COLOR QUANTIZATION BY MORPHOLOGICAL HISTOGRAM PROCESSING

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Abstract: In a previous paper it was proposed a graylevel quantization method by morphological histogram processing. This paper introduces the extension of that quantization method to color images. Considering an image under the RGB color space model, this extension reduces the number of colors in the image by partitioning an 3-D histogram, similar to the RGB color space, in rectangular parallelepiped regions, through a iterative process. Such partitioning is done, in each iteration, by application of the graylevel quantization method to the longest dimension of the current region which has the greatest volume. The final classified color space is used to quantize the image. This paper also shows the comparison of the proposed method to the classical median cut one.

1 INTRODUCTION

Color reduction techniques are fundamental in digital image processing and computer graphics. Image quantization by color reduction (Gonzalez and Woods, 1992; Heckbert, 1982; Soille, 1996) has been applied to solve problems of image display, image compression, image simplification and segmentation. (Gomes and Velho, 1994).

Color quantization also provides the reduction of *flat zones* (connected regions of pixels with constant color) in the image, such a connected filter (Crespo et al., 1997; Salembier and Serra, 1995; Heijmans, 1999; Meyer, 1998). The reduction of flat zones does not introduct borders in the image, but, by supressing some borders, two of more flat zones may be joined in one. Flat zone reduction is usually applied to image compression, image segmentation (Meyer and Beucher, 1990; Beucher and Meyer, 1992) and in the reduction of the statistics in an image in order to simplify the number of samples used in pattern recognition techniques (Hirata Jr. et al., 1999; Flores et al., 2000; Flores et al., 2002).

In a previous paper (Flores and Lotufo, 2001), it was proposed a method which gives not only an image simplification in terms of graylevel reduction but also in terms of flat zone reduction. The proposed method is given by application of a set of morphological operators to the image histogram. The main motivation behind the project of this operator is that each object in the image has a significative graylevel distribution. So, to simplify an object in the image, that is enough to classify its corresponding distribution in the histogram.

In that paper it was also proposed a method to reduce an image to n graylevels. It consists in to choose n regional maxima in the processed histogram and to filter the other peaks. The chosen maxima will provide the classification of the graylevels in the histogram by application of watershed operator. The major drawback of the method proposed in (Flores and Lotufo, 2001) is the choice of the n regional maxima. In that paper, the chosen regional maxima was the n highest ones. Note that, by far, it is not the best criterion to choose the regional maxima.

In a following paper (Flores et al., 2006), it

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was proposed the application of *dynamics* (Grimaud, 1992) to select the regional maxima in order to achieve a better graylevel reduction. Dynamics consists in a valuation of extrema of the image by a measure of contrast that does not consider the size or shape of valleys and peaks. It is usually applied to find markers to morphological segmentation and achieve hierarchical segmentation (Meyer, 1996). The results achieved by the application of dynamics as regional maxima criterion showed itselt far better than the simple choice of the highest peaks. Both, the visual quality of the resulting images and the flat zones reduction are better when dynamics is applied.

This paper introduces the extension of the quantization method by morphological histogram processing to color images. Considering an image under the RGB color space model, this extension reduces the number of colors in the image to at most *n* colors by partitioning the RGB color space in rectangular parallelepiped regions. Such partition is an iterative process where, in each iteration, one region is split in at most *k* rectangular parallelepiped regions. The splitting of a region is done by choosing the longest side of the parallelepipe (one of the three dimensions, red, green or blue), computing the histogram along this side and applying the graylevel quantization method to this histogram; the result gives where the region must be split and how many regions are created in that iteration.

This paper is organized as follows: section 2 presents some preliminar definitions used in this paper. Section 3 introduces the color quantization by morphological histogram processing. Section 4 presents some experimental results and section 5 concludes this paper with a brief discussion.

2 DEFINITIONS

Let $E \subset \mathbb{Z} \times \mathbb{Z}$ be a rectangular finite subset of points. Let $K = [0,k]$ be a totally ordered set. Denote by $Fun[E,K]$ the set of all functions $f : E \to K$. An *image* is one of these functions (called graylevel functions). Particularly, if $K = [0,1]$, f is a binary image. An *image operator* (*operator*, for simplicity) is a mapping $\psi : Fun[E,K] \to Fun[E,K]$.

