TRACKING CONTROL DESIGN FOR A CLASS OF AFFINE MIMO TAKAGI-SUGENO MODELS

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Abstract: When controlling Takagi-Sugeno fuzzy systems, verification of some sector conditions is usually assumed. However, setpoint changes may alter the sector bounds. Alternatively, setpoint changes may be considered as an offset addition in many cases, and hence affine Takagi-Sugeno models may be better suited to this problem. This work discusses a nonconstant change of variable in order to carry out offset-ellimination in a class of MIMO canonical affine Takagi-Sugeno models. Once the offset is cancelled, standard fuzzy control design techniques can be applied for arbitrary setpoints. The canonical models studied use as state representation a set of basic variables and their derivatives. Some examples are included to illustrate the procedure.

1 INTRODUCTION

In the last decade, design of fuzzy controllers based on the so-called Takagi-Sugeno TS models (Takagi and Sugeno, 1985) has reached maturity (Sala et al., 2005). TS models express the behaviour of a system via a convex interpolation of local (homogeneous) linear models, where the interpolation functions are fuzzy membership functions with and add-1 conditions. In particular, designs using the Linear Matrix Inequality framework (Tanaka and Wang, 2001; Guerra and Vermeiren, 2004) have become widespread. Part of the success of such techniques is due to the existence of systematic methodologies for TS fuzzy identification (Takagi and Sugeno, 1985; Tanaka and Wang, 2001; Babuska, 1998; Nelles et al., 2000).

One particular characteristic of the mainstream TS control design framework is that all of the local models must share the same equilibrium point, usually set to $x = 0$ for convenience. This is not a severe problem, as the identification procedures above referred need only be applied with a constant change of variable, used in the context of Taylor linearisation in control design for decades. Once that change of variable is carried out, global stability and performance regarding reaching $x = 0$ from any initial conditions can be proved. The reader is referred to (Tanaka and Wang, 2001; Guerra and Vermeiren, 2004) for details on the methodology.

An affine structure for TS models was also originally addressed in (Takagi and Sugeno, 1985), which considers local models without the shared equilibrium point. This structure, to be denoted as Takagi-Sugeno-Offset (TSO) may originate either directly from the identification process or when considering tracking tasks with varying setpoints in ordinary TS models. Indeed, in the latter case, the change of variable needed to transform the new operating point into $x = 0$ should involve changing the shape of the membership functions and the parameters of the local models. Otherwise, the resulting models lose the shared equilibrium point.

The above mentioned control methodologies must be adapted to TSO models. Some ideas appear in (Kim and Kim, 2002; Johansson, 1999), where quadratic Lyapunov functions and S-procedure LMIs (Boyd et al., 1994) are used to prove stability of the origin. However, setpoint changes are not considered, and the division into ellipsoidal zones of the operating regime results in a cumbersome procedure which involves considering the different regions of overlap of the antecedent membership functions.

As an alternative, this paper presents a particular

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class of TSO fuzzy models stemming from a canonical representation which, basically, takes the controlled outputs and its derivatives as the chosen set of state variables. This canonical representation has a clear physical insight and stems from usual canonical forms in linear and nonlinear systems (Antsaklis Panos and Michel Anthony, 1997; Slotine and Li, 1991).

Once the canonical TSO models are introduced, an offset-removing transformation for state feedback is discussed, which is the main result of this work. The offset-removing transformation will allow to express the original TSO model as an ordinary TS form on the transformed variables, enabling standard fuzzy TS control design techniques to be used in such a system.

The structure of the paper is as follows. Section 2 will present the definitions for the canonical TSO framework. Section 3 will present results on the equilibrium point of the canonical representation and the main offset-removing transformation. Some examples illustrating the approach will be given in Section 4, and a conclusion section will close the paper.

2 CANONICAL TSO MODELS

In this section, basic definitions of the fuzzy systems under study will be presented, which generalise the classical Takagi-Sugeno fuzzy system used in most current design techniques (Tanaka and Wang, 2001; Guerra and Vermeiren, 2004; Sala et al., 2005) described by

$$\dot{x} = \sum_{i=1}^{n} \mu_i(z)(A_i x + B_i u) \qquad \sum_i \mu_i(z) = 1 \tag{1}$$

where z is assumed to be a set of accessible variables which may include some or all of those ones comprising the state vector x plus external scheduling ones, and $\mu_i(z)$ are denoted as antecedent membership functions.

