RESOURCE ALLOCATION FOR OFDM SYSTEMS IN THE PRESENCE OF TIME-VARYING CHANNELS

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Keywords: OFDM, resource allocation, time-varying channel, channel estimation, channel prediction.

Abstract: Adaptive resource allocation schemes for OFDM systems designed assuming static channels are known to experience significant performance loss when the channel is time-varying. The inaccuracy in channel state information (CSI) due to outdated channel estimates has been recognized as the main reason for this problem. To mitigate this effect, we present robust bit and power allocation schemes based on the predicted channel state information. Simulation results show that channel prediction schemes can significantly improve the performance of resource allocation algorithms over time-varying channels.

1 INTRODUCTION

Adaptive resource allocation has been shown to result in significant performance improvement for OFDM systems over frequency selective channels (Chow et al., 1995; Pan et al., 2004; Krongold et al., 2000; Gao and Naraghi-Pour, 2006). In these studies it is assumed that the transmitter has *complete* and *perfect* knowledge of the channel state information (CSI). Then for each subcarrier a proper size of modulation signal set and transmit power is selected according to the channel frequency response such that the desired quality of service (QoS) can be achieved with the maximum spectral efficiency. In practice, however, CSI is obtained at the receiver through channel estimation and the noise in the received signal may cause estimation errors. Furthermore, the wireless channel is often time-varying and thus transmission and processing delay will make the CSI estimates outdated. Previous research (Ye et al., 2002; Leke and Cioffi, 1998; Wyglinski et al., 2004; Falahati et al., 2004) has confirmed that the performance of most adaptive modulation schemes assuming perfect knowledge of CSI will degrade significantly even with moderate errors in the estimated CSI. While the estimation error can be suppressed using efficient channel estimation techniques, for time-varying channels, the difficulties due to outdated CSI estimates remain.

In (Eyceoz et al., 1997) it is shown that the state of a frequency-flat fading channel can be reliably predicted from the previous observations across a long range of time. This motivates the prediction of frequency-selective channels for the OFDM system since, using OFDM, the wideband channel is transformed into a number of flat-fading sub-channels. In this paper we adopt the idea of CSI prediction and propose to perform resource allocation based on the predicted CSI.

The remainder of paper is organized as follows. Some preliminary results are presented in Section 2, in which we also motivate the need for channel prediction in resource allocation over time-varying channels. In Section 3, different channel predictors using Wiener filter or adaptive filters are discussed. Bit and power allocation based on the predicted CSI is discussed in Section 4 and the simulation results are presented in Section 5. Finally, conclusions are drawn in Section 6.

Naraghi-Pour M. and Gao X. (2007).
RESOURCE ALLOCATION FOR OFDM SYSTEMS IN THE PRESENCE OF TIME-VARYING CHANNELS.
In *Proceedings of the Second International Conference on Wireless Information Networks and Systems*, pages 117-124
DOI: 10.5220/0002150301170124

2 PRELIMINARY ANALYSIS

2.1 Channel Model

The equivalent lowpass impulse response of a time-varying, frequency-selective multipath fading channel can be written as

$$c(\tau;t) = \sum_{i=0}^{D-1} r_i(t)\delta(\tau-\tau_i) \tag{1}$$

where we assume that the path gains $r_i(t)$ are wide-sense stationary uncorrelated processes (WSS-US) (Proakis, 2000; Stuber, 2000). The WSS-US assumption implies that

$$E[r_i(t_1)r_j^*(t_2)] = \begin{cases} 0 & i \neq j \\ E[|r_i|^2]\rho(t_1-t_2) & i = j \end{cases} \tag{2}$$

where $\rho(t)$ denotes the normalized autocorrelation function of $\{r_i(t)\}$ and $E(\cdot)$ denotes expectation.

Consider an N-tone OFDM signal transmitted over the channel defined by (1). It is assumed that, when compared with the rate of the OFDM blocks, the variation of $r_i(t)$ is slow for all i. Thus, in this case, the inter-carrier interference (ICI) can be ignored, and the OFDM outputs can be represented by $y_{n,k} = h_{n,k}x_{n,k} + v_{n,k}$ for all $n = 0,1,\cdots,N-1$ and $k = \cdots,1,2,\cdots$, where $x_{n,k}$ and $y_{n,k}$ are, respectively, the n^{th} transmitted and received complex symbols of the k^{th} block. The sequence $\{h_{n,k}\}_{n=0}^{N-1}$ represents the channel frequency response (CFR) and $\{v_{n,k}\}_{n=0}^{N-1}$ is a sequence of iid complex Gaussian random variables with zero mean and fixed variance σ_v^2 for all k and n.

The size of the cyclic prefix (CP) used in this system is denoted by L, and the sampling time is T_S.

