ESTIMATION OF STATE AND PARAMETERS OF TRAFFIC SYSTEM

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Abstract: This paper deals with the problem of traffic flow modelling and state estimation for historical urban areas. The most important properties of the traffic system are described. Then the model of the traffic system is presented. The weakness of the model is pointed out and subsequently rectified. Various estimation and identification techniques, used in the traffic problem, are introduced. The performance of various filters is validated, using the derived model and synthetic and real data coming from the center of Prague, with respect to filter accuracy and complexity.

1 INTRODUCTION

Intelligent traffic control is one possible way how to preserve or to improve capacity of current light controlled network. Generally, the problem can be solved by setting proper parameters of the signal lights. However, the suitable setting of these parameters is conditioned by exact knowledge of the current traffic situation at an intersection or micro-region[1].

Nowadays, when many intersection arms are equipped by detectors[2], the traffic situation can be sufficiently described by measurable intensity, occupancy, instant speed and hardly measurable queue length. Unfortunately, the knowledge of the queue length seems to be advantageous for a design of traffic control which can be based on the minimisation of the queue lengths (Kratochvílová and Nagy, 2004).

The key problem, either for estimation or control, is to specify the model of a micro-region. It is very interesting that the traffic situation can be described by a linear state space model (SSM) (Homolová and Nagy, 2005), where the directly immeasurable queue lengths are included in the state. Unfortunately, there are also some unknown parameters in the SSM, which cannot be determined from physical properties of the traffic situation and they have to be estimated as well.

Generally, there are two possibilities how to estimate the state and the parameters. The first possibility is based on an off-line identification of unknown parameters: prediction error methods (Ljung, 1999), instrumental variable methods (Söderström and Stoica, 2002), subspace identification methods (Viberg, 2002)) and subsequently on an on-line estimation of the state by the well-know Kalman Filter (KF) (Anderson and Moore, 1979). However, off-line identified time variant or invariant parameters represent the average values rather than the actual (true) parameters. The second possibility is based on the concurrent on-line estimation of the state and the parameters by suitable nonlinear estimation methods. There are two main groups of estimation methods for nonlinear systems, namely local and global methods (Sorenson, 1974). Although, the global methods are more sophisticated than local methods, they have significantly higher computation demands. Due to the computational efficiency, the stress will be mainly laid on the local methods, namely on the local derivative-free filtering methods (Nørgaard et al., 2000; Julier et al., 2000; Duník et al., 2005) and partially will be laid on a global method based on the Gaussian sums (Duník et al., 2005).

[1] One micro-region consists of several intersections with some detectors on the input and output roads. There must be at least one signal-controlled intersection.

[2] Detector is a inductive loop built in a cover of road, which is activated by a passing vehicle.

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The aim of this paper is to apply and to compare the various estimation techniques in the area of the estimation of queue lengths and parameters of traffic system and to choose a suitable estimation technique with respect to the estimation performance and computational demands.

2 TRAFFIC MODEL

2.1 Traffic Model and its Parameters

This paper deals with the estimation of immeasurable queue length[3] on each lane[4] of controlled intersections in a micro-region. Lane can be equipped by one detector on the output and three detectors on the input: (i) detector on stop line, (ii) outlying detector, (iii) strategic detector. Ideally, each lane has all three types of the detectors but in real traffic system, the lane is usually equipped by one or two types of such detectors, due to the constrained conditions. The strategic detectors, which are most remote from a stop line, give the best information about the traffic flow at present.

The detector is able to measure following quantities:

- *Intensity* $I_{i,t}$ is the amount of passing unit vehicles on arm i per hour $[uv/h]$.
- *Occupancy* $O_{i,t}$ is the proportion of the period when the detector is occupied by vehicles [%].

Traffic flow can be influenced by signal lights setting. A signal scheme can be modified by split, cycle time and offset:

- *Cycle time* is time required for one complete sequence of signal phases $[s]$.
- *Split* z_t is proportional duration of the green light in a single cycle time [%].
- *Offset* is the difference between the start (or end) times of green periods at adjacent signals $[s]$.

