EFFICIENT PLACEMENT OF WIRELESS BASE-STATIONS IN URBAN ENVIRONMENT

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Abstract: This work proposes a novel approach for placing wireless base-stations in urban environments. The new approach solves the placement problem using 2-D convolution. Convolution searches for the best locations of base stations based on highest consumption criteria. Convolution theorem is then used to substantially cut the computation load of the proposed approach. The approach allows simple user-interface and arbitrary demand and supply patterns of power. Simulations show that the new approach can be used to efficiently model and place wireless base-stations.

1 INTRODUCTION

Placing wireless base-stations is a complex process which usually involves many parameters and requires long time to solve. Modeling the placement problem using conventional optimization techniques is complex and need expert knowledge of the considered technique. Moreover, the solution approach usually take long time for moderate grid-size resolution.

One of the first studies to solve the placement problem was in (Sherali and Rappaport, 1996). In this study, the solution of single as well as multiple transmitters problem was considered. The problem was modeled as a nonlinear program and then three nonlinear optimization algorithms were considered to solve this model. The work in (Hao Q., 1997) formulated the placement problem as a large-scale combinatorial optimization model. The model is then solved using the simulated-annealing approach. The Hata's propagation model (Hata, 1980) was used to determine the transmission loss in this study. Similar model was developed in (Calegari, 1997). The model in this work was solved using generic algorithms resulting in sub-optimal solutions. The work in (Park and Park, 2002) considered the determination of both BSs placement as well as transmission power. A simple weighted objective function was established. The real-coded generic algorithm was then used to obtain the solution. The solution takes into account the interference situation to determine the appropriate transmission power. Transmission power in wireless systems can also be adjusted via well-developed power control techniques (Aldajani and Sayed, 2003).

In this work, we present a new approach for placing wireless base stations in urban environment. The approach uses convolution as a core process to come up with a minimum number of base-stations such that the minimum coverage level is achieved. Fast algorithms for computing the convolution are then used to substantially reduce the computation load.

2 PROBLEM FORMULATION

The objective of the placement problem is to minimize the total number of base-stations N such that the net power inside a 2-D Euclidian space Γ is at least equal to the power threshold α at all locations. In other words,

$$p(x,y) \geq \alpha \qquad \forall \;\; x,y \in \Gamma \tag{1}$$

where $p(x,y)$ is the *net* power at the point with coordinates (x,y).

To find an expression for $p(x,y)$, let us define the quantity $s_n(x,y)$ as the power supplied by the n^{th} BS to the mobile station at location (x,y). This quantity

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indicates simply the signal strength at location (x,y) due to station n. This quantity is dependent mainly on the radio propagation and path loss model of the transmitter antenna. For example, for omni-directional antenna in free-space, the quantity $s_n(x,y)$ is given by the well-known Friis equation (Janaswamy, 2000)

$$s_n(x,y) = \overline{p}_n G_t G_r \left(\frac{\lambda}{4\pi h_n(x,y)}\right)^2 \tag{2}$$

where $\overline{p}_n$, G_t, G_r, and λ are the transmission power, transmitter gain, receiver gain, and wave length respectively. $h_n(x,y)$ is the distance between the point (x,y) and the base-station n.

Let us also introduce the term $d(x,y)$ to represent the demand level at point (x,y). This quantity can be used to model different priorities of coverage and extra signal attenuations at location (x,y).

Then we can define the net power p at location (x,y) as follows

$$p(x,y) = \max_{n=[1,N]} \{s_n(x,y)\} - d(x,y). \tag{3}$$

We assume here that the mobile station at location (x,y) will connect to the base station that delivers the *maximum* signal power.

The objective of the placement problem is to find the minimum number of base-stations and their locations that will satisfy the power constraint (1) where $p(x,y)$ is given by (3).