Let $\mathbf{h}(k) = [h_{0,k},\cdots,h_{N-1,k}]^T$. Then $\mathbf{h}(k) = \mathbf{F}_L\mathbf{g}(k)$, where $\mathbf{F}_L$ is the first L columns of the N-point DFT transform matrix and $\mathbf{g}(k) = [g_{0,k},\cdots,g_{L-1,k}]^T$ is the discrete time channel impulse response (CIR) for block k. We have

$$g_{l,k} = \sum_{i=0}^{D-1} r_i(kT_B)p(lT_S-\tau_i),\ \forall l = 0,1,\cdots,L-1, \tag{3}$$

where $p(\tau)$ denote the composite impulse response of the analog components in the OFDM system, T_S denotes the sampling time, and where $T_B = (N+L)T_S$. It is inferred from (3) that, although $r_i(t)$ are uncorrelated, the elements of $\mathbf{g}(k)$ are correlated. Therefore, in order to achieve optimal channel prediction, all elements of the outdated CIRs should be used to predict each element of the current CIR.

2.2 Motivation

In this section we illustrate the effect of outdated channel state information and motivate the need for channel prediction. For the purpose of resource allocation, a straightforward approach is to take the most recent estimate of CSI from the receiver, which is not only noisy but also outdated, as the predicted value of the current CSI, i.e., In other words, let

$$\tilde{\mathbf{g}}(k) := \hat{\mathbf{g}}(k-d) = \mathbf{g}(k-d) + \mathbf{e}(k-d),$$

where $\hat{\mathbf{g}}(k-d)$ is the estimated CIR at time $k-d$, $\tilde{\mathbf{g}}(k)$ is our prediction of CIR at time k and dT_B is the associated delay and $\mathbf{e}(k-d)$ is the channel estimation error.

The normalized mean square error (NMSE), defined by $\text{NMSE} := E(\|\tilde{\mathbf{g}}(k)-\mathbf{g}(k)\|^2)/E(\|\mathbf{g}(k)\|^2)$, is used to measure the difference between $\mathbf{g}(k)$ and $\tilde{\mathbf{g}}(k)$, where $\|\cdot\|$ is the L_2-norm. It is easy to show that $\text{NMSE} = \text{NMSE}_{\tilde{\mathbf{g}}(k)}$. Using the WSS assumption of the channel, the NMSE of the outdated CIR can be calculated as follows

$$\text{NMSE} = \frac{2E(\|\mathbf{g}\|^2) + E(\|\mathbf{e}\|^2) - 2\text{Re}\left[\sum_{l=0}^{L-1} E(g_{l,k-d}^* g_{l,k})\right]}{\text{E}(\|\mathbf{g}\|^2)} \tag{4}$$

Using (2) it can be shown that

$$\begin{aligned} E[g_{l,k-d}^* g_{l,k}] &= \\ \sum_{i=0}^{D-1}\sum_{j=0}^{D-1} & E[r_i^*((k-d)T_B)r_j(kT_B)p^*(lT_S-\tau_i)p(lT_S-\tau_j) \\ &= \rho(dT_B)\sum_{i=0}^{D-1} E[|r_i|^2]\,|p(lT_S-\tau_i)|^2 \\ &= \rho(dT_B)E[g_{l,k}^* g_{l,k}] \end{aligned} \tag{5}$$

Using (5) and (5) we can write

$$\text{NMSE} = 2\left[1-\text{Re}(\rho(dT_B))\right] + E(\|\mathbf{e}\|^2)/E(\|\mathbf{g}\|^2) \tag{6}$$

Equation (6) shows that if the outdated channel estimate is used as a prediction of the current CSI, the associated NMSE is not only determined by the signal to noise ratio (SNR) of the channel estimation method, but also the autocorrelation function $\rho(\cdot)$. It turns out that in this case the effect of the channel autocorrelation function is more significant than that of the estimation error.

For Rayleigh fading channels, $\rho(t) = J_0(2\pi f_m t)$ (Stuber, 2000), where $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind and f_m denotes the maximum Doppler frequency shift. Another commonly used autocorrelation function is $\rho(t) = e^{-\lambda f_m t}$. For

$\lambda \approx 2.8634$ this model has the same coherence time[1] as the Rayleigh fading model. The NMSEs calculated from (6) using these two correlation functions have been plotted with respect to $f_m dT_B$ in Figure 1, where the SNR in the CIR estimation, namely the value of $E(\|\mathbf{g}\|^2)/E(\|\mathbf{e}\|^2)$, is fixed to be 25dB. This figure illustrates that, when a delayed version of the estimated CIR is used for future CIR prediction, a small delay may result in large errors in CIR prediction even in cases where the estimation error is at an acceptable level (SNR=25dB). For comparison, in the same figure, we show the results of the CIR prediction using different predictors, which will be discussed in the next section.