The geometry of intersections and drivers demands determine other quantities which are needed for a construction of the traffic model. These quantities are valid for a long time period. They are:

- *Saturated flow* S is the maximal flow rate achieved by vehicles departing from the queue during the green period at cycle time $[uv/h]$.

[3]Queue length ξ_t is a number of vehicles waiting to proceed through an intersection (given in unit vehicles [uv] or meters [m]) per cycle time. It is supposed that 1 uv = 6 m.

[4]Each intersection arm consists of one or more traffic lanes.

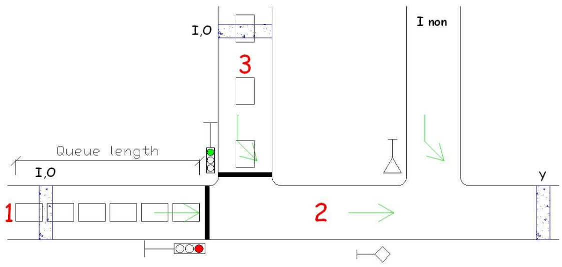

Figure 1: The micro-region: three-arm intersection with one unmeasured input.

- *Turning rate* $\alpha_{h,g}$ is the ratio of cars going from the h-th arm to the g-th arm [%].

The basic idea, which lies on the background of the model design, is the traffic flow conservation principle (Homolová and Nagy, 2005): "the new queue is equal to the remaining queue plus arrived cars minus departed cars".

The basic methodology of the traffic model design will be shown on a specific example. The micro-region consists of one three-arm controlled intersection with one unmeasured input, see Figure 1. Intersection is comprised of two one-way input arms (No.1 and 3) and one output arm (No. 2). The input arms are equipped by the strategic detectors and the output arm is equipped by the output detector. The unmeasured flow enters to the road No. 2 before the output detector. For the sake of the simplicity, the constant cycle time with two phases is considered.

In this case, the traffic system is described by the following model given by (1), (2), where $b_{i,t} = (1-\delta_{i,t}) \cdot I_{i,t} - \delta_{i,t} S_i$. Parameter $\delta_{i,t}$ is Kronecker function $(0\,,1)$, $\delta_{i,t} = 1$ if queue exist (on arm i at time t) and $\delta_{i,t} = 0$ in otherwise. Parameters $\kappa_{i,t}$, ϑ_t describe the relation between occupancy and queue length and parameter $\beta_{i,t}$ describes the relation between current and previous occupancy. The parameter $\lambda_{i,t}$ can be understood as a correction term to omit a zero occupancy. $I_{i,t}$ and $O_{i,t}$ are the input intensity and occupancy, respectively, measured by the input detectors. $Y_{i,t}$ is output intensity which is measured on the output detector. Mention that the subscript i stand for the number of intersection arm. The state and measurement noises are currently supposed to be Gaussian, i.e. $p(w_k) = \mathcal{N}\{w_k : 0, Q_k\}$ and $p(v_k) = \mathcal{N}\{v_k : 0, R_k\}$. The noise covariance matrices Q_k and R_k can be identified off-line by means of e.g. the prediction error method (Ljung, 1999) or the method based on the multi-step prediction (Šimandl and Duník, 2007). On-line noise covariance matrices estimation, so called adaptive filtering, has not been used due to the extensive computational demands.

Generally, traffic model can be described in matrix

$$\underbrace{\begin{bmatrix} \xi_{1,t+1} \\ \xi_{3,t+1} \\ O_{1,t+1} \\ O_{3,t+1} \end{bmatrix}}_{x_{t+1}} = \underbrace{\begin{bmatrix} \delta_{1,t} & 0 & 0 & 0 \\ 0 & \delta_{3,t} & 0 & 0 \\ \kappa_{1,t} & 0 & \beta_{1,t} & 0 \\ 0 & \kappa_{3,t} & \vartheta_t & \beta_{3,t} \end{bmatrix}}_{A_t} \cdot x_t + \underbrace{\begin{bmatrix} -b_{1,t} & 0 \\ 0 & -b_{3,t} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}}_{B_t} \cdot \underbrace{\begin{bmatrix} z_{1,t} \\ z_{2,t} \end{bmatrix}}_{z_t} + \underbrace{\begin{bmatrix} I_{1,t} \\ I_{3,t} \\ \lambda_{1,t} \\ \lambda_{3,t} \end{bmatrix}}_{F_t} + w_t \quad (1)$$