2.1 Discretization of the Model

The variables $p(x,y)$, $s_n(x,y)$, and $d(x,y)$ are discretized in 2-D Euclidian space to form the matrices P, S_n, and D respectively. Therefore, the optimization problem can be written in matrix format as

$$\min \quad N \tag{4}$$

subject to

$$P_N = \max_{n=[1,N]} \{S_n\} - D \geq \alpha \tag{5}$$

where P_N is the power pattern matrix of size $(I \times J)$ after assigning N base-stations, S_n is the power supply matrix of the n^{th} BS, and D is the demand pattern matrix. Notice that this constraint states that all the elements of the matrix P_N should be greater than the power threshold α.

The matrix S_n can be broken down into the convolution of two matrices as follows

$$S_n = X_n \otimes A \tag{6}$$

where the symbol $\otimes$ indicates the 2-dimensional convolution. The matrix A of size $(I_A \times J_A)$ is a fixed propagation pattern matrix of the transmitter radio antenna. The matrix X_n indicates the location of base-station n. If we denote this location by the coordinates (u_n, v_n) then X_n has all its elements equal to zero except at (u_n, v_n) where it equals to "1". In other words,

$$X_n(i,j) = \begin{cases} 1 & \text{at} \quad (u_n, v_n) \\ 0 & \text{elsewhere.} \end{cases} \tag{7}$$

Expression (6) means simply shifting the elements of the A matrix by (u_n, v_n).

Notice that minimizing the number of base-stations N is equivalent to minimizing the summation norm of the location matrices X_n for all base-stations. In view of this fact, the optimization problem can finally be written as

$$\min \quad \|\sum_{n=1}^{N} X_n\| \tag{8}$$

subject to

$$P_N = \max_{n=[1,N]} \{X_n \otimes A\} - D \geq \alpha. \tag{9}$$

3 SOLUTION OF THE PLACEMENT PROBLEM

A flow chart of the proposed algorithm is shown in Fig. 1. To determine the amount of power consumptions associated with placing a BS at a certain grid point, the antenna propagation matrix A is convolved with the existing power pattern P_{n-1} that resulted from previously assigned BSs, i.e.,

$$Y_n = A \otimes P_{n-1} \qquad P_0 = -D. \tag{10}$$

The role of the convolution here is as follows. For each point on the current power pattern P_{n-1}, the antenna propagation A is centered at that point and dot-multiplied with the intersecting sector of P_{n-1}. The multiplication values are then summed up and the answer is stored at the corresponding point in Y_n. This convolution process is repeated for all other points in P_{n-1}.

The coordinates that correspond to the minimum value of the matrix Y_n indicates the highest consumption. This point is chosen as the location of the n^{th} base-station,

$$(u_n, v_n) = \underset{(i,j)}{\text{argmin}} \;\; Y_n. \tag{11}$$

Once a new base-station location is chosen, the location matrix X_n is constructed from (7). The power matrix is then updated as follows