3 CHANNEL PREDICTION

Channel prediction can be performed for either CFR or CIR. However, given the fact that the length of the CFR sequence (N) is often much larger than that of CIR (L), complexity of the predictor will be significantly lower for CIR prediction.

Due to the limited capacity of the feedback channel, it is assumed that only a down-sampled version (by a ratio of $d > 1$) of the CIR estimates at the receiver are fed back to the transmitter. Let $\hat{\mathbf{g}}(k-d), \hat{\mathbf{g}}(k-2d), \cdots, \hat{\mathbf{g}}(k-Md)$ be the CIR estimations available to the transmitter. The objective is to find an optimal prediction of $\mathbf{g}(k)$ based on these outdated samples. We rearrange the elements of these vectors into a single LM-by-1 vector $\mathbf{u}(k) := [\mathbf{u}_{0,k}^T, \cdots, \mathbf{u}_{L-1,k}^T]^T$, where $\mathbf{u}_{l,k} = [\hat{g}_{l,(k-d)}, \cdots, \hat{g}_{l,(k-Md)}]^T$. Then the linear minimum mean-squared error (MSE) prediction of $\mathbf{g}(k)$ is given by the Wiener-Hopf equation as follows.

$$\tilde{\mathbf{g}}_{opt}(k) = \left(\mathbf{R}_{\mathbf{uu}}^{-1}\mathbf{P}_{\mathbf{ug}}\right)^H \mathbf{u}(k) \tag{7}$$

where $\mathbf{R}_{\mathbf{uu}} = E[\mathbf{u}(k)\mathbf{u}(k)^H]$, $\mathbf{P}_{\mathbf{ug}} = E[\mathbf{u}(k)\mathbf{g}^H(k)]$, and $(\cdot)^H$ and $(\cdot)^{-1}$ denote, respectively, the matrix transpose conjugate and matrix inversion operations (Haykin, 2002). The definition of $\mathbf{u}(k)$ implies that $\mathbf{R}_{\mathbf{uu}} = [\mathbf{A}(l_1,l_2)]_{0\le l_1,l_2\le L-1}$, where $\mathbf{A}(l_1,l_2) = E[\mathbf{u}_{l_1,k}\mathbf{u}_{l_2,k}^H]$ is a square matrix. Similarly, $\mathbf{P}_{\mathbf{ug}} = [\mathbf{b}(l_1,l_2)]_{0\le l_1,l_2\le L-1}$, in which $\mathbf{b}(l_1,l_2) = E[\mathbf{u}_{l_1,k}g_{l_2,k}^*]$ is a column vector. The MSE associated with this predictor is given by

$$\mathrm{NMSE}_{opt} = 1 - \mathrm{Tr}(\mathbf{P}_{\mathbf{ug}}^H\mathbf{R}_{\mathbf{uu}}^{-1}\mathbf{P}_{\mathbf{ug}})/E[\|\mathbf{g}\|^2] \tag{8}$$

where $\mathrm{Tr}(\mathbf{A})$ is the trace of the matrix $\mathbf{A}$.

[1]The time over which the correlation coefficient is above 0.5

In many cases, when $l_1 \neq l_2$, the cross-correlation between g_{l_1,k_1} and g_{l_2,k_2} is relatively small and can be ignored. Thus, assuming $E(g_{l_1,k_1}g_{l_2,k_2}^*) = 0$ for $l_1 \neq l_2$, the matrices $\mathbf{R}_{\mathbf{uu}}$ and $\mathbf{P}_{\mathbf{ug}}$ can be rewritten as $\mathbf{R}_{\mathbf{uu}} = \mathrm{diag}\{\mathbf{A}(0,0), \cdots, \mathbf{A}(L-1,L-1)\}$ and $\mathbf{P}_{\mathbf{ug}} = \mathrm{diag}\{\mathbf{b}(0,0), \cdots, \mathbf{b}(L-1,L-1)\}$. Thus, (7) and (8) can be simplified as follows.

$$\tilde{\mathbf{g}}_{sub}(k) = \left\{\mathbf{b}^H(l,l)\mathbf{A}^{-1}(l,l)\mathbf{u}_{l,k}\right\}_{l=0}^{L-1} \tag{9}$$

$$\mathrm{NMSE}_{sub} = 1 - \sum_{l=0}^{L-1}\mathbf{b}^H(l,l)\mathbf{A}^{-1}(l,l)\mathbf{b}(l,l)/E[\|\mathbf{g}\|^2] \tag{10}$$

The structure in (9) is computationally much more efficient than that in (7) and will be adopted for the remainder of this paper.