Let $N(x)$ be the set containing the *neighbourhood* (Flores and Lotufo, 2001) of x, $x \in E$. We define a *path* from x to y, $x,y \in E$ as a sequence $P(x,y) = (p_0, p_1, ..., p_n)$ from E, where $p_0 = x$, $p_n = y$ and $\forall i \in [0, n-1], p_i \in N(p_{i+1})$.

A *connected subset* of E is a subset $X \subset E$ such that, $\forall x,y \in X$, there is a path C entirely inside X.

Let $f \in Fun[E,K]$. A *flat zone* of f is a connected subset $X \subset E$, such that $f(x) = f(y)$, $\forall x,y \in X$.

Definition 1 *The inf - reconstruction and sup - reconstruction operators are given, respectively, by,* $\forall f,g \in Fun[E,K]$,

$$\rho_{B,g}(f) = \delta^{\infty}_{B,g}(f)$$
$$\rho^{*}_{B,g}(f) = \varepsilon^{\infty}_{B,g}(f)$$

where $B \subset E$ *is the structuring element,* $n \in \mathbb{Z}_+$ *and* $\delta^n_{B,g}$ *and* $\varepsilon^n_{B,g}$ *are, respectively, the n-conditional dilation and the n-conditional erosion operators (Heijmans, 1994).* $\delta^{\infty}_{B,g}(f)$ ($\varepsilon^{\infty}_{B,g}(f)$) *means that the dilation (erosion) is applied till idempotency.*

Let $\tau_i : Fun[E,K] \to Fun[E,[0,1]]$, $i \in K$, be a threshold function, where $\tau_i(f)(x) = 1$, if $f(x) \geq i$, and $\tau_i(f)(x) = 0$, otherwise.

Definition 2 *Let* $f \in Fun[E,K]$. *A regional maximum is a flat zone Z such that* $f(z) > f(n)$, $z \in Z$, $n \in N, N \in \mathcal{F}_Z$, *where* $\mathcal{F}_Z$ *is a set of all flat zones adjacent to Z (Flores and Lotufo, 2001). The regional maxima of f is found by application of a operator* $\mu^{\max}_{B_c} : Fun[E,K] \to Fun[E,[0,1]]$, *given by*

$$\mu^{\max}_{B_c}(f) = \tau_1(\rho_{B_c,(f+1)}(f)) \vee \tau_k(f)$$

where $B_c \subset E$ *is the structuring element defining connectivity.*

A regional miminum is a flat zone Z such that $f(z) < f(n)$, $z \in Z$, $n \in N, N \in \mathcal{F}_Z$, *where* $\mathcal{F}_Z$ *is a set of all flat zones adjacent to Z.*

2.1 Dynamics

Dynamics (Grimaud, 1992; Meyer, 1996) is a transformation which valuates the extrema of an image according to a contrast measurement. One advantage of application of dynamics is that, while some methods such as morphological filters need a size parameter to evaluate constrast, the dynamics measurement does not take in account the size and the shape of image structures.

The evaluation of constrast of a regional minimum is a good way to provide markers to application of watershed operator in the morphological segmentation framework: an hierarchical segmentation may be achieved by selecting the regional minima which dynamics is higher than a thresholding value and assigning markers to them (Meyer, 1996).

Definition 3 *Let* $x,y \in E$. *The dynamics* Dyn_f *of a path* $P(x,y)$ *on an image* $f \in Fun[E,K]$ *is given by,*

$$Dyn_f(P(x,y)) = \{\bigvee |f(x_i) - f(x_j)| : x_i, x_j \in P(x,y)\}.$$

i.e., the dynamics of P(x,y) is given by the difference in altitude between the points of highest and lowest altitude of P(x,y).

Grimaud (Grimaud, 1992) also defines the dynamics between two points $x,y \in E$ on an image $f \in Fun[E,K]$ as

$$Dyn_f(x,y) = \{\bigwedge Dyn_f(P(x,y)) : P(x,y),\}$$

where $P(x,y)$ is a path between x and y. However, it will not be applied here, since the histogram is an 1-D signal and, therefore, there is only one path between any two points from the domain of histogram function. So, it will be considered here that $Dyn_f(x,y) = Dyn_f(P(x,y))$.

Definition 4 *Let $a(Z) \in K$ be the altitude of a regional minimum Z in f. The dynamics of Z is given by,*

$$Dyn(Z) = \{\bigwedge Dyn_f(x,y), x \in Z, y \in M : a(M) < a(Z)\}.$$

i.e., the dynamics of Z is given by the dynamics of the path with the lowest dynamics that links Z to a point y thats belongs to a catchment basin which regional minimum has an altitude lower than Z.