Definition 1 *Canonical local model with offset*
Let us have a system with p inputs, p outputs and n states defined by:

$$\begin{aligned} \dot{x} &= A \cdot x + B \cdot u + R \\ y &= C \cdot x \end{aligned} \tag{2}$$

where the state vector is assumed to be partitioned according to the following structure:

$$x = [x_{11}\ x_{12} \ldots x_{1r_1}\ x_{21}\ x_{22} \ldots x_{2r_2}\ \ldots\ x_{p1}\ x_{p2} \ldots x_{pr_p}]^T \tag{3}$$

i.e., *p blocks of size $r_1, \ldots, r_p$ respectively, $r_1 + \cdots + r_p = n$, compatible with the block structure in matrices A, B, C, R to be described below.*

Let us define an auxiliary matrix with dimension $q \times (q-1)$ as:

$$T_q = [0_{(q-1)\times 1}\ I_{q-1}] \tag{4}$$

where I_{q-1} denotes the identity matrix with size $(q-1) \times (q-1)$ and $0_{(q-1)\times 1}$ the zero matrix with size $(q-1) \times 1$. Also, the notation $[\ ]_{s\times t}$ will denote a matrix with dimension $s \times t$ with arbitrary elements. Then, matrices in (2) have the structure:

$$A = \left[\begin{array}{c|c|c|c} T_{r_1} & 0_{(r_1-1)\times r_2} & \ldots & 0_{(r_1-1)\times r_p} \\ \hline \multicolumn{4}{c}{[\ ]_{1\times n}} \\ \hline 0_{(r_2-1)\times r_1} & T_{r_2} & \ldots & 0_{(r_2-1)\times r_p} \\ \hline \multicolumn{4}{c}{[\ ]_{1\times n}} \\ \hline \multicolumn{4}{c}{\ldots} \\ \hline 0_{(r_p-1)\times r_1} & \ldots & 0_{(r_p-1)\times r_{p-1}} & T_{r_1} \\ \hline \multicolumn{4}{c}{[\ ]_{1\times n}} \end{array}\right] \tag{5}$$

$$B = \left[\begin{array}{c} 0_{(r_1-1)\times p} \\ \hline [\ ]_{1\times p} \\ \hline 0_{(r_2-1)\times p} \\ \hline [\ ]_{1\times p} \\ \hline \vdots \\ \hline 0_{(r_p-1)\times p} \\ \hline [\ ]_{1\times p} \end{array}\right] \tag{6}$$

$$C = \left[\begin{array}{c|c|c|c|c|c|c} \begin{matrix} 1 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{matrix} & [\ ]_{p\times(r_1-1)} & \begin{matrix} 0 \\ 1 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{matrix} & [\ ]_{p\times(r_2-1)} & \ldots & \begin{matrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 1 \end{matrix} & [\ ]_{p\times(r_p-1)} \end{array}\right] \tag{7}$$

$$R = \left[\begin{array}{c} 0_{(r_1-1)\times p} \\ \hline [\ ]_{1\times p} \\ \hline 0_{(r_2-1)\times p} \\ \hline [\ ]_{1\times p} \\ \hline \vdots \\ \hline 0_{(r_p-1)\times p} \\ \hline [\ ]_{1\times p} \end{array}\right] \tag{8}$$

Note 1 *The above system structure is similar to the well-known reachable canonical form (Antsaklis Panos and Michel Anthony, 1997). For instance, a canonical SISO system:*

$$A = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & \ldots & 1 \\ -a_1 & -a_2 & -a_3 & \ldots & -a_q \end{bmatrix} \tag{9}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & \ldots & b \end{bmatrix}^T \tag{10}$$

$$C = \begin{bmatrix} 1 & c_2 & c_3 & \dots & c_q \end{bmatrix} \tag{11}$$

$$R = \begin{bmatrix} 0 & 0 & 0 & \dots & r \end{bmatrix}^T \tag{12}$$

conforms to the above structure.

Definition 2 *Canonical Fuzzy Takagi-Sugeno-Offset model.*
A Fuzzy canonical Takagi-Sugeno-Offset model will be defined according to the following structure:

$$\begin{aligned} \dot{x} &= \sum_i^m \mu_i(z)(A_i \cdot x + B_i \cdot u + R_i) \\ y &= \sum_i^m \mu_i(z) C_i x \end{aligned} \tag{13}$$

where each of the component models has matrices A_i, B_i, R_i y C_i which follow the structure in Definition 1 and $\mu_i(z)$ are the membership functions, which are assumed to verify $\sum_i \mu_i(z) = 1$.