$$\underbrace{\begin{bmatrix} Y_{2,t+1} \\ O_{1,t+1} \\ O_{3,t+1} \end{bmatrix}}_{y_{t+1}} = \underbrace{\begin{bmatrix} -\alpha_{1,2} & -\alpha_{3,2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{C} \cdot x_{t+1} + \underbrace{\begin{bmatrix} \xi_{1,t} \\ \xi_{3,t} \\ 0 \\ 0 \end{bmatrix}}_{G_t} + v_{t+1} \quad (2)$$

form as follows:

$$x_{t+1} = A_t x_t + B_t z_t + F_t + w_t \quad (3)$$
$$y_{t+1} = C x_{t+1} + G_t + v_{t+1} \quad (4)$$

The last comment deals with the system initial condition. The starting time is chosen at early morning hours, when it can be supposed that there is no traffic in the micro-region and thus the system initial state is perfectly known and it is $x_0 = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T$.

2.2 Nonlinearities in Traffic Model

The traffic model becomes nonlinear in two main cases. The first case is when the traffic system has one or more unmeasured inputs or outputs (and the particular intensities should be estimated). The second case is when the parameters $\kappa_{i,t}$, $\beta_{i,t}$, $\lambda_{i,t}$, and ϑ_t, which cannot be determined from the physical properties and from construction dispositions of the micro-region, are estimated as a part of the state.

To find an actual estimates of the parameters, it is necessary to estimate them on-line. One of the possibilities is to extend the system state with these parameters $\tilde{x}_t = [x_t^T, \kappa_{1,t}, \kappa_{3,t}, \beta_{1,t}, \beta_{3,t}, \lambda_{1,t}, \lambda_{3,t}, \vartheta_t]^T$ (Anderson and Moore, 1979). This extension inevitably also leads to a nonlinear SSM

$$\tilde{x}_{t+1} = f_t(\tilde{x}_t, z_t) + w_t \quad (5)$$
$$y_{t+1} = \tilde{C}\tilde{x}_{t+1} + G_t + v_{t+1} \quad (6)$$

and to an application of appropriate nonlinear estimation techniques. The variables with tildes stand for the variables which had to be modified due to the extension of the state.

It should be also mentioned that the concurrent estimation of the state and parameters is also advantageous for the unusual traffic situations, e.g. accidents, when the estimator adapts the model and the estimated results are significantly more exact towards the model with invariant parameters.

3 STATE ESTIMATION TECHNIQUES

This section is devoted to a brief introduction of possible estimation methods which can be used for the estimation of traffic system state. With respect to the nature of this problem only, a special part of the estimation problem will be considered, namely the filtering.

The aim of the filtering is to find a state estimate in the form of the probability density function of the state x_t at the time instant t conditioned by the measurements $y^t = [y_0, y_1, \ldots, y_t]$ up to the time instant t, i.e. the conditional pdf $p_{x_t|y^t}(x_t|y^t)$ is looked for. General solution to the filtering problem is given by the Bayesian Recursive Relations (BRRs) (Anderson and Moore, 1979).

The exact solution of the BRRs is possible only for a few special cases, e.g. for linear Gaussian system (with known parameters). In other cases, such as linear system with unknown parameters, nonlinear and/or non-Gaussian systems, it is necessary to apply some approximative method, either local or global.

The local methods are often based on approximation of the nonlinear functions in the state or measurement equation so that the technique of the Kalman Filter design can be used for the BRRs solution. This approach causes that all conditional probability density functions (pdfs) of the state estimate are given by the first two moments. This rough approximation of posterior estimates induces local validity of the state estimates and consequently impossibility to ensure the convergence of the local filter estimates. The resulting estimates of the local filters are suitable mainly for point estimates. On the other hand, the advantage of the local methods can be found in the simplicity of the BRRs solution. Generally, there are two main approaches in the local filter design. The first possibility is to approximate the nonlinear function in the model by means of the Taylor expansion first or second order, which leads e.g. to the Extended Kalman Filter, or by means of the Stirling's polynomial interpolation, which leads to the Divided Difference Filter first or second order, abbreviated as (DD1), (DD2) or together as (DDFs) (Nørgaard et al., 2000; Duník et al., 2005). The second possibility, often used in the local filter design, is based on the approximation of state estimates by a set of deterministically chosen points. This method is known as the Unscented Transformation and its application in the local filter design leads to e.g. the Unscented Kalman Filter (UKF) (Julier et al., 2000; Duník et al., 2005).

