# BLIND TWO-THERMOCOUPLE SENSOR CHARACTERISATION

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Abstract: Thermocouples are one of the most popular devices for temperature measurement in many mechatronic implementations. However, large wire diameters are required to withstand harsh environments and consequently the sensor bandwidth is reduced. This paper describes a novel algorithmic compensation technique based on blind deconvolution to address this loss of high frequency signal components using the outputs from two thermocouples. In particular, a cross-relation blind deconvolution for parameter estimation is proposed. A feature of this approach, unlike previous methods, is that no *a priori* assumption is made about the time constant ratio of the two thermocouples. The advantages, including small estimation variance, are highlighted using results from simulation studies.

## 1 INTRODUCTION

There is a growing trend towards the integration of different types of sensors and actuators with information processing (Isermann, 2005). Commercial and industrial applications increasingly demand dynamic temperature measurement when advanced mechatronic components are incorporated. In the automotive industry for example, accurate and reliable measurement of exhaust gas temperature is required for the regeneration of diesel particulate filters (DPF), and for the evaluation of the combustion performance of internal combustion engines (Kee and Blair, 1994).

Fast response temperature measurement can be performed using techniques such as Coherent Anti-Stokes Spectroscopy, Laser-Induced Fluorescence and Infrared Pyrometry. However, these are expensive, difficult to calibrate and maintain and are therefore impractical for wide-scale deployment outside the laboratory (Hung *et al.*, 2005a). Thermocouples are widely used for temperature measurement due to their high permissible working limit and good linear temperature dependence. In addition, their low cost, robustness, ease of installation and reliability means that there are many situations in which thermocouples are indeed the only suitable choice. Unfortunately, their design involves a compromise between robustness and speed of response which poses major problems when measuring temperature fluctuations with high frequency signal components.

To remove the effect of the sensor on the measured quantity in such conditions, compensation of the thermocouple measurement is desirable. Usually, this compensation involves two stages: thermocouple characterisation followed by temperature reconstruction. Reconstruction is a process of restoring the unknown fluid temperature from thermocouple outputs using either software techniques or hardware. This paper will concentrate on the first stage, since effective and reliable characterisation is essential for achieving satisfactory temperature reconstruction.

In an attempt to improve existing characterisation of thermocouples, this paper proposes a novel technique based on the cross-relation method (Liu *et al.*, 1993) from the field of blind deconvolution put forward by Sato (1975). Compared to other algorithms, simulations suggest



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that the proposed method gives estimations with lower variance even in environments with moderate amount of noise.

This paper is organised as follows: Section 2 introduces the background of two-thermocouple characterisation. Section 3 proposes the cross-relation method and shows how it can be applied to this problem. Simulation results are presented in Section 4 while conclusions follow in Section 5.

# 2 DIFFERENCE EQUATION SENSOR CHARACTERISATION

## 2.1 Thermocouple Modelling

Assuming some criteria regarding to the construction of thermocouples are satisfied (Forney and Fralick, 1994; Tagawa and Ohta, 1997), a first-order lag model with time constant $\tau$ and unity gain can represent the frequency response of a fine-wire thermocouple (Petit, 1982). This simplified model can be written mathematically as

$$T_{\text{fluid}}(t) = T(t) + \tau \dot{T}(t) \; . \qquad (1)$$

Here the original fluid temperature $T_{\text{fluid}}$ can be reconstructed if $\tau$ , the thermocouple output $T(t)$ and its derivative are available. In practice, this direct approach is infeasible as $T(t)$ contains noise and its derivative is difficult to estimate accurately. More importantly, it is generally not possible to obtain a reliable *a priori* estimate of $\tau$ , related to their thermocouple bandwidth $\omega_B$

$$\tau = \frac{1}{\omega_B} , \qquad (2)$$

which, in turn, is a function of thermocouple wire diameter $d$ and fluid velocity $v$

$$\omega_B \propto \sqrt{\frac{v}{d^3}} \; . \qquad (3)$$

Hence, $\tau$ varies as a function of operating conditions. Clearly, a single-thermocouple does not provide sufficient information for *in situ* estimation. Equation (3) highlights the fundamental trade-off that exists when using thermocouples. Large wire diameters are usually employed to withstand harsh environments such as engine combustion systems, but these results in thermocouples with low bandwidth, typically $\omega_B$ < 1 Hz. In these situations high frequency temperature transients are lost with the thermocouple output significantly attenuated and phase-shifted compared to $T_{\text{fluid}}$. Consequently, appropriate compensation of the thermocouple measurement is needed to restore the high frequency fluctuations.