The notation below will be used as shorthand for fuzzy summations

$$\widetilde{\Omega}(z) = \sum_i \mu_i(z)\Omega_i \tag{14}$$

Then, the fuzzy system in Definition 2 may be written as:

$$\begin{aligned} \dot{x} &= \widetilde{A}(z) \cdot x + \widetilde{B}(z) \cdot u + \widetilde{R}(z) \\ y &= \widetilde{C}(z) x \end{aligned} \tag{15}$$

by using $\widetilde{A}(z) = \sum_i^n \mu_i(z) \cdot A_i$, $\widetilde{B}(z) = \sum_i^n \mu_i(z) \cdot B_i$, etc.

3 OFFSET-ELLIMINATION PROCEDURE

Proposition 1 *Given a system with the structure in Definition (1), for constant inputs $u = u^{eq}$, the equilibrium values of the state variables verify*

$$x_{ij}^{eq} = 0 \quad i = 1,\dots,p \;\; j = 2,\dots,r_i \tag{16}$$

Proof: With $u = u^{eq}$, the equilibrium equation is:

$$0 = Ax^{eq} + Bu^{eq} + R \tag{17}$$

Using the canonical matrix structure, the states $x_{i2} \dots x_{ir_i}$ $\forall$ i verify:

$$0_{(r_i-1)\times 1} = I_{(r_i-1)\times(r_i-1)} \begin{bmatrix} x_{i2}^{eq} \\ \vdots \\ x_{ir_i}^{eq} \end{bmatrix} + 0_{(r_i-1)\times p} \begin{bmatrix} u_1^{eq} \\ \vdots \\ u_p^{eq} \end{bmatrix} + 0_{(r_i-1)\times 1} \tag{18}$$

Hence,

$$0 = I_{(r_i-1)\times(r_i-1)} \cdot \begin{bmatrix} x_{i2}^{eq} \\ \vdots \\ x_{ir_i}^{eq} \end{bmatrix} \quad i = 1,\dots,p \tag{19}$$

finally obtaining (16).

Proposition 2 *Given a system with the structure in Definition 1, with constant input $u = u^{eq}$, the equilibrium values for the state vector and the output are related, in the form:*

$$x^{eq} = [y_1^{eq} \; 0_{1\times(r_1-1)} \; y_2^{eq} \; 0_{1\times(r_2-1)} \; \dots \; y_p^{eq} \; 0_{1\times(r_1-1)}]^T$$

$$y^{eq} = [y_1^{eq} \; \dots \; y_p^{eq}]^T$$

Proof: Replacing x^{eq} in the output equation,

$$y^{eq} = C \cdot x^{eq} \tag{20}$$

given the structure of C (7) and the results from the previous proposition, stating that only the states corresponding to the columns of the identity may be nonzero, we have:

$$y^{eq} = \begin{bmatrix} x_{11}^{eq} \\ x_{21}^{eq} \\ \vdots \\ x_{p1}^{eq} \end{bmatrix} \tag{21}$$

and finally,

$$x^{eq} = [y_1^{eq} \; 0_{1\times(r_1-1)} \; y_2^{eq} \; 0_{1\times(r_2-1)} \; \dots \; y_p^{eq} \; 0_{1\times(r_p-1)}]^T$$

Lemma 1 *Given a canonical fuzzy system with the structure in Definition 2,* i.e.,

$$\begin{aligned} \dot{x} &= \widetilde{A}(z) \cdot x + \widetilde{B}(z) \cdot u + \widetilde{R}(z) \\ y &= \widetilde{C}(z) x \end{aligned} \tag{22}$$

defining an auxiliary input

$$u_{est}(z, y_{ref}) = (\widetilde{C}(z)\widetilde{A}(z)^{-1}\widetilde{B}(z))^{-1}(-y_{ref} - \widetilde{C}(z)\widetilde{A}(z)^{-1}\widetilde{R}(z)) \tag{23}$$

under suitable invertibility assumptions, and carrying out the change of variable

$$\hat{x} = x - x_{ref} \tag{24}$$

$$\hat{u} = u - u_{est}(z, y_{ref}) \tag{25}$$

where

$$x_{ref} = [y_{ref,1} \; 0_{1\times(r_1-1)} \; y_{ref,2} \; 0_{1\times(r_2-1)} \; \dots \; \dots \; y_{ref,p} \; 0_{1\times(r_1-1)}]^T \tag{26}$$