$$P_n = G_n - D, \qquad n = 1, 2, ..., N. \tag{12}$$

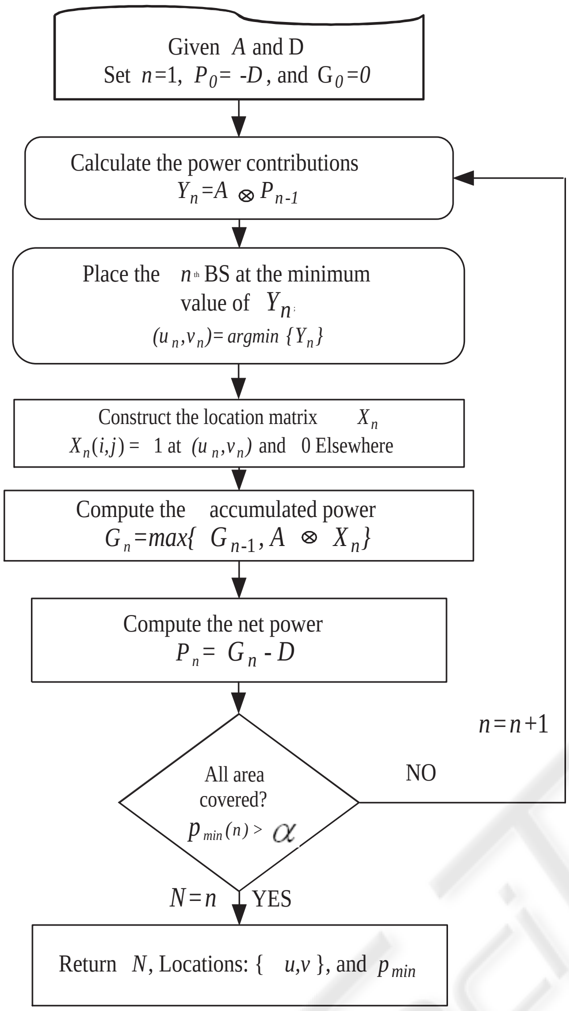

Figure 1: The proposed solution algorithm.

where G_n is the power pattern supplied by the stations 1 to n. This matrix can be computed iteratively from

$$G_n = \max\ \{G_{n-1}, S_n\} = \max\ \{G_{n-1}, A \otimes X_n\}, \qquad G_0 = \mathbf{0} \tag{13}$$

3.1 Penalizing Boundaries of the Demand Grid

Since the design space is always provided as a confined rectangular region, the demand matrix D need to be surrounded by a negative frame value w as shown in Fig. 2 to penalize the boundaries of the grid. The purpose of the penalty w is to push the locations of the BSs inward and therefore increase the coverage efficiency. In case of a tie, the algorithm will pick the value that results in higher p_{min}. In this way, not only the number of stations will be minimized but also the minimum power will be maximized reflecting an improved over-all coverage.

$$D = \begin{bmatrix} w & w & w & w & w & w & w \\ w & -5 & -5 & 0 & 0 & 0 & w \\ w & -5 & -5 & 0 & 0 & 0 & w \\ w & -5 & 0 & +10 & +10 & +5 & w \\ w & 0 & 0 & +10 & +10 & +5 & w \\ w & 0 & 0 & +10 & +10 & +5 & w \\ w & w & w & w & w & w & w \end{bmatrix}$$

Figure 2: Numerical example of the demand matrix D surrounded by the penalty frame value w.

3.2 Verification of the Solution

Since analytical solution for the placement problem given by (8) and (9) is not available, we follow two numerical approaches to verify the proposed solution. First, the algorithm is implemented on simple models where solutions are known and the results are then compared (Park and Park, 2002). Second, solution is verified by performing an exhaustive search on all possible locations.

4 SIMULATIONS

Matlab was used to implement the algorithm on a 2.1GHz personal computer with 256MB of memory. The Matlab program provides a friendly User Interface (UI). It inputs a color-coded map, similar to that of Fig. 3, in a common image format (JPEG) and then constructs the corresponding demand pattern matrix D. It also inputs the propagation pattern matrix A. It then computes the number of base-stations and their locations and then shows them on the color-coded image. The program also returns the final minimum power p_{min}, and the percentage coverage (PC) of each assigned base-station.

In our simulations, the size of the matrices D and A is fixed to 41×61 for each (corresponds to 2501 possible locations). Furthermore, the power threshold is arbitrarily fixed in all simulations to the normalized value $\alpha = 1\%$.

