the vanishing face.

## 4 RAINING LOCALITY

This section describes the way of representing raining localities of intensity, density, movement and appearance, which is one of the main contributions of the proposed method.

### 4.1 Raindrop Generation with Local High-density Distribution

In the situation of shower or heavy rain, we can see cluster of raindrops with high intensity and high density moving. This paper develops a simple model for rendering these non-uniform characteristics of raining scene. It is assumed that there are two kinds of area in a raining scene. The first area has temporally-and-spatially uniform characteristics, while the other

area does not. We employ a raindrop-generating model that deals with these different areas separately. To be more precise, this proposed model generates raindrops uniformly in the RP discussed in section 3 and synthesizes local high-density distributions of raindrops. This synthesis procedure provides blocky placements of raindrop in the rendered space. The local high-density distributions of raindrops should be moved over the course of time due to its nature property. On the other hand, as described in section 3, this proposed method only tracks the trajectories of raindrop in the rectangle parallelepiped and renders them. Consequently, in the local high-density distributions of raindrops, one needs to consider only raindrops whose initial positions are included in the generating face of the RP. However, for example, in producing an animation sequence where the view direction gradually moves upward from initial horizontal direction, the raindrop-generating face changes as the viewing line tilts. Therefore, we define local high-density distribution on another plane that is independent of the raindrop-generating face on the RP.

Figure 6 illustrates the raindrop-generating procedure of the proposed method. As shown in Figure 6(a), we define a plane  $S$  that is parallel to the  $xz$ -plane of the world-coordinate system. As shown in Figure 6(b), the local high-density distribution is given as a form of Gauss distribution centered at the reference point  $Q$  on the plane  $S$ . Since all raindrops have to be generated on the raindrop-generating face of the RP in the proposed method, the raindrops generated in the local high-density distribution are projected onto the raindrop-generating face and are merged with the raindrops normally generated in the RP. The reference point  $Q$  is regarded as a particle that is influenced by the wind effect and is moved like a particle in the wind field. The computing method of the wind vector influencing the reference  $Q$  is the same with the computing method of the wind vector influencing the raindrops that is described in the next subsection. Since local high-density distributions can be processed independently, it is possible to move several distributions simultaneously in a scene.

This paper classifies raining types into three categories, that is, gentle rain, hard rain and heavy rain. Each type has fundamental properties described in Table 1. The proposed method assigns one of the fundamental properties of raining to the raindrops generated in the RP and assigns a heavier property to the raindrops generated in the local high-density distribution. Consequently, two groups of raindrops in the scene have distinctive differences in the terminal velocity and their size, in addition to the raindrop-generating density.

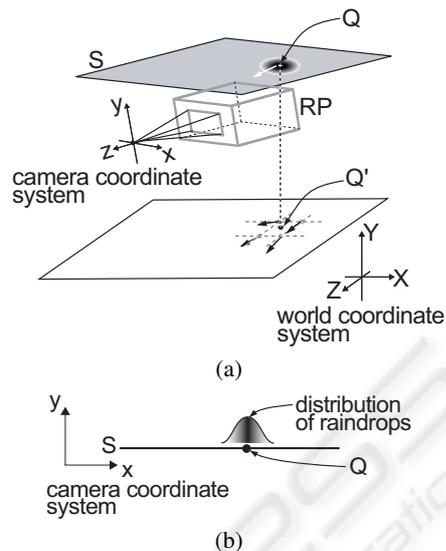


Figure 6: Local high-density distribution of raining.

## 4.2 Local Wind Influence

We assume that the raindrops are influenced just horizontally by the wind field and recognizes only two factors as external forces on a raindrop as described with the prior condition (1) in section 2.1, that is, the vertical gravity and the horizontal wind influence. As shown in Table 1, the vertical downward velocity is assumed to be equivalent of a constant terminal velocity depending on the degree of raining intensity. On the other hands, the horizontal velocity is determined based on the two-dimensional fluid simulation. These vertical-and-horizontal results are utilized for computing the trajectories of raindrop. It should be noticed that this simplified procedure is very efficient for computing the unsteady wind field and its influence to the raindrop movement.

We employ a solving technique of Navier-Stokes equations suggested by Stam (Stam, 1999) for simulating two-dimensional wind field. A simulation field is configured on the scene with enough area to deal with changes in the field of view and the calculus of finite differences is applied to its mesh representation. At the mesh points, the wind velocity vectors are computed at the frame rate. The wind velocity vector at the sub-mesh points can be interpolated linearly based on four values at the nearest surrounding mesh points and it is used as the wind velocity vector,  $V_w$  in Figure 4.

Table 1: Fundamental properties of three types of raining intensity.

type	gentle	hard	heavy
rainfall (mm/h)	4.0	15.0	40.0
diameter (mm)	1.0	1.5	2.0
terminal velocity (m/s)	4.0	5.0	6.0

### 4.3 Raindrop Rendering Considering Illumination Locality

It is indispensable to reflect the occluding condition of the sky and cloudiness on the trajectories of raindrop with a simplified method. Since the image-based method using the environment mapping technique for representing the reflection to the surface of raindrops leads to the shortage problem of dynamic range as discussed in section 2, the illuminance due to skylight is utilized for computing the reflection on the raindrop surface. The following parts describe a simplified rendering model for representing the influence of the lighting environment on a raindrop, the computation of the illuminance due to skylight, and the procedure of producing an illuminance map.