and

$$y_{ref} = [y_{ref,1} \; \dots \; y_{ref,p}] \tag{27}$$

is a user-defined vector, then $\hat{x} = 0$, $\hat{u} = 0$ is the new equilibrium point for the system in the variables $\hat{x}$, $\hat{u}$ and, moreover, the transformed system has the equations:

$$\dot{\hat{x}} = \sum_i \mu_i(z)(A_i\hat{x} + B_i\hat{x}) \tag{28}$$

i.e., *it is a standard Takagi-Sugeno fuzzy system* (1), *where the offset terms have dissapeared. The variables $\hat{x}$, $\hat{u}$ will be denoted as* incremental.

Proof:

Let us denote by S_{z_0} the local affine system formed by the result of evaluating $\widetilde{A}$, $\widetilde{B}$, $\widetilde{C}$ and $\widetilde{R}$ at a particular (arbitrary) point z_0:

$$\begin{aligned}\dot{\xi} &= \widetilde{A}(z_0)\cdot\xi + \widetilde{B}(z_0)\cdot u + \widetilde{R}(z_0)\\ y &= \widetilde{C}(z_0)\xi\end{aligned} \tag{29}$$

which has the canonical structure of definition (1).
Let us compute the input $u_{est} = u_{est}(z_0, y_{ref})$ so that output y_{ref} is an equilibrium point for the above system S_{z_0}, by using the equilibrium equation:

$$0 = \widetilde{A}(z_0)\cdot\xi_{ref} + \widetilde{B}(z_0)\cdot u_{est} + \widetilde{R}(z_0) \tag{30}$$

$$y_{ref} = \widetilde{C}(z_0)\cdot\xi_{ref} \tag{31}$$

On the following the dependence on z_0 will be omitted for notational simplicity. Carrying out some operations,

$$\begin{aligned}\widetilde{B}\cdot u_{est} &= -\widetilde{A}\cdot\xi_{ref} - \widetilde{R}\\ \widetilde{C}\widetilde{A}^{-1}\widetilde{B}\cdot u_{est} &= -\widetilde{C}\xi_{ref} - \widetilde{C}\widetilde{A}^{-1}\widetilde{R} = -y_{ref} - \widetilde{C}\widetilde{A}^{-1}\widetilde{R}\\ u_{est} &= (\widetilde{C}\widetilde{A}^{-1}\widetilde{B})^{-1}(-y_{ref} - \widetilde{C}\widetilde{A}^{-1}\widetilde{R})\end{aligned} \tag{32}$$

Let's now obtain the equilibrium state ξ_{ref}. Indeed, as $u_{est}(z_0, y_{ref})$ ensures that the output y_{ref} is an equilibrium point, and as S_{z_0} has the structure 1, then Proposition 2 ensures that ξ_{ref} is:

$$\xi_{ref} = [y_{ref,1}\ 0_{1\times r_1}\ y_{ref,2}\ 0_{1\times r_2}\ \ldots\ y_{ref,p}\ 0_{1\times r_1}]^T$$

identical to the state x_{ref} defined in (26). As z_0 in (30) is an arbitrary point, then

$$0 = \widetilde{A}(z)\cdot x_{ref} + \widetilde{B}(z)\cdot u_{est}(z, y_{ref}) + \widetilde{R}(z) \quad \forall\, z \tag{33}$$

Carrying out the change of variable

$$\hat{x} = x - x_{ref}$$

$$\hat{u} = u - u_{est}(z, y_{ref})$$

and using (33), the system equations may be written as

$$\begin{aligned}\dot{x} &= \widetilde{A}x + \widetilde{B}u + \widetilde{R} = \widetilde{A}x + \widetilde{B}\hat{u} + \widetilde{B}u_{est} + \widetilde{R}\\ \dot{x} &= \widetilde{A}x + \widetilde{B}\hat{u} - \widetilde{A}x_{ref} - \widetilde{R} + \widetilde{R} = \widetilde{A}(x - x_{ref}) + \widetilde{B}\hat{u}\\ \dot{x} &= \widetilde{A}\hat{x} + \widetilde{B}\hat{u}\end{aligned}$$

If y_{ref} is considered as a constant setpoint ($\dot{y}_{ref} = 0$), then $\dot{x}_{ref} = 0$ and, hence $\dot{\hat{x}} = \dot{x}$. So the fuzzy system in the new variables results in:

$$\dot{\hat{x}} = \widetilde{A}\hat{x} + \widetilde{B}\hat{u} = \quad \textstyle\sum_i \mu_i(z)(A_i\hat{x} + B_i\hat{x})$$

whose equilibrium point is $\hat{x} = 0$, corresponding to x_{ref} (and output y_{ref}) in the original non-incremental variables. □

The system in the new variables has its offset term removed and a standard fuzzy controller may be designed on it, such as the ones in (Tanaka and Wang, 2001) using LMI techniques, which will be used in the examples below. The control action for the original system will be computed by adding to the resulting control action the term $u_{est}(z, y_{ref})$.