Dynamics computation can be implemented by using tree of critical lakes (Meyer, 1996) or based on flooding simulations algorithms (Grimaud, 1992).

Given the dynamics of a regional minimum Z, some metrics can be used to evaluate such minimum:

1. depth of the catchment basin which the minimum is contained (given by the dynamics of the minimum itself);
2. area of the catchment basin;
3. volume of the catchment basin;

Let us denote by $Dyn_i(f)(Z)$ the function that computes to Z from f an value given by the metric $i \in \{1,2,3\}$ introduced above. $Dyn_i(f)(Z)$ will be used to evaluate the significant distributions in the histogram, as will be explained below.

Note that two catchment basin which have the same depth may have different volume or area measurements. Classification of regional minima in an image can be achieved by application of such metrics.

2.2 Graylevel Quantization by Morphological Histogram Processing

For a complete description of the graylevel quantization by morphological histogram processing, see (Flores et al., 2006; Flores and Lotufo, 2001). It consists in an application of a set of morphological operators to the image histogram. Since each object in the image has a significative graylevel distribution, that is enough to classify its corresponding distributions to simplify them.

Basically, the method computes all regional maxima in the histogram of the graylevel image and filters all unnecessary regional maxima located in the significative distributions in the histogram (Fig. 2). The filtered image is negated and the watershed operator is applied, resulting in a pre-classification (Fig. 3).

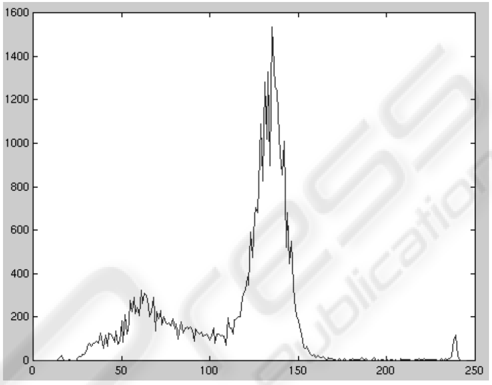


Figure 1: Histogram.

Given an image with the regional maxima from the filtered histogram (Fig. 1) labeled with their respective graylevels. Its inf-reconstruction conditioned to the pre-classification achieved by the watershed operator gives classification of all graylevel classes (Fig. 3). The classified histogram is used as a look-up table in order to reduce the graylevels.

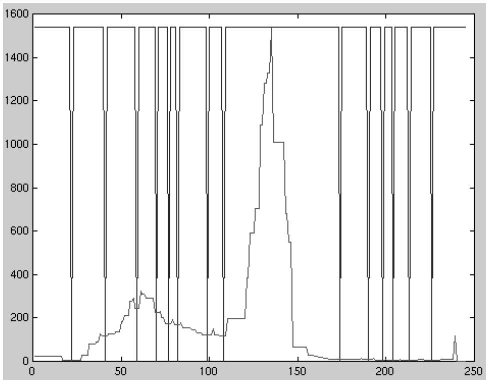


Figure 2: Pre-classification (filtered histogram and the watershed result).

As a consequence of such processing we have a reduction in the graylevels appearing in the image. In other words, the proposed filter is a mapping $\psi : Fun[E,K_1] \rightarrow Fun[E,K_2]$, where $|K_2| < |K_1|$.

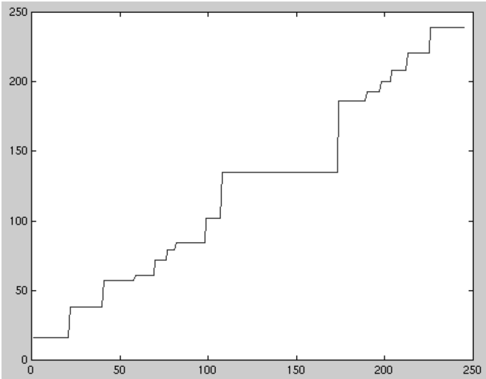


Figure 3: Classified graylevels.

2.3 The Use of Dynamics in Graylevel Quantization by Morphological Histogram Processing

When the operator ψ is applied to an image f, it reduces the graylevels appearing in f to the number of regional maxima of the filtered histogram. However, it is possible to reduce the graylevels to a smaller number, by adding a parameter n which gives the number of graylevels to appear in $\psi(f)$. In this subsection we will present a way to select the n most significant regional maxima of the filtered histogram by application of dynamics.