3.1 Adaptive Channel Prediction

Using the L predictors in (9) requires the matrices $\{\mathbf{A}(l,l)\}_{l=0}^{L-1}$ and the vectors $\{\mathbf{b}(l,l)\}_{l=0}^{L-1}$. However, for a time-varying channel, frequent computation of these will be unrealistic. In this case, a more realistic approach is to use an adaptive prediction scheme such as least mean-square (LMS), recursive least-squares (RLS) or a Kalman filter, to replace each of the L Wiener filters in (9). In this case, the filter coefficients can be computed recursively. For the LMS predictor, the processing of the l^{th} branch of the predictor can be represented by

$$\tilde{g}_{l,k} = \mathbf{w}_{l,k}^H\mathbf{u}_{l,k} \tag{11}$$

$$\mathbf{w}_{l,k+d} = \mathbf{w}_{l,k} + \nu\left(\hat{g}_{l,k-d} - \mathbf{w}_{l,k}^H\mathbf{u}_{l,k-d}\right)^*\mathbf{u}_{l,k-d} \tag{12}$$

where $\mathbf{w}_{l,k}$ denotes the filter coefficients of the l^{th} branch at time $t = kT_B$, and ν is a positive constant denoting the step size. The choice of ν affects the convergence properties and the performance of the predictor and this has been thoroughly discussed in (Haykin, 2002).

Figure 1 also shows the performance of the optimal and suboptimal Wiener predictors ((7), (9)) and the LMS predictor ((11)-(12)) in terms of NMSE assuming a 12-ray channel model following (1), $D = 12$. The path delays $\{\tau_i\}$, which are in the interval $[0,5]$ μsec, and the power gain for each path are provided in Table 2.1 of (Stuber, 2000). All paths are assumed to undergo Rayleigh fading and have the normalized autocorrelation function $\rho(t) = J_0(2\pi f_m t)$. The OFDM parameters used are $N = 64$, $L = 16$, $T_S = 0.625\mu s$ ($T_B = 50\mu s$), and $p(\tau)$ is set to be the raised-cosine function with a roll-off factor of 0.35. The SNR of CIR estimations is set to 25dB, as in Section 2.2. In this case, d is fixed to be 10 and f_m varies between 0 and 240Hz. It is clear that, the two Wiener predictors have very close performance and are better than the LMS predictor. Moreover, even the LMS predictor shows significant improvement over that of using the outdated CSI.

4 BIT AND POWER ALLOCATION

In this section we consider the problem of optimal bit and power allocation for OFDM systems using the predicted values of CIR. Since the discussion is focused on a single block, the subscript indicating the block number is dropped. Let P_n and β_n, respectively, denote the power and the number of bits allocated to subcarrier n. The objective is to minimize the power allocated to the OFDM block while satisfying the BER (ε_{target}) and data rate (R_{target} bits per block) requirements. In this section the prediction error is treated as additive noise and is measured by NMSE.

4.1 Resource Allocation with Gaussian Prediction Error

We assume that the CIR prediction error $\mathbf{e} = (\tilde{\mathbf{g}} - \mathbf{g})$ is a complex-valued Gaussian random vector such that $E[\mathbf{e}] = \mathbf{0}_{L\times 1}$ and $E[\mathbf{e}\mathbf{e}^H] = \sigma_e^2 \mathbf{I}_{L\times L}$. This assumption is justified in light of the fact that the predictor is linear and that all fading components of the channel are assumed to follow a Gaussian distribution. In (Gao and Naraghi-Pour, 2006), we discussed the problem of bit and power allocation for the case of perfect CSI (i.e., $\sigma_e^2 = 0$). In this section, this is extended to the case of $\sigma_e^2 \neq 0$.

The instantaneous bit error probability for subcarrier n can be written as

$$\mathrm{BER}_n = c_1 \exp\left[-P_n |h_n|^2 q(\beta_n)\right] \tag{13}$$

where $q(\cdot)$ is a known function of β_n. It should be noted that the BER in (13) is evaluated using the channel CFR $\{h_n\}$, whereas the bit and power allocation is performed using the predicted values of CFR, namely $\{\tilde{h}_n\}$, which in turn is obtained from an N-point DFT of CIR predictions $\{\tilde{g}_n\}$.

For $\sigma_e^2 \neq 0$, h_n and thus BER_n are random variables. In this case, a constraint regarding system BER requirement is proposed as follows:

$$E(\mathrm{BER}_n | \tilde{h}_n) = \varepsilon_{target}, \ \forall n = 0, \cdots, N-1. \tag{14}$$