The global methods are rather based on approximation of the conditional pdf of the state estimate of some kind to accomplish better state estimates.

Due to the higher computational demands of the global methods, the main stress will be laid on the local methods especially on the derivative-free local methods, namely the DD1, the DD2, and the UKF. The derivative-free methods were chosen because of there is no need of computations of derivatives of nonlinear functions (Duník et al., 2005) which is tedious especially for high dimensional systems like traffic systems. However, some attention will be paid on the Gaussian sum approach as a representative of global methods. Moreover, the KF with off-line identification methods will be considered as well.

4 ANALYSIS OF MODEL

In the previous sections, the model design, estimation and identification techniques were discussed and it was also mentioned that the quality of the model affects the estimation performance of all filters. From the more detailed analysis of the traffic model, it is evident that the estimated state has a backward impact on the model through the parameter δ_t. That is the main weakness of the model because δ_t depends on the queue length which is estimated. In other words, δ_t can be understood as a parameter which switches between two models representing peak and off-peak hours. The problem arises in the situations when the traffic flow is in the transition from off-peak to peak hours. Then, δ_t can be switched from 0 to 1 although the real traffic flow still corresponds to value 0 and vice versa, due to non-exact state estimate.

There are two possibilities how to rectify this problem. The first one is based on the modification of the state equation(s) describing the evolution of the queue length (the first two equations in (1)). The discontinuous equation is approximated by the continuous approximation based on the hyperbolic tangent, as it is depicted in Figure 2, where the relation

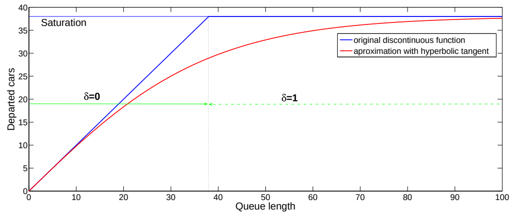

Figure 2: The approximation of the discontinuous function in the state equation.

between queue lengths and number of departed cars is shown. Then, the continuous approximation prevents from the bad switching of the models. Note that such approximation is done for all intersection arms. The second possibility is based on the Gaussian sum method and on the multi-model approach. It is still assumed that parameter δ_t belongs into the discrete set $\{0,1\}$ but at each time instant both values are used and the most probable value is looked for and then chosen. In case of more arms, all possible combinations of $\delta_{i,t}$ are tested and the most probable combination is chosen.

5 NUMERICAL ILLUSTRATIONS

In this section, the different micro-regions, either synthetic or real, will be described and the estimation task will be performed on each of them.

5.1 Synthetic Micro-regions

Micro-region with short queues: Let a micro-region consisting of one four-arm controlled intersection be considered, see Figure 3. All input roads are equipped by the strategic detectors. For estimation, the data from real traffic network supplemented with synthetic data was used. The queue lengths, supposed to be "true", and the missing output intensities were determined with simulation software AIMSUN[5].

The traffic model was built analogously to the model (1), (2). The original state has dimension $dim(x_t) = 8$ (queues and occupancies on four arms) and extended state has dimension $dim(\tilde{x}_t) = 24$ (original state and unknown parameters $\kappa_{i,t}$, $\beta_{i,t}$, $\lambda_{i,t}$, $\vartheta_{i,j,t}$, $i,j = 1,\ldots,4$).

All local filters show very similar estimation performance in the traffic problem. The reason can be found in a absence from significant nonlinearities (Duník et al., 2005). Thus the results of the DD1 will be presented only.

[5] AIMSUN is a simulation software tool which is able to reproduce the traffic condition of any traffic network.