We will denote by $\psi_n : Fun[E,K_1] \to Fun[E,K_2]$, $|K_2| < |K_1|$, $|K_2| = n$, the operator which performs the reduction of the graylevels in the image to n graylevels.

Remember that the original histogram was filtered in order to preserve the highest regional maximum among a set of regiona maxima belonging to the same distribution. Let h_f and $\eta(\cdot)$ be, respectively, the original and the filtered histograms.

Let $D_i : Fun[K,\mathbb{Z}_+] \to Fun[K,\mathbb{Z}_+]$ be the function given by,

$$D_i(\eta)(x) = \begin{cases} Dyn_i(\nu(\eta))(Z) : x \in Z, \text{ if } \mu_B^{\max}(\eta)(x) = 1 \\ 0, \text{ otherwise} \end{cases},$$

where ν is the negation operator and Z is one of the regional minima of the negation of $\eta(\cdot)$. In other words, if x belongs to a regional maximum in $\eta(\cdot)$, $D_i(\eta)(x)$ will be equal to the dynamics (see section 2.1) of the regional minimum where x is located in the negation of $\eta(\cdot)$. i is the criterium chosen to evaluate $\eta(\cdot)$: depth (1), area (2) or volume (3).

Let m be the number of regional maxima in $\eta(\cdot)$. Let Q be the set defined by

$$Q = \{q_i \in K : D_i(\eta)(q_i) > 0 \text{ and } D_i(\eta)(q_i) \geq D_i(\eta)(q_{i+1})), i = 1, \cdots, m-1\}.$$

(i.e. Q is a sequence of all computed dynamics, according to criterion i, in decreasing order).

Let $\sigma_n : Fun[K,\mathbb{Z}_+] \times n \to Fun[K,\mathbb{Z}_+]$ be the mapping, given by, $\forall x \in K$, $\forall n \in \mathbb{Z}_+$,

$$\sigma_n(x) = \begin{cases} \max(h_f), \text{ if } x \in Q \\ 0, \text{ otherwise} \end{cases}.$$

Let $\eta_n : Fun[K,\mathbb{Z}_+] \times n \to Fun[K,\mathbb{Z}_+]$ be the mapping, given by, $\forall x \in K$, $\forall n \in \mathbb{Z}_+$,

$$\eta_n = \nu(\rho^*_{B,\nu(\eta)}(\sigma_n)).$$

By applying the operator η_n, the n regional maxima of $\eta(\cdot)$ which have the highest dynamics are selected. The function $\eta_n(\cdot)$ contains just n regional maxima and they are responsible for the classification of n classes (given by application of watershed operator). The remaining peaks are removed.

The method proposed in subsection 2.2 can be now extended to reduce an image to n graylevels, by adding the dynamics step introduced in this subsection to the framework, before the application of the watershed operator.

3 THE PROPOSED METHOD

The color quantization method proposed in this paper receives, as input data:

- The original image to be quantize (under the RGB color space model);
- A positive integer n : the number of colors the original image will be reduced to.
- A positive integer k : the maximum number of regions a parallelepiped region will be split in each iteration. If a region is split without the control of this parameter, the splitting method may compute many points and it may lead to the splitting of many regions in a single iteration and to a bad quantization result. With this parameter, no region is split in more than k regions in a single iteration (i.e., it is possible that the method finds less than k splitting points in a iteration. If it is the case, the method uses the points it found).

The starting parallelepiped region is a 3-D histogram from the original color image. This 3-D histogram is a discrete RGB color space cube, which dimensions are related, respectively, to red, green and blue. Each color appearing in the original image has a corresponding point inside this cube, and each point stores how many pixels the color appears in the original image. All other points in the cube are valued zero.

Color quantization by morphological histogram processing is the iterative process describe below:

1. Compute the starting region (the 3-D histogram from the original image);
2. $i \leftarrow 1$ (the current number of regions);
3. Find, among the i current parallelepiped regions, that one which have the greatest volume (let us call it R_i);
4. Take the longest dimension of the parallelepiped region R_i;
5. Let $L_i = [a_i, b_i]$ be the interval that defines the longest dimension of R_i;
6. For all $l \in L_i$, let $R_i(l)$ be the slice of the region that contains all the points projected in l (for instance, if the longest dimension of R_i corresponds to the red band, $R_i(l)$ contains all points (l,x,y), such that $(l,x,y) \in R_i$);
7. Compute the 1-D histogram along the longest dimension of R_i. The 1-D histogram $h_i : L_i \rightarrow \mathbb{Z}_+$ is given by, for all $l \in L_i$,
$$h_i(l) = \sum_{x \in R_i(l)} \texttt{value}(x),$$
where $\texttt{value}(x)$ is the number of pixels that the color x appears in the original image.
8. Apply the graylevel quantization by morphological histogram processing to h_i. Reduce it to, at most, k classes. The choice of the k peaks is done by computing the most significative volume dynamics. The classification of h_i gives the points where R_i must be split. Let s be the number of regions that R_i will be split;
9. Split R_i;
10. Let $i \leftarrow i + s - 1$;
11. if $i < n$, go to Step 3.
Otherwise, stop.

The result of this algorithm is the classification of the RGB color space model in at most n parallelepiped regions. The color to be assigned to a region is given by the centroid point of all color points that belongs to the region.

The classified color space also works as a look-up table. To quantize a color from the original image, just check the region where the color belongs to and, then, change the color by the one assigned to that region.

Figures 4 and 5 show the application of the color quantization method to reduce Fig. 4 (a) to, respectively, $n = 64$ and $n = 16$ colors. The original image (Fig. 4 (a)) has 47915 distinct colors.

Figure 4 (b-c) shows the quantization to 64 colors with parameter $k = 2$ and $k = 4$, respectively. The visual result is a few better in Fig. 4 (b) than the result shown in Fig. 4 (c).

The quantization of Fig. 4 (a) to 16 colors is shown in Fig. 5. Figures 5 (b-e), show, respectively, the results achieved by the following choices of k: 2 (Fig. 5 (b)), 3 (Fig. 5 (c)), 4 (Fig. 5 (d)) and 5 (Fig. 5 (e)). In this example, the lower the value of k, the better the visual result.

There is a trade off in the choice of k: the lesser the value, the better the visual result. However, the greater the value of k, the faster the method converges to the results. If there is no much difference in the visual quality provided by several k values, as in the case shown in Fig. 4, the higher values may be a good choice.

4 EXPERIMENTAL RESULTS

The color quantization by morphological histogram processing is, at a few points, similar to the median cut algorithm (Heckbert, 1982), a classical quantization method in computer graphics and image processing context. Some experiments were done in order to compare the color quantization method proposed in this paper to the median cut one.

The results of each experiment will be assessed qualitatively, by assessing the visual quality of the resulting images, and quantitatively, by analyzing the results of a quantization error function. The quantization error function (Braquelaire and Brun, 1997) used in this paper is given by,
$$E = \sum_{i=1}^{n} \sum_{c \in C_i} f(c) \left\| c - c_i \right\|^2,$$
where n is the number of colors the image was quantized to, C_i is the set of colors in the original image image converted to color c_i, and $f(c)$ is the number of pixels in the image which color is c.

The goal in the first experiment is to reduce Fig. 6 (a) to $n = 256$ colors using the proposed method and the median cut to assess the visual quality of the results provided by them. Figure 6 (a) has 89648 distinct colors and $288 \times 451 = 129888$ pixels.

Figure 6 (b-c) show, respectively, the results provided by the color quantization by morphological histogram processing (using $k = 5$) and the median cut. Both visual results are very good, but the result provided by the morphological method (Fig. 6 (b)) still retained some small details from the original image.

The quantization errors computed to both quantized images are very close to each other. The error computed from the proposed method was $2.2968 \cdot 10^7$

(a)

(b)

(c)

Figure 4: Reduction to $n = 64$ colors: (a) Original Image. (b) $k = 2$. (c) $k = 4$.

(mean error of 176.8342 per pixel). The error computed from the median cut image was $2.4003 \cdot 10^7$ (mean error of 184.8007 per pixel). The proposed method achieved an error smaller than the achived by the median cut one.

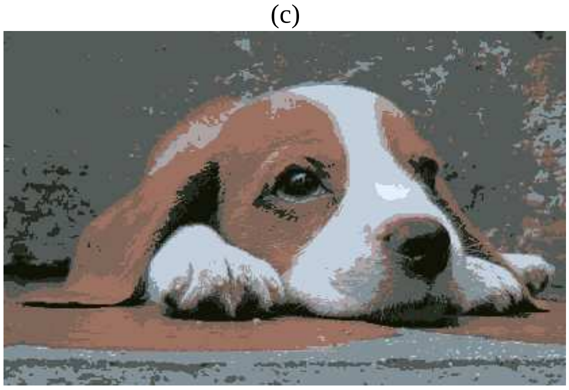
The second experiment consists in to reduce the same original image (Fig. 6 (a)) to $n = 16$ colors using the morphological method proposed in this paper and the median cut technique. Again, the visual quality of the results provided by them will be assessed.