Using the assumption on channel prediction error being Gaussian, it can be shown that, given $\tilde{h}_n$, h_n is a complex-valued Gaussian random variable with mean $\tilde{h}_n$ and variance $L\sigma_e^2$. Using (13), (14) can be rewritten as:

$$\frac{c_1}{1 + L\xi_n\sigma_e^2} \exp\left[-\frac{|\tilde{h}_n|^2 \xi_n}{1 + L\xi_n\sigma_e^2}\right] = \varepsilon_{target}, \tag{15}$$
$$n = 0, 1, \cdots, N-1,$$

where $\xi_n = q(\beta_n)P_n$. The left hand side of (15) is monotone decreasing in ξ_n. Thus for a given ε_{target}, there exists a unique ξ_n^* satisfying (15). Let $|h_n^\dagger|^2 := \ln(\frac{c_1}{\varepsilon_{target}})/\xi_n^*$. Then P_n and β_n satisfy (15) if and only if

$$c_1 \exp[-P_n |h_n^\dagger|^2 q(\beta_n)] = \varepsilon_{target}.$$

Consequently $|h_n^\dagger|^2$ can be defined as the *effective power gain* of subcarrier n, and can be calculated as follows:

$$|h_n^\dagger|^2 =$$
$$L^2\sigma_e^4 \ln\left(\frac{c_1}{\varepsilon_{target}}\right) \Big/ \left[\frac{|\tilde{h}_n|^2}{\Psi^{-1}\left\{\frac{\varepsilon_{target}}{c_1}\Psi\left[\frac{|\tilde{h}_n|^2}{L\sigma_e^2}\right]\right\}} - L\sigma_e^2\right] \tag{16}$$

where $\Psi(x) = xe^x$ and $\Psi^{-1}(\cdot)$ is the inverse function of $\Psi(x)$.

By treating $|h_n^\dagger|^2$ as the perfect power gain for subchannel n, the bit and power allocation problem can be solved using known algorithms developed for the case of perfect CSI (Gao and Naraghi-Pour, 2006; Krongold et al., 2000). However, unlike the approach of directly using $|\tilde{h}_n|^2$, the proposed method guarantees that the BER satisfies the requirement in (14).

4.2 Resource Allocation with Arbitrary Channel Prediction Error

Our discussion in the previous section assumed that the values of CFR, $\{h_n\}$, are predicted. However, in bit and power allocation algorithm it is $\{|h_n|^2\}$, the channel power gain, (or more specifically the SNR of each subcarrier) that is required. As argued in (Falahati et al., 2004), using the square magnitude of $\tilde{h}_n$ as a prediction of the channel power gain underestimates the true power and results in a biased estimate. Consequently in (Falahati et al., 2004), an unbiased quadratic power prediction method from (Ekman et al., 2002) has been used for optimal rate and power allocation.

In this section we consider the problem of optimal bit and power allocation in OFDM assuming an arbitrary distribution for the channel CFR. In particular let $X_n := |h_n|^2$ and $Y_n := |\tilde{h}_n|^2$. It is assumed that the joint PDF of X_n and Y_n, denoted by $f_{X_n,Y_n}(x,y)$, is known, whether $\tilde{h}_n$ or $|\tilde{h}_n|^2$ is obtained through channel prediction. For all n, both X_n and Y_n are viewed as complex random variables. As before, P_n and β_n are, respectively, the power and the number of bits allocated to the n^{th} subchannel.

In the optimal bit and power allocation P_n and β_n are determined by the value of Y_n. Since BER also depends on the value of X_n, (see (13)), as in the previous section the BER constraint is considered as follows.

$$\begin{aligned} E\left(\text{BER}_n | Y_n = y\right) &= \int_0^\infty c_1 e^{-q(\beta_n)P_n x} f_{X_n|Y_n}(x;y)dx \\ &= \varepsilon_{target}, \ \forall n = 1, \cdots, N. \quad (17) \end{aligned}$$

Define

$$Z_n(z;y) = \int_0^\infty e^{-zx} f_{X_n|Y_n}(x;y)dx$$

Then, for each y, $Z_n(z;y)$ is a monotone decreasing function of z. Let $Z_n^{-1}(\cdot;y)$ denote its inverse function such that $Z_n(Z_n^{-1}(x;y);y) \equiv x$. Thus (17) can be rewritten as

$$P_n = \frac{1}{q(\beta_n)} Z_n^{-1}\left(\frac{\varepsilon_{target}}{c_1};y\right) \quad (18)$$

Let $\Omega = \{b_0, b_1, \cdots, b_{\mathcal{M}}\}$ be the set of integers that β_n can assume. Divide the interval $[0,\infty)$ into $\mathcal{M}$ consecutive subintervals with the boundary points $0 = \varphi_{0,n} < \varphi_{1,n} < \cdots < \varphi_{\mathcal{M},n} < \varphi_{\mathcal{M}+1,n} = \infty$. Then, let $\beta_n = b_m$ if the value of Y_n falls in the interval $(\varphi_{m,n}, \varphi_{m+1,n}]$. Finally, calculate P_n from (18) with $\beta_n = b_m$. The same procedure will be performed for all n to obtain resource allocation for the entire block. From (18), the transmit power of the OFDM block is given by