Note also that, with an ordinary TS system and a setpoint $y = 0$, the result is $u_{est} = 0$, hence the proposed framework encompasses the standard one. For the canonical systems, it is more powerful, however, as setpoint changes can be immediately accommodated as the examples below will illustrate.

4 EXAMPLES

In this section, a set of examples showing the possibilities of the proposed approach will be presented. First, the control of a standard TS fuzzy system with no offset will be extended to varying operation points, in order to compare with the results applying usual methodologies involving a constant change of variable. Then, a second example will illustrate the proposed methodology in a MIMO case.

Example 1 *Let us have a standard, offset-free system defined by:*

$$\begin{aligned}\dot{x} &= \sum_{i=1}^{2} \mu_i(z)(A_i x + B_i u)\\ y &= Cx\end{aligned} \tag{34}$$

with the two models given by:

$$A_1 = \begin{bmatrix} 0 & 1 & 0\\ 0 & 0 & 1\\ 1 & 2 & 1\end{bmatrix} \tag{35}$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0\\ 0 & 0 & 1\\ 4 & 8 & 4\end{bmatrix} \tag{36}$$

$$B_1 = B_2 = \begin{bmatrix} 0 & 0 & 1\end{bmatrix}^T \tag{37}$$

$$C = \begin{bmatrix} 1 & 0 & 0\end{bmatrix} \tag{38}$$

and membership functions $\mu_i(z)$, defined on $z = x_1 + 2x_2 + x_3$ as the trapezoidal partition depicted in Figure 1. Figure 2 shows the nonlinearity in the system as a function of z. Note that conditions to design a standard PDC controller (Tanaka and Wang, 2001) for the TS system to reach the origin are fulfilled. For instance, an LMI methodology

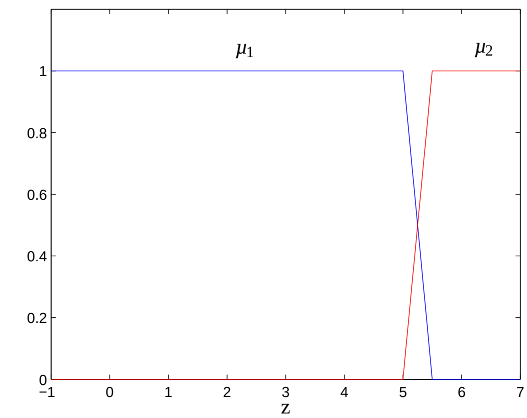

Figure 1: Membership functions.

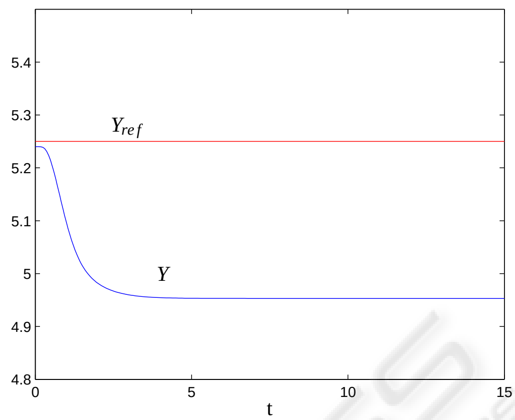

Figure 3: System output Y.

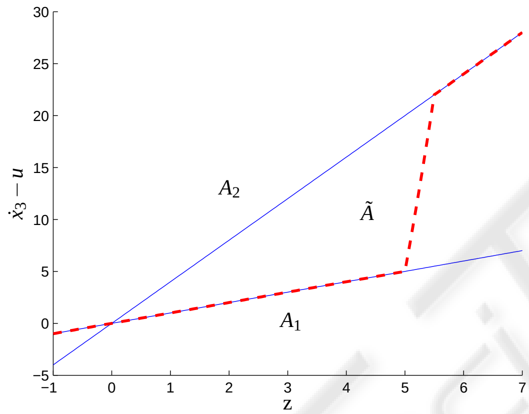

Figure 2: Nonlinearity in $\dot{x}_3 - u$.