The results provided by the color quantization by morphological histogram processing (using $k = 3$)

(a)

(b)

(c)



(c)

Figure 5: Reduction to $n = 16$ colors: (a) $k = 2$. (b) $k = 3$. (c) $k = 4$. (d) $k = 5$.

Figure 6: Reduction to $n = 256$ colors: (a) Original Image. (b) Color Quantization by Morphological Histogram Processing ($k = 5$). (c) Median Cut.

and the median cut are shown, respectively, in Fig. 7 (a-b). Both methods present results with a strong loss of quality in the visualization, but the result provided by the quantization method proposed in this paper (Fig. 7 (a)) has a visual quality far better than the provided by the median cut (Fig. 7 (b)).

The difference between the quantization errors is more evident in this experiment. Quantization error computed from the proposed quantization method was $1.4460 \cdot 10^8$ (mean error of 1113.2730 per pixel). Median cut error was $2.2306 \cdot 10^8$ (mean error of

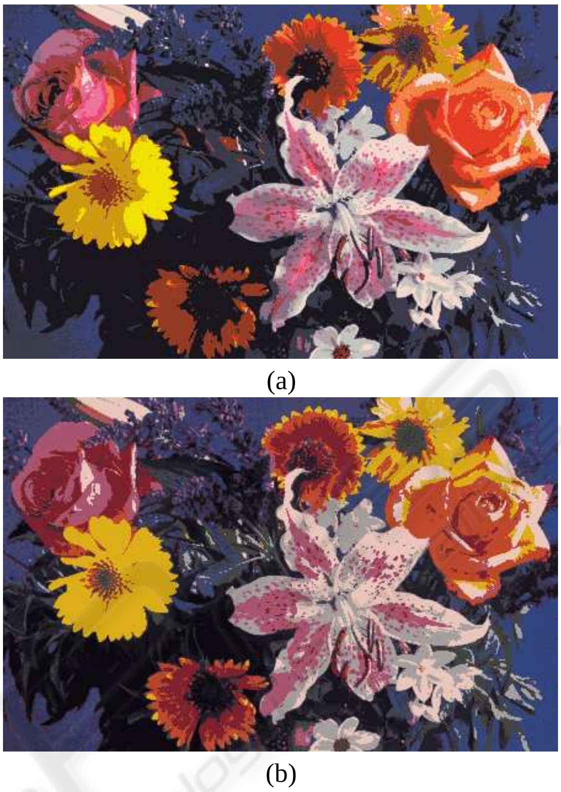


Figure 7: Reduction to $n = 16$ colors: (a) Color Quantization by Morphological Histogram Processing ($k = 3$). (b) Median Cut.

1717.3413 per pixel). In this experiment, the error achieved by the proposed quantization method was far lower than the error achieved by the median cut method.

5 CONCLUSIONS

Color quantization by morphological histogram processing, the extension of the graylevel processing proposed in a previous paper, reduces the colors of an image (under the RGB color space model) in a iterative process where a 3-D histogram computed from the image is split in at most n regions. This classified ”color space” is used as a look-up table to do the image quantization.

The splitting of regions is an iterative process. In each iteration, one region is split in at most k rectangular parallelepiped regions; it is done by choosing the longest side of the parallelepipe region, computing the histogram along this side and applying the graylevel method to this histogram. It provides how many regions are created in that iteration and where the region must be split.

The method introduced in this paper depends on a parameter k, that is the maximum number of regions a parallelepipe will be split in each iteration. The dependence of the parameter k is a drawback of the proposed method and some way to impose an automatic k value should be studied. Some quantization results using several k values are shown and discussed in the paper.

Experiments were done in order to compare the quantization method proposed in this paper with the classical median cut technique. In the first experiment, both methods provided good visual quality results but the morphological methods still retained a few details from the original image. The second experiment showed a strong loss of information in the application of both methods, but the color quantization introduced in this paper provided a better visual result. More, quantitative analysis was done in both experiments and the quantization error given by the application of the proposed quantization method was lower than the error given by the median cut one.

Future works include the choice of new criteria to choose the most significant peaks in the filtered histogram and the automatic choice of the k parameter.

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