**(1) Raindrop Rendering Model.** The intensity of light arriving at the viewpoint comes through a raindrop,  $I_V$ , is expressed as the following equation based on the intensity of light reflected at a point on the raindrop,  $I_{Ri}$ , and the intensity of light transmitted through the rain drop,  $I_{Ti}$ .

$$I_V = \sum_i (r_i I_{Ri} + (1 - r_i) I_{Ti}) \omega_i, \quad (1)$$

where  $r_i$  is a reflection ratio at a point  $x_i$  on the raindrop,  $\omega_i$  is a contribution of  $x_i$  ( $\sum_i \omega_i = 1$ ). It is well known that the computation cost to obtain both  $I_{Ri}$  and  $I_{Ti}$  is heavy. Then we set the following assumptions in order to reduce computation cost for rendering raindrop considering locality of illumination:

- Since the major part of the  $I_V$  consists of energy which comes from the sky, the intensity of  $I_V$  is proportional to the illuminance due to skylight at the location of the raindrop.
- Only the light comes from background of a raindrop whose direction is the same as  $I_V$ ,  $I_P$  in Figure 7, is considered as a transmitted component.

Then, equation 1 can be re-written as follows,

$$I_V = r_c \alpha_{sky} E_{sky} + (1 - r_c) I_{Tc}, \quad (2)$$

where  $\alpha_{sky}$  is the coefficient to transform from illuminance due to skylight,  $E_{sky}$  to intensity of light,  $I_{Tc}$  is intensity of light which arrives at the viewpoint

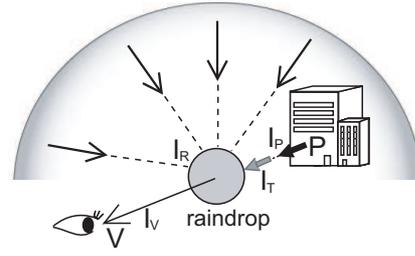


Figure 7: Raindrop rendering model.

passing through the center of the raindrop,  $r_c$  is the reflection ratio of the light corresponding to  $I_{Tc}$ .

**(2) Computation of Illuminance due to Skylight.** The proposed method calculates the illuminance due to skylight,  $E_{sky}$  considering with sky occlusion in the same manner of (Tadamura et al., 1993) with the following equation.

$$E_{sky} = \sum_{i=0}^N \sum_{j=0}^M \tau(i, j) L(i, j) H(i, j) \Delta S_{ij}, \quad (3)$$

where  $\tau(i, j)$  is an attenuation ratio due to raining related to the sky-element  $(i, j)$ . Since it is assumed that there is no directional difference of attenuation,  $\tau(i, j)$  can be a constant.  $L(i, j)$  is a luminance at the sky-element  $(i, j)$ ,  $H(i, j)$  is a blocking function that if the sky-element  $(i, j)$  is blocked (black pixels in Figure 8(b) and (c)) it gives 0 otherwise (gray pixels) 1.  $\Delta S_{ij}$  is the area onto which the sky-element  $(i, j)$  is projected. In the proposed method, the sky luminance  $L(i, j)$  is computed by ALL SKY MODEL (Igawa et al., 2001) that can consider the location of the sun even in the overcast-sky (see the details in (Igawa et al., 2001)). Figure 8(a) and (b) show the typical occluding condition of the sky, at a place surrounded by tall buildings and a place where there is no tall building, respectively. It should be noted that the surrounding buildings determine the occluding conditions. Therefore, the light intensity coming through a raindrop at the two places differs greatly.

**(3) Illuminance Map.** Computing the illuminance at all positions of raindrops with the above procedure results in a huge load. Therefore, we employ a scheme for reducing the computation load without keeping the locality of the light intensity coming through a raindrop based on the occluding condition of the sky. In this scheme, at the mesh points used for two-dimensional wind simulation, the illuminance is computed in advance and the obtained illuminance is stored in the data table. This data table is referred as an illuminance map.

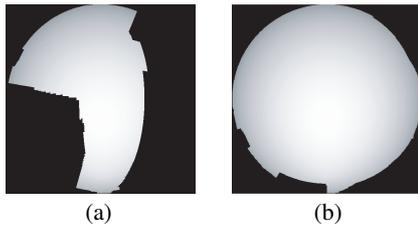


Figure 8: Examples of the sky occlusion; (a) surrounded by tall buildings; (b) no tall building.