(Tanaka and Wang, 2001) may be applied. To achieve a decay rate α, the following LMIs must be verified:

$$-XA_1^T - A_1X + M_1^TB_1^T + B_1M_1 - 2\alpha X > 0 \quad (39)$$
$$-XA_2^T - A_2X + M_2^TB_2^T + B_2M_2 - 2\alpha X > 0 \quad (40)$$
$$-XA_1^T - A_1X - XA_2^T - A_2X + M_1^TB_2^T + \quad (41)$$
$$+B_2M_1 + M_2^TB_1^T + B_1M_2 - 4\alpha X > 0 \quad (42)$$

where

$$X = P^{-1}, \; M_1 = F_1X, \; M_2 = F_2X \quad (43)$$

being P a quadratic matrix defining a Lyapunov function and F_1 and F_2 the state feedback gains to be implemented, *i.e.*, the control action:

$$\hat{u} = -(\mu_1(z)F_1 + \mu_2(z)F_2)\hat{x} \quad (44)$$

A set of LMI conditions (decay $\alpha = 1$) for the above system yields the controller:

$$F_1 = \begin{bmatrix} 27.5203 & 28.7108 & 8.4221 \end{bmatrix} \quad (45)$$
$$F_2 = \begin{bmatrix} 30.5203 & 34.7108 & 11.4221 \end{bmatrix} \quad (46)$$
$$\tilde{F}(z) = \sum_{i=1}^{2} \mu_i(z)F_i \quad (47)$$

which, as expected, behaves correctly when reaching the origin (figure not shown for brevity).

However, when trying to stabilise the system around a new operating point ($u_{ref} = -13.125$, $y_{ref} = 5.25$, $x_{ref} = (5.25, 0, 0)^T$), Lyapunov conditions no longer hold as the linearised model at that point has slopes out of those given by the vertices of the model above: in order to use a standard methodology in that case, redefining the local models would be needed. Indeed, with the constant change of variable $\hat{u} = u + 13.125$, $\hat{x} = x - x_{ref}$ (the usual one to achieve $x = 0$ as the operation point in many linear and fuzzy techniques), the resulting controller

$$u = -13.125 + \tilde{F}(z)(x - (5.25, 0, 0)^T) \quad (48)$$

yields an unstable equilibrium point: Figure 3 shows how initial conditions in the vicinity of the desired target drift away to another region of the state space.

On the contrary, the proposed methodology in this work provides a controller valid for all operating points with no modifications of the LMI conditions. Figure 4 shows the non-constant u_{est} calculated with lemma 1, which replaces the constant value -13.125 in (48) above. In that way, the resulting loop has the desired operating point as a stable equilibrium with the desired decay rate, as shown in Figure 5.

Example 2 *This example will demonstrate the methodology on a 5th order MIMO Takagi-Sugeno-Offset system with two unstable local models given by:*

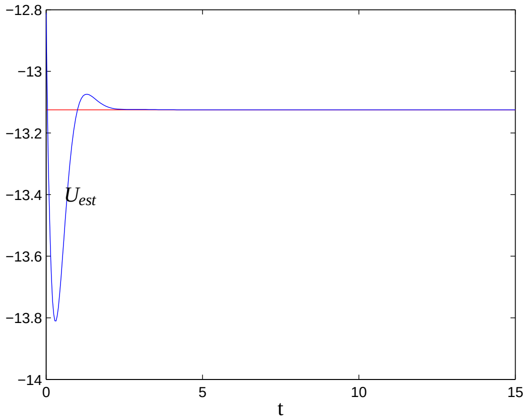

Figure 4: Non-constant u_{est} control action.