To test the algorithm, we considered the placement problem in (Park and Park, 2002). In this case, a configuration of seven hexagonal cells is to be covered with omni-directional antennas having the same radius as that of the cells. The solution for this problem is obvious: exactly seven BS are needed which

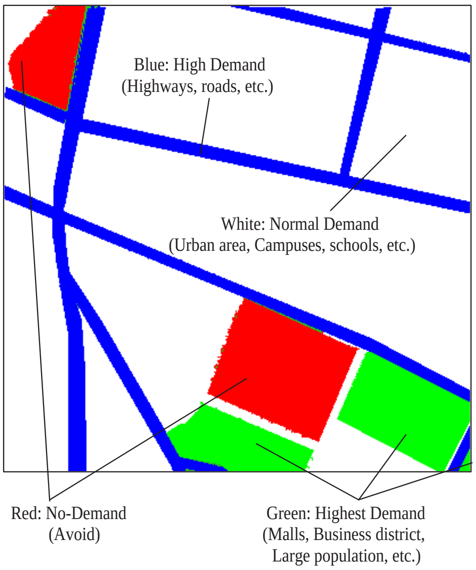

Figure 3: Example of designing the demand levels on a real map using color codes.

Table 1: Example of color codes and their corresponding values inside the demand matrix D .

Color	Demand	Value inside D
Green	*Highest*	$+20$
Blue	*High*	$+10$
White	*Normal*	0
Red	$No-Demand$	-20

should be located at the centers of the cells. To implement the proposed approach, the edges of the seven cells are drawn using popular drawing software and then feeded directly to the algorithm. The edges are mapped as negative values in D. The results are shown in Fig. 4. The algorithm achieved 99.7% coverage in seven iterations. Solving the same problem with Genetic Algorithm (GA), for example, would need more than 1000 generations to get the same coverage (Park and Park, 2002). Furthermore, the fact that the model can be built by simply drawing the cell boundaries and feeding the drawing to the algorithm makes the proposed approach much more attractive when compared to the cumbrous modeling process demanded by the GA.

In another experiment, the color-coded map of Fig. 3 is used in simulation to build a different non-trivial demand matrix D. The numerical weights assigned to the four colors in this example are listed in Table 1. An omni-directional propagation matrix A is used here. Fig. 5 shows the resulting placement

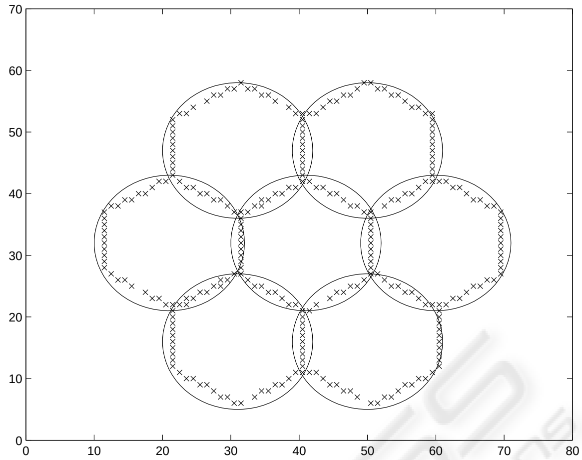

Figure 4: Solution of the hexagonal cells problem.

of the base-stations. In this case, six base-stations were sufficient to meet the coverage requirement. Notice that as expected, the first base-station was located at the green region (corresponds to very high demand). Also, the algorithm avoided the placement of any base-station at the red region (correspond to no-demand region). The minimum power returned by the algorithm is $p_{min} = 1.0724$ which is just above the required power threshold $\alpha = 1$. The optimal frame value w^* in this example is -86. The percentage coverage and accumulated percentage coverage for this example are shown in Fig. 6. This information can play very useful role for cell planners. Through this information, they have a choice to eliminate those stations on the map that have negligible coverage. The second base-station covered about 40% of the area while the 6th one covered about 1% only. This means that if 99% total coverage is sufficient, then the 6th station can simply be removed. The algorithm returned the results in less than 2 minutes.

5 COMPUTATION COMPLEXITY

From the discussions above, the proposed scheme has an outer loop as well as an inner loop. The outer loop searches for the optimal frame value w^* while the inner loop implements Fig. 1 to find the location of the base-stations.

For the outer loop, a simple line search was found sufficient to find w^*. The search is limited to the integer values in the range $[w_{min}, 0]$. Still, more efficient search algorithm could be adopted to find this value.