## 2.2 Two-Thermocouple Sensor Characterisation

In 1936 Pfriem suggested using two thermocouples with different time constants to obtain *in situ* sensor characterisation. Since then, various thermocouple compensation techniques incorporating this idea have been proposed in an attempt to achieve accurate and robust fluid temperature compensation (Forney and Fralick, 1994; Tagawa and Ohta, 1997; Kee *et al.*, 1999; Hung *et al.*, 2003, 2005a, 2005b). However, the performance of all these algorithms deteriorates rapidly with increasing noise power, and many are susceptible to singularities and sensitive to offsets (Kee *et al.*, 2006). It would be very useful from the implementation point of view to know when the characterisations are not reliable.

Some of these two-thermocouple methods rely on the restrictive assumption that the ratio of the thermocouple time constants $\alpha$ ($\alpha < 1$ by definition) is known *a priori*. Hung *et al.* (2003, 2005a, 2005b) develop difference equation methods that do not require any *a priori* assumption about the time constant ratio.

The equivalent discrete time representation for the thermocouple model (2) is:

$$T(k) = aT(k-1) + bT_{\text{fluid}}(k-1) \, , \qquad (4)$$

where $a$ and $b$ are difference equation ARX parameters and $k$ is the sample instant. The discrete time equivalent of $\alpha$ is defined as

$$\beta = b_2 / b_1 \, , \quad \beta < 1 \, . \qquad (5)$$

Here subscripts 1 and 2 are used to distinguish between signals from different thermocouples. Assuming ZOHs and a sampling interval $\tau_s$ , the parameters of the discrete and continuous time thermocouple models are related by

$$a = \exp(-\tau_s/\tau), \quad b = 1 - a\,. \tag{6}$$

Since two sets of (4) are available from each thermocouple outputs $T_1(k)$ and $T_2(k)$, a beta model (Hung, *et al.*, 2005) can be formulated by eliminating $T_{\text{fluid}}$ from (4) to become

$$\Delta T_2^k = \beta \Delta T_1^k + b_2 \Delta T_{12}^{k-1}, \tag{7}$$

where the pseudo-sensor output $\Delta T_2^k$ and inputs $\Delta T_1^k$ and $\Delta T_{12}^{k-1}$ are defined as

$$\begin{aligned} \Delta T_1^k &= T_1(k) - T_1(k-1) \\ \Delta T_2^k &= T_2(k) - T_2(k-1) \\ \Delta T_{12}^{k-1} &= T_1(k-1) - T_2(k-1). \end{aligned} \tag{8}$$

For an *M*-sample data set (7) can be expressed in ARX vector form

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\theta}, \tag{9}$$

with $\mathbf{Y} = \Delta\mathbf{T}_2^k$, $\mathbf{X} = [\Delta\mathbf{T}_1^k \; \Delta\mathbf{T}_{12}^{k-1}]$, and $\boldsymbol{\theta} = [\beta \; b_2]^T$. Here $\Delta\mathbf{T}_1^k$, $\Delta\mathbf{T}_2^k$ and $\Delta\mathbf{T}_{12}^{k-1}$ are vectors containing *M*-1 samples of the corresponding composite signals $\Delta T_1^k$, $\Delta T_2^k$ and $\Delta T_{12}^{k-1}$.

Due to the form of the composite input and output signals, the noise terms in the **X** and **Y** data blocks are no longer independent. The result is that conventional least-squares and total least-squares both generate biased parameter estimates even when the measurement noise on the thermocouples is independent. It has been shown that generalised total least-squares (GTLS) on the other hand, can produce unbiased parameter estimate $\hat{\boldsymbol{\theta}}$ that outperforms other difference equation based methods. One of the reasons can be traced back to the use of $\beta$, which enhanced the model stability during parameter estimation (McLoone *et al.*, 2006).

Unfortunately, the $\beta$−GTLS approach occasionally returns unreasonable $\hat{\boldsymbol{\theta}}$ estimates as will be illustrated in Section 4. This is caused by the sensitivity of GTLS to violations in the underlying theoretical assumptions with composite signals (Huffel and Vandewalle, 1991), plus ill-conditioning of the noise correlation matrix. The blind deconvolution approach is considered here to isolate these invalid $\hat{\boldsymbol{\theta}}$.

## 3 BLIND SENSOR CHARACTERISATION

One of the best known deterministic blind deconvolution approaches is the method of cross-relation (CR) proposed by Liu *et al.* (1993). Such techniques exploit the information provided by output measurements from multiple systems of known structure but unknown parameters, for the same input signal.