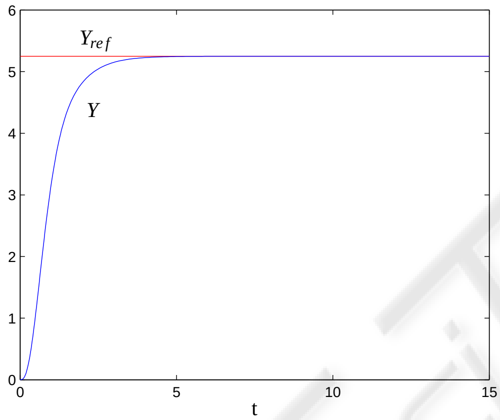

Figure 5: System output Y with non-constant u_{est}.

$$\begin{aligned} \dot{x} &= \sum_{i=1}^{2} \mu_i(z)(A_i(x - x_{i0}) + B_i(u - u_{i0})) \\ y &= \sum_{i=1}^{2} \mu_i(z) C_i x \end{aligned} \tag{49}$$

where

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 4 & 2 & -5 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & -1 & -5 & -5 & -1 \end{bmatrix} \tag{50}$$

$$B_1 = \begin{bmatrix} 0 & 0 & 2 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}^T \tag{51}$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 5 & 1 & 0.1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & -1 & -1 & -2 & -1 \end{bmatrix} \tag{52}$$

$$B_2 = \begin{bmatrix} 0 & 0 & 10 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}^T \tag{53}$$

$$C_1 = \begin{bmatrix} 1 & 0.1 & 0.05 & 0 & -0.1 \\ 0 & -0.2 & 0.5 & 1 & -0.1 \end{bmatrix} \tag{54}$$

$$C_2 = \begin{bmatrix} 1 & 0.2 & 0.1 & 0 & -0.2 \\ 0 & -0.1 & -0.5 & 1 & 0.2 \end{bmatrix} \tag{55}$$

$$x_{10} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \tag{56}$$

$$u_{10} = \begin{bmatrix} 0 & 0 \end{bmatrix}^T \tag{57}$$

$$x_{20} = \begin{bmatrix} 2 & 0 & 0 & 2 & 0 \end{bmatrix}^T \tag{58}$$

$$u_{20} = \begin{bmatrix} 1 & 3 \end{bmatrix}^T \tag{59}$$

and the membership functions $\mu_i(z)$, defined on $z = x_1 + x_4$ as:

$$\mu_1(z) = \begin{cases} 1 & z < 0 \\ 1 - 0.25z & 0 \le z \le 4 \\ 0 & z > 4 \end{cases} \tag{60}$$

being $\mu_2(z) = 1 - \mu_1(z)$.

Let us group the different equilibrium points of each local model into an offset term:

$$R_i = -A_i x_{i0} - B_i u_{i0} \tag{61}$$

Hence, the system follows the structure in Definition 2. So the control action $u_{est}(z, y_{ref})$, computed via (23), and the equilibrium state are

$$x_{ref} = [y_{ref1}\ 0\ 0\ y_{ref2}\ 0]^T \tag{62}$$

$$u_{est} = (\widetilde{C}\widetilde{A}^{-1}\widetilde{B})^{-1}(-y_{ref} - \widetilde{C}\widetilde{A}^{-1}\widetilde{R}) \tag{63}$$

where y_{ref1} and y_{ref2} are arbitrary user-defined setpoints for the two plant outputs. As usual, the change of variable removes the offset terms so the system may be expressed as $\dot{\hat{x}} = \sum_{i=1}^{2} \mu_i(z)(A_i\hat{x} + B_i\hat{u})$. For a decay of $\alpha = 0.5$, the LMI Control Toolbox in Matlab obtains:

$$F_1 = \begin{bmatrix} 2.305 & -1.104 & -0.758 & 2.048 & 1.181 \\ 19.95 & 48.12 & 3.575 & -22.27 & -11.56 \end{bmatrix}$$

$$F_2 = \begin{bmatrix} 1.985 & -1.718 & -0.798 & 2.338 & 1.257 \\ -0.792 & 45.89 & 12.89 & -31.82 & -15.57 \end{bmatrix}$$

Hence the actual control action to be applied to the plant, after inverting the change of variable is:

$$u = u_{est}(z, y_{ref}) - (\mu_1(z)F_1 + \mu_2(z)F_2)(x - x_{ref}) \tag{64}$$

Figure 6 shows the system response approaching a setpoint $y_{ref} = [1\ 1]^T$. The usefulness of the obtained controller for setpoint changes is shown in Figure 10.

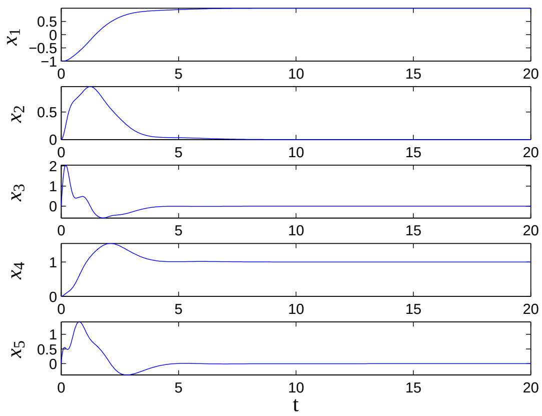

Figure 6: Time response of the state variables.