## 5 RESULTS AND DISCUSSIONS

We implemented the proposed method described above on a PC with Intel Pentium 4 2.8GHz processor, nVIDIA GeForce 6600GT and 2GB RAM. In order to obtain a raining animation, we prepared sequence images which were generated by a 3D-CG package software as back-ground images for rain. At first, we will show an example for dealing with locality of wind. Figure 9(a) is a scene obtained by applying conventional method, that is, a stable wind blows with uniform direction and velocity over the scene. Figure 9(b) is a frame of an animation created by the proposed method treating with the temporal-spatial locality of unsteady wind by solving Navier-Stokes equation. Two portions of the image surrounded by white broken line ellipses,  $P$  and  $Q$ , in Figure 9(b) show the difference of the trajectory of raindrops. The direction of the trajectory of the raindrops in  $Q$  are quite different from the same portion of the Figure 9(a). Figure 10 shows a frame of an animation for demonstrating the usefulness of non-uniform density and intensity of the rain distribution. We can see the high density raindrops in the vicinity of the center of the image. The animation allows us to investigate temporal movement of the high density raindrops. Figure 11 demonstrates the advantage of rendering a raindrop with illuminance due to skylight. The skylight around the sun is the brightest in the whole sky even in the overcast sky. Figure 11(a) is an example for a case that the high intensity portion of the sky is not occluded by the buildings because of high solar altitude, while bright portion is almost occluded by the buildings because of low solar altitude in (b). These figures are noticed that illumination due to skylight deeply affects on appearance of raindrop.

It takes about one second per frame to add rain trajectories to the background image with our proposed method. Since the proposed method draws the rain streak with CPU, more than 70% of computation time is consumed for scan-conversion of the rain streaks.



(a) A scene with uniform wind.



(b) A scene with non-uniform wind.

Figure 9: Effects of dealing with wind locality.



Figure 10: A scene with temporal-spatial localities of rain intensity and density.

## 6 CONCLUSIONS

We have proposed a method for rendering the trajectory of raindrops that can deal with both the temporal-spatial localities of raining and arbitrary camera movements. These localities include wind velocity and direction, intensity and density of the raindrops and light intensity coming through the raindrop. We employed 3-D wrapping-around of raindrops for efficient calculation of the trajectory of the raindrops. 2-D wind field was simulated by solv-



(a) High solar altitude



(b) Low solar altitude

Figure 11: Comparison of the results with different solar altitude.

ing Navier-Stokes equations and used for achieving to represent temporal-spatial localities of wind. We used a numerical sky luminance distribution model which can consider the location of the sun for the overcast-sky.

In order to use the proposed method to the real-time application such as games and drive simulators, we need to accelerate the rain streak drawing process by using GPU. In future work, we will modify the proposed method for utilizing GPU ability.

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## REFERENCES

Feng, Z., Tang, M., Dong, J., and Chou, S. (2005a). Real-time rain simulation in cartoon style. In *9th International Conference on Computer Aided Design and Computer Graphics*.

- Feng, Z., Tang, M., Dong, J., and Chou, S. (2005b). Real-time rendering of raining animation based on the graphics hardware acceleration. In *9th International Conference on Computer Supported Cooperative Work in Design*, pages 734–739.
- Garg, K. and Nayar, S. (2006). Photorealistic rendering of rain streaks. In *SIGGRAPH'06*, pages 996–1002.
- Igawa, N., Koga, Y., Matsuzawa, T., and Nakamura, H. (2001). Development of all sky model of sky radiance distribution under various atmospheric conditions. In *Istanbul 2001 International Lighting Congress 2*, pages 331–333.
- Kaneda, K., Ikeda, S., and Yamashita, H. (1999). Animation of water droplets moving down a surface. *The journal of visualization & computer animation*, 10(1):15–26.
- Reeves, T. W. (1983). Particle systems - a technique for modeling a class of fuzzy objects. *ToG*, 2(2):359–376.
- Rousseau, P., Jolivet, V., and Ghazanfarpour, D. (2006). Realistic real-time rain rendering. *Computers & Graphics*, 30(4):507–518.
- Sato, T., Dobashi, Y., and Yamamoto, T. (2002). A method for real-time rendering of water droplets taking into account interactive depth of field effects. In *IWEC'02*, pages 110–117.
- Sims, K. (1990). Particle animation and rendering using data parallel computation. *Computer Graphics*, 24(4):405–413.
- Stam, J. (1999). Stable fluids. In *SIGGRAPH'99*, pages 121–128.
- Starik, S. and Werman, M. (2003). Efficient simulation of rain in videos. In *Texture 2003 Nice*, pages 95–100.
- Tadamura, K., Nakamae, E., Kaneda, K., Baba, M., Yamashita, H., and Nishita, T. (1993). Modeling of skylight and rendering of outdoor scenes. *Computer Graphics forum*, 12(3):189–200.
- Tatarchuk, N. (2006). Artist-directable real-time rain rendering in city environments. In *SIGGRAPH2006 Course Note 26*, chapter 3, pages 23–64. ACM.
- Wang, L., Lin, Z., Fang, T., Yang, X., and Kang, S. (2006). Real-time rendering of realistic rain. In *SIGGRAPH'06 Sketches*.