This new approach to characterisation of thermocouples is completely different from those in Section 2. As commutation is a fundamental assumption for the method of cross-relation, the thermocouple models are both assumed to be linear. This is reasonably realistic as long as the thermocouples concerned are used within well-defined temperature ranges. Nonetheless, linearisation can easily be carried out using either the data capture hardware or software, even if the thermocouple response is nonlinear. Further, the approach requires constant model parameters, therefore the fluid flow velocity $v$ is assumed to be constant, such that the two thermocouple time constants $\tau_1$ and $\tau_2$ are time-invariant.

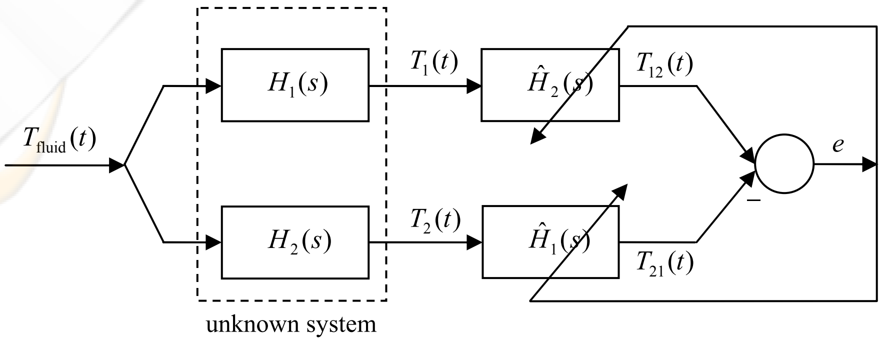


Figure 1: Two-thermocouple cross-relation characterisation.

## 3.1 Two-Thermocouple Sensor Characterisation

By exploiting the commutative relationship between linear systems, a novel two-themocouple characterisation scheme is proposed as follows. Since the fluid temperature $T_{\text{fluid}}$ is unknown, the two thermocouple output signals $T_1$ and $T_2$ are passed through two different synthetic thermocouples as shown in Fig. 1. These are also modelled by (1) and can be expressed in first-order transfer function as:

$$\hat{H}_1(s) = \frac{1}{1+s\hat{\tau}_1}, \quad \hat{H}_2(s) = \frac{1}{1+s\hat{\tau}_2}, \tag{10}$$

where $\hat{H}$ is the estimate of the thermocouple transfer function $H$. The unknown thermocouple time constant parameters can then be estimated as $\hat{\tau}_1$ and $\hat{\tau}_2$ using the cross-relation method, illustrated in Fig. 1. Here the cross-relation error signal, $e = T_{12}(t) - T_{21}(t)$ is used to define a mean-square-error cost function

$$\begin{aligned} J_2(\hat{\tau}_1, \hat{\tau}_2) &= E\{e^2\} \\ &= E\{[T_{12}(t) - T_{21}(t)]^2\}, \forall\, \hat{\tau}_1, \hat{\tau}_2 . \end{aligned} \tag{11}$$

Equation (11) is then minimised with respect to $\hat{\tau}_1$ and $\hat{\tau}_2$ to yield the estimates of the unknown thermocouple time constants. Clearly, the cross-relation cost function $J_2(\hat{\tau}_1, \hat{\tau}_2)$ is zero when $\hat{\tau}_1 = \tau_1$ and $\hat{\tau}_2 = \tau_2$. In practice it will not be possible to obtain an exact match between $T_{12}$ and $T_{21}$ due to measurement noise and other factors such as thermocouple modelling inaccuracy and violations of the assumption that the two thermocouples are experiencing identical environmental conditions.

Xiu *et al.* (1995) suggest that one of the necessary conditions for multiple finite-impulse-response channels to be identifiable is that their transfer function polynomial do not share common roots. Applying this condition to the two-thermocouple characterisation problem corresponds to requiring that the time constants, and hence the diameters (3), of the thermocouples are different, that is

$$\tau_1 \neq \tau_2 \qquad \Rightarrow \qquad d_1 \neq d_2 . \tag{12}$$

Not surprsingly, this requirement is consistent with all other two-thermocouple characterisation techniques mentioned in Section 2. Thus, cross-relation deconvolution converts the problem of sensor characterisation into an optimisation one.

## 3.2 Cost Function

A 3-D surface plot and a contour map of a typical $J_2(\hat{\tau}_1, \hat{\tau}_2)$ cost function are shown in Figs. 2 and 3. Unfortunately, $J_2(\hat{\tau}_1, \hat{\tau}