$$\begin{aligned} \bar{P} &= \sum_{n=1}^{N} \int_0^\infty \frac{1}{q(\beta_n)} Z_n^{-1}\left(\frac{\varepsilon_{target}}{c_1};y\right) f_{Y_n}(y)dy \\ &= \sum_{n=1}^{N}\sum_{m=1}^{\mathcal{M}} \frac{1}{q(b_m)} \int_{\varphi_{m,n}}^{\varphi_{m+1,n}} Z_n^{-1}\left(\frac{\varepsilon_{target}}{c_1};y\right) f_{Y_n}(y)dy, \end{aligned} \quad (19)$$

and the data rate is given by

$$\bar{R} = \sum_{n=1}^{N}\sum_{m=1}^{\mathcal{M}} b_m \int_{\varphi_{m,n}}^{\varphi_{m+1,n}} f_{Y_n}(y)dy = R_{target} \quad (20)$$

As mentioned previously, the bit and power allocation algorithm attempts to minimize the total power assigned to an OFDM block subject to the constraints on the BER and the data rate per OFDM block. It can be shown that there exist optimal boundary values $\{\varphi^*_{m,n}\}$ such that the transmit power in (19) can be minimized subject to (17) and rate constraint in $\bar{R} = R_{total}$. This problem can be solved using the method of Lagrange multipliers.

The Lagrange cost function is

$$\begin{aligned} &J(\varphi_{1,1}, \cdots, \varphi_{\mathcal{M},N}) \\ &= \sum_{n=1}^{N}\sum_{m=1}^{\mathcal{M}} \frac{1}{q(b_m)} \int_{\varphi_{m,n}}^{\varphi_{m+1,n}} Z_n^{-1}\left(\frac{\varepsilon_{target}}{c_1};y\right) f_{Y_n}(y)dy \\ &+\Lambda\left[\sum_{n=1}^{N}\sum_{m=1}^{\mathcal{M}} b_m \int_{\varphi_{m,n}}^{\varphi_{m+1,n}} f_{Y_n}(y)dy - R_{target}\right] \quad (21) \end{aligned}$$

where Λ is the Lagrange multiplier. The necessary conditions for optimality are given by

$$\left.\frac{\partial J}{\partial \varphi_{m,n}}\right|_{\varphi_{m,n}=\varphi^*_{m,n}} = 0, \ \forall m,n \quad (22)$$

which yield

$$Z_n^{-1}\left(\frac{\varepsilon_{target}}{c_1};\varphi^*_{m,n}\right) = -\frac{\Lambda(b_m - b_{m-1})}{\left[\frac{1}{q(b_m)} - \frac{1}{q(b_{m-1})}\right]} \quad (23)$$
$$\text{for } n = 1, \cdots, N; \ m = 1, \cdots, \mathcal{M}.$$

For a given value of Λ, we can obtain $\{\varphi^*_{m,n}\}$ by solving the above equations. For a given set of thresholds $\{\varphi^*_{m,n}\}$ we can obtain the value of Λ, from (20). In practice, a recursive numerical method can be used to solve for $\{\varphi^*_{m,n}\}$ and Λ.

Example 4.1 Suppose that the prediction of channel power gain is perfect, i.e., $Y_n = X_n$ for all n. In this case, $f_{X_n|Y_n}(x;y) = \delta(x-y)$ and $Z_n(z;y) = e^{-zy}$. Thus (23) can be reduced to

$$\varphi^*_{m,n} = \frac{\ln(\frac{\varepsilon_{target}}{c_1})}{\Lambda}\left[\frac{\frac{1}{q(b_m)} - \frac{1}{q(b_{m-1})}}{b_m - b_{m-1}}\right], \ \forall m, \ n. \quad (24)$$

Equation (24) shows that, for each subcarrier, the ratio of the optimal threshold values are fixed. A result which coincides with earlier results in (Gao and Naraghi-Pour, 2006; Krongold et al., 2000). Moreover, the values given by (24) are the same as those obtained in (Krongold et al., 2000), where these optimal threshold values are derived using a different approach. ∎

Example 4.2 Suppose $\tilde{h}_n = h_n + \eta_n$, where h_n and η_n are independent complex Gaussian random variables satisfying $h_n \sim \mathcal{CN}(0,\theta^2)$ and $\eta_n \sim \mathcal{CN}(0,\sigma_\eta^2)$, where θ^2 and σ_η^2 are given. When conditioned on $\tilde{h}_n$, h_n is a complex-valued Gaussian random variable with mean $\tilde{h}_n$ and variance σ_η^2. It is clear that given $Y_n = |\tilde{h}_n|^2$, $X_n = |h_n|^2$ has a *non-central* chi-square distribution with two degrees of freedom. Accordingly, $f_{X_n|Y_n}(x;y)$ can be written as

$$f_{X_n|Y_n}(x;y) = \frac{1}{\sigma_\eta^2}\exp\left(-\frac{x+y}{\sigma_\eta^2}\right) I_0\left(\frac{\sqrt{xy}}{\sigma_\eta^2/2}\right) \quad (25)$$

where $I_0(\cdot)$ denotes the zeroth order modified Bessel function of the first kind. For the conditional distribution defined in (25), the function Z_n can be rewritten as follows:

$$Z_n(z;y) = \frac{\exp\left[-(\frac{yz}{1+\sigma_\eta^2 z})\right]}{1+\sigma_\eta^2 z} \quad (26)$$