In the inner loop represented by Fig 1, the only

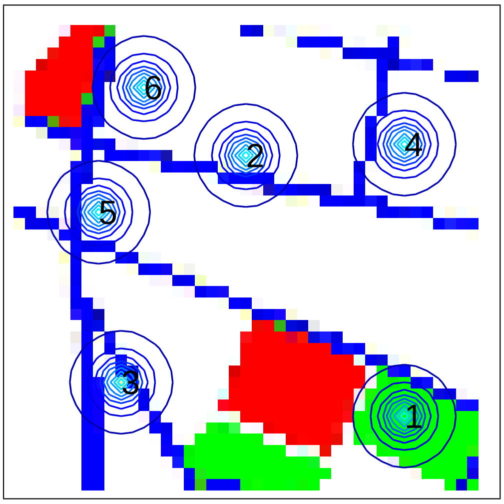

Figure 5: Result of the placement problem with omni-directional antenna and a given demand map.

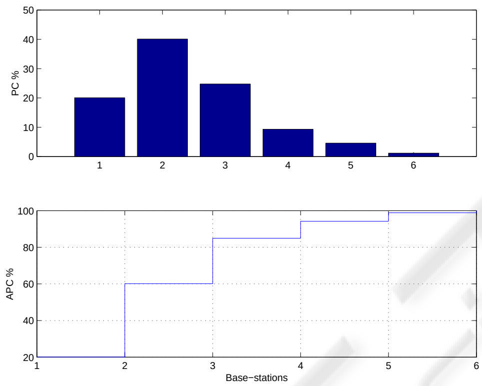

Figure 6: Percentage coverage (PC) and accumulative percentage coverage (APC) for the base-stations in Fig. 5.

computationally expensive operation is the convolution $A \otimes P_{n-1}$. As mentioned in the motivation section, the cost of the convolution can be reduced substantially from m^2 to $m\log(m)$ using available fast convolution techniques (Bernardini and Mian, 1996; Chiang and Chew, 2005). This feature makes the proposed solution feasible even for large grid-sizes.

Table 2 summarizes the order of complexity for the proposed approach. In Fig. 7, we show the simulated and theoretical time needed to assign one base-station using fast convolution, averaged over 100 runs.

Furthermore, the sparsity of the matrix P_n can be utilized to reduce the computations of the convolution. This matrix is usually full of zeros. Finally, the search space reduces as new stations are assigned. This can also reduce the computation load significantly.

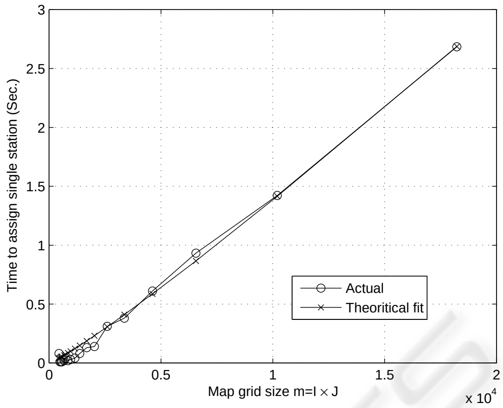

Figure 7: Simulated and theoretical time needed to assign single station as a function of the grid size $m = I \times J$, using fast convolution. The theoretical plot is obtained from $\beta m \log(m)$, where β is a processing constant.

Table 2: Order of complexity for the proposed approach.

Method used for convolution	**Order of Complexity**
Direct	m^2
Fast	$m\log(m)$

6 CONCLUSION

In this work, we proposed a new approach for placing the wireless base-stations. The new approach simplifies modeling and solution of the placement problem. Modeling the problem is performed by drawing color-codes on the map. Solution is obtained through the convolution process which searches for the highest consumption areas. Convolution theorem is then used to substantially reduce the computation load. Simulations of the proposed approach showed its efficiency and flexibility in solving the placement problem.

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