The local filter will be applied on three variants of model: (A) standard model illustrated by (1), (2), (B) model with continuous approximation of δ functions, and (C) multi-model approach. The variant (A) works with δ as Kronecker function. In the variant (B), the switching of parameter δ is replaced by the approximation with hyperbolic tangent. In the variant (C), the model includes Kronecker function δ as well but switching is replaced by multi-modal approach (by "brute" force) where the state of models with all combinations of δ functions are estimated.

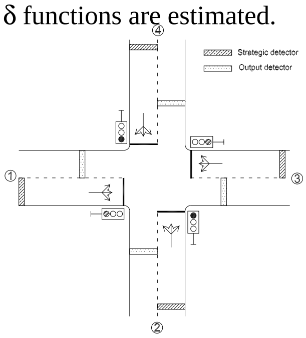

Figure 3: The micro-region: one four-arm controlled intersection.

Table 1 shows a comparison of all three variants with respect to the estimation performance for different types of traffic flows and to the computation load. The weak traffic flow is characterised by small intensities and on the other hand working days are rather characterised by high intensities. The estimation performance is measured by the Mean Square Error (MSE) of estimates queue length on one arm in $[uv^2]$. The average queue length is about 20 cars in all arms.

The best estimation performance, with respect to the MSE, shows the approach (C). On the other hand, the utilisation of the original model (A) leads to the least computational demand. With respect to Table 2, where the maximal differences between real and estimate queue are given, the best approach is multi-model. The same case is with the number of unsuitable values, which are defined as $\xi_{real} + 4 < \xi_{est}$.

For the needs of traffic control, the estimation method should be sufficiently exact (with small number of estimates which exceed allowable bound) and computational efficient. From the results, the best choice seems to be approach (B) and the DD1.

Table 1: Comparison of the different approaches with respect to the function δ (criterion no. 1).

	2 days (weekend) (1920 samples)		5 days (workweek) (4800 samples)	
	MSE	Time	MSE	Time
(A)	10.9	26 s	12.5	62 s
(B)	9.4	49 s	7.9	115 s
(C)	8.9	404s	7.1	858 s

Table 2: Comparison of the different approaches with respect to the function δ (criterion no. 2).

	Maximal difference	No. of unsuitable values (from 19200 data)	[%]
(A)	13.3	3284	17.0
(B)	15.7	1183	6.1
(C)	24.0	1086	5.7

Mention that the KF for micro-region with all measured arms provides little bit worse but comparable results with local filters and approach (A).

Micro-region with long queues: The typical micro-region has some arms unequipped by the detector. This situation together with a long queue lengths on the access roads will be illustrated in this part.

Let a micro-region consisting of one one-way three-arm controlled intersection and one unmeasured input given by (1), (2) be considered, see Figure 1. The two input roads have strategic detectors and one output road is equipped by an output detector. Moreover, the long queues on the access road will be considered to illustrate the situation with permanently engaged detectors which are not able to provide sufficient information about the current traffic flow.

For this simulation, the real data was used but the intensities were artificially increased. The "true" queue lengths were computed subsequently by means of simulation.

The four-dimensional state equation (1) includes eight a priory unknown parameters (κ_t, β_t, λ_t, ϑ_t for each measured input road). All these parameters and the immeasurable intensity can be estimated by the local filters. The KF is able to estimate the original state only and so for the application of the KF, the model was identified off-line by the prediction error method (Ljung, 1999) and the unmeasured intensity was considered as long time average value.

The results of the DD1 were compared with the KF results. The "true" and estimated queue lengths of both filters are depicted in Figure 4. It clearly shows the estimation improvement of the DD1, which is significantly better and perfectly matches the "true" queue on arm no. 3 contrary to the KF results.

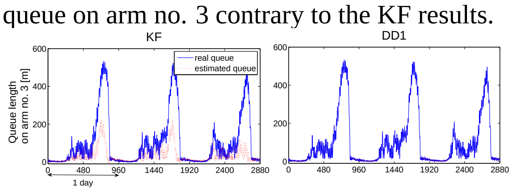

Figure 4: Comparison of the KF and local filter in the problem of queue estimation in the micro-region with unmeasured input intensity.