In this case, (23) can be simplified as

$$\varphi_{m,n}^* = \left[\frac{1}{\Lambda\Delta_m} + \sigma_\eta^2\right] \ln\left(\frac{c_1/\varepsilon_{target}}{1+\Lambda\sigma_\eta^2\Delta_m}\right) \quad (27)$$

$$\text{for all } n = 1,\cdots,N;\ m = 1,\cdots,\mathcal{M}\ .$$

where $\Delta_m = -(b_m - b_{m-1})/(\frac{1}{q(b_m)} - \frac{1}{q(b_{m-1})})$. It is observed that for $\sigma_\eta^2 = 0$, (27) reduces to (24). In general, the Lagrange multiplier Λ is determined by the constraint in (20). Specifically, if σ_η^2 and θ^2 are identical for all n, then (20) can be rewritten as follows:

$$\sum_{m=1}^{\mathcal{M}} \frac{b_m}{\theta^2}\left(e^{-\varphi_m^*/\theta^2} - e^{-\varphi_{m+1}^*/\theta^2}\right) = \frac{R_{target}}{N} \quad (28)$$

■

5 SIMULATION RESULTS

In this section, we first illustrate through simulation the efficacy of the resource allocation schemes proposed in Section 4. Meanwhile, the 12-ray channel model described in Section 3.1 has been used in this simulation, and the value of f_m is set to 240Hz. The other system parameters are the same as those in Section 3.1, in which $d = 10$. The set of modulation schemes used in this case are QPSK, 16QAM, 64QAM and 256QAM, and the target data rate is $R_{target} = 4N$. From Figure 1, the NMSE of the LMS predictor for $f_m = 240$Hz is about -15dB. Thus, for simplicity, we ignore the design of the channel predictor by assuming its output is a (Gaussian) noisy version of the CIR with NMSE=-15dB. In other words, the predicted CIR used for resource allocation is generated by adding to $\mathbf{g}(k)$ (which is calculated using (3)), a sequence of iid complex-valued Gaussian random variables, whose variance is determined by NMSE.

The allocation methods proposed in Sections 4.1 and 4.2 are compared with the results in (Gao and Naraghi-Pour, 2006) (which assumes perfect knowledge of the CSI). We should point out that if knowledge of the CSI is perfect, then the method in (Gao and Naraghi-Pour, 2006) is optimal. For these three approaches, the BER values measured from simulation are plotted vs. the target BER values in Figure 2. It is clear that both methods in Section 4 meet the BER requirement while the approach in (Gao and Naraghi-Pour, 2006) does not.

The spectral efficiency of the above three allocation methods is compared in Figure 3, where the results corresponding to the perfect CSI case (NMSE= $-\infty$ dB) is also plotted. For the method in Section 4.1, since, in the case of perfect CSI, the effective power gain $|h_n^\dagger|^2$ in (16) equals the predicted value $|\tilde{h}_n|^2$, then the scheme in Section 4.1 is equivalent to that in (Gao and Naraghi-Pour, 2006). Thus, only two curves exist in the case of perfect CSI. In the case of perfect CSI, the approach in Section 4.2 outperforms that in Section 4.1. This can be explained as follows: the method in Section 4.1 requires each OFDM block to transmit R_{target} bits, while the method in Section 4.2 has a more relaxed condition (20) and should result in higher efficiency. However, it should be noted that the scheme in Section 4.1 is easier to implement. For the case of imperfect CSI (NMSE=-15dB), the resource allocation scheme in Section 4.1 has about 2.5dB improvement over the method in (Gao and Naraghi-Pour, 2006) for $\varepsilon_{target} = 10^{-4}$, and the method in Section 4.2 is about 3.5dB better than that in (Gao and Naraghi-Pour, 2006).

We also simulated a complete OFDM system with both channel prediction and resource allocation. The same channel model and system parameters as those in previous simulations have been used in this system, where the range of f_m is $0-150$Hz and $R_{target} = 2N$. For every 20 OFDM blocks, one channel estimation is sent back to the transmitter, i.e., $d = 20$ in this case. The channel predictor used here is the LMS predictor in (11)-(12) with M=5.