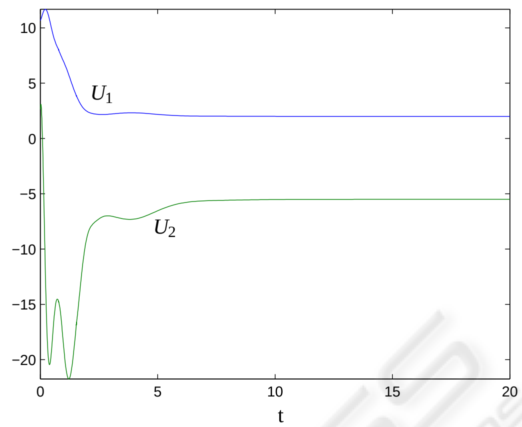

Figure 9: Actual overall control action U.

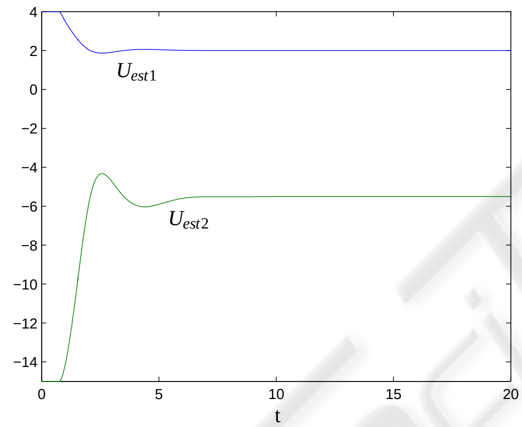

Figure 7: Offset removing term U_{est}.

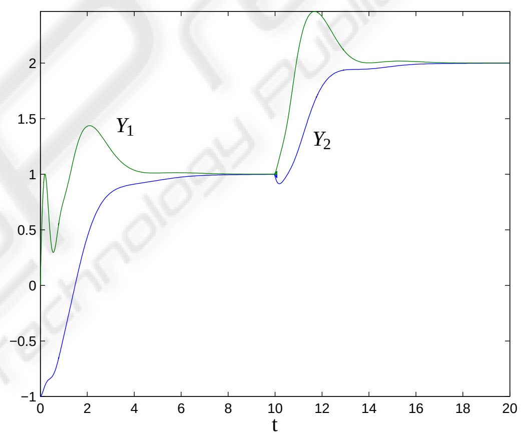

Figure 10: System output Y with a setpoint change to $Y_{ref} = (2,2)^T$.

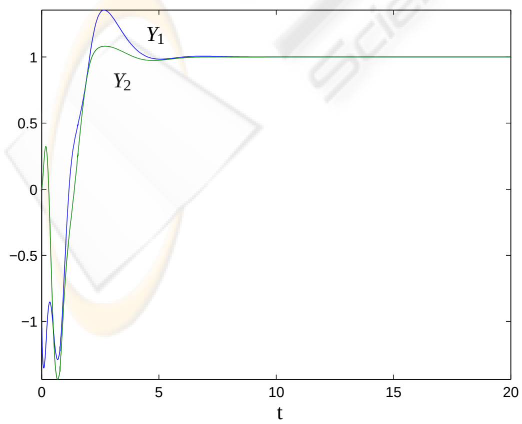

Figure 8: System output Y.

5 CONCLUSIONS

This paper presents an offset-ellimination change of variable which applies to fuzzy Takagi-Sugeno-offset models with local linear models with a particular canonical structure. The canonical structure may be obtained, for instance, by taking as state variables the outputs and its derivatives.

As a result, a transformed system with equilibrium at $\hat{x} = 0$ is obtained. The difference with standard changes of variable is that it is non-constant in time. As a result, the offset is neatly removed and the resulting transformed system has the same representation for any desired setpoint, and well-known control design techniques for fuzzy non-offset Takagi-Sugeno systems may be directly applied independently of the chosen setpoint. Interestingly, as a particular case, the procedure applies to setpoint changes in standard offset-free Takagi-Sugeno models.

The presented results apply to a state feedback setting. Further research should be devoted to generalising the procedure to situations with output feedback and noisy measurements.

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