5.2 Real Micro-region

For the last experiment, the estimation of queue lengths was tested on data from the micro-region in

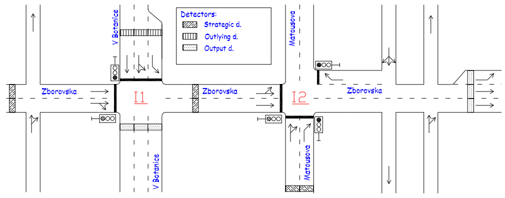

Figure 5: The micro-region: two four-arm controlled and three uncontrolled intersections.

Prague including two four-arm controlled intersections and three uncontrolled ones. The arms are one-way and they consist of several lanes, see Figure 5. Two input arms are equipped by strategic detectors and one input arm by outlying detector. Output detectors are in two arms.

For queue estimation, the state space model (3) and (4) without any approximation was used. The extended state is $dim(\tilde{x}_t) = 50$.

The input and output intensities, occupancies and green times were measured on the real traffic net during several months with sample period 90 sec. The "true" queue length was again determined with simulation software AIMSUN. Comparing of the estimated states and simulated ones shows that the estimation depends on the type of input detector. In lanes, which are equipped by the strategic detector, was usually $MSE \approx 17$, the error is about 10% with respect to the maximal queues. On the other hand, in lanes, equipped by the outlying detectors only, the good results was only in cases where the queue did not exceed the outlying detector. This is residual queue and for evaluation of traffic situation is not interesting.

The experiments show that the nonlinear estimation methods are a sufficient tool for estimation of the queue lengths, even in a real network. The dimension of the state, which is extended due to the estimation of parameters or intensities, increases the computation time, however, the computational demands remains still feasible (namely the DD1 and model variant (B)).

6 CONCLUSION

The problem of the queue length estimation, which is hardly measurable quantity, was considered in this paper. For the queue estimation, the mathematical model was presented, which describes the micro-region including its physical properties and taking into account behaviour of drivers. The disadvantage of the model was highlighted and two possible solutions of that were proposed. The theoretical results were illustrated by the numerical examples based on the synthetic or real data. It was shown that the Kalman Filter is suitable for situations where all quantities are measured. In other cases, it is advantageous to use a nonlinear filters for concurrent estimation of the state and parameters or possibly other unmeasurable quantities together with improved model.

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REFERENCES

Anderson, B. D. O. and Moore, S. B. (1979). *Optimal Filtering*. Englewood Cliffs, New Jersey: Prentice Hall Ins.

Duník, J., Šimandl, M., Straka, O., and Král, L. (2005). Performance analysis of derivative-free filters. In *Proceedings of the 44th IEEE Conference on Decision and Control, and European Control Conference ECC'05*, pages 1941–1946, Seville, Spain. ISBN: 0-7803-9568-9, ISSN: 0191-2216.

Homolová, J. and Nagy, I. (2005). Traffic model of a microregion. In *Preprints of the 16th IFAC World Congress*, pages 1–6, Prague, Czech Republic.

Julier, S. J., Uhlmann, J. K., and Durrant-White, H. F. (2000). A new method for the nonlinear transformation of means and covariances in filters and estimators. *IEEE Transactions On AC*, 45(3):477–482.

Kratochvílová, J. and Nagy, I. (2004). Traffic control of microregion. In Andrýsek, J., Kárný, M., and Kracík, J., editors, *CMP'04: Multiple Participant Decision Making, Theory, algorithms, software and app.*, pages 161–171, Adelaide. Advanced Knowledge Int.

Ljung, L. (1999). *System identification: theory for the user*. UpperSaddle River, NJ: Prentice-Hall.

Nørgaard, M., Poulsen, N. K., and Ravn, O. (2000). New developments in state estimation for nonlinear systems. *Automatica*, 36(11):1627–1638.

Söderström, T. and Stoica, P. (2002). Instrumental variable methods for system identification. *Circuits, Systems, and Signal Processing*, 21(1):1–9.

Sorenson, H. W. (1974). On the development of practical nonlinear filters. *Inf. Sci.*, 7:230–270.

Viberg, M. (2002). Subspace-based state-space system identification. *Circuits, Systems, and Signal Processing*, 21(1):23–37.

Šimandl, M. and Duník, J. (2007). Multi-step prediction and its application for estimation of state and measurement noise covariance matrices. Technical report. University of West Bohemia in Pilsen, Department of Cybernetics.