Figure 4 illustrates the simulation results. A reference system without channel prediction, which uses the most recent channel estimation has also been simulated. The results of this case are labeled as "non-pred." in this figure. The system having channel prediction and the proposed resource allocation clearly outperforms the reference system under all Doppler frequencies. In this case, the efficiency of the proposed system is almost as good as that of perfect CSI for Doppler frequencies up to 100Hz. On the other hand, the reference system suffers tremendous performance loss even for $f_m = 50$Hz. For $\varepsilon_{target} = 10^{-3}$, the proposed system results in 2dB improvement over the reference system for $f_m = 50$Hz, 8dB improvement for $f_m = 100$Hz, and 10dB improvement for $f_m = 150$Hz.

6 CONCLUSION

The problem of resource allocation with imperfect CSI has been considered for OFDM systems and time-varying channels. The outdated CSI is identified as the main source of difficulty in achieving the performance enhancement promised by resource allocation techniques. With the aid of channel prediction, a bit and power allocation scheme has been proposed to overcome this difficulty. The simulation results confirm that, using the proposed method, the system performance for slowly time-varying channels (e.g., $f_m \leq 100$Hz in our simulation) can be very close to that of time-invariant channels.

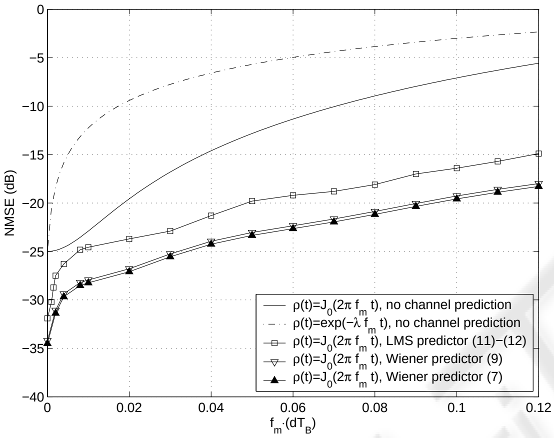

Figure 1: Performance of channel prediction in terms of NMSE.

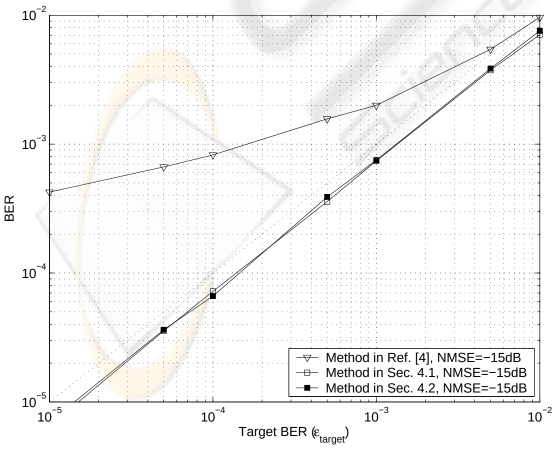

Figure 2: Comparison between the measured BER and the target BER.

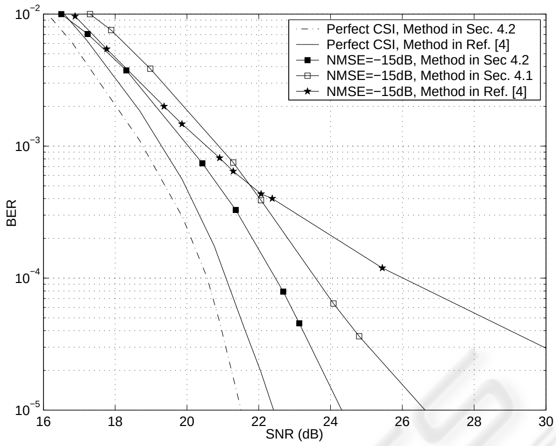

Figure 3: Performance of resource allocation schemes for imperfect CSI.

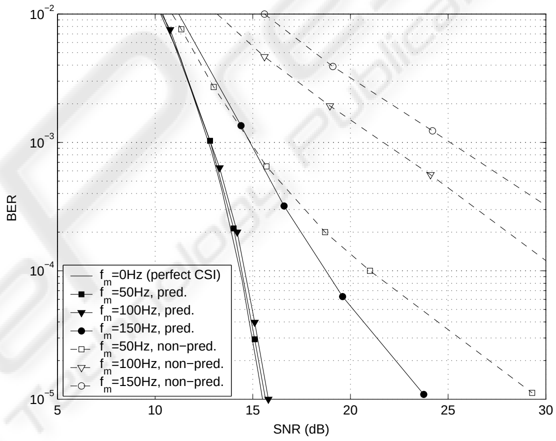

Figure 4: A comparison of predictive and non-predictive resource allocation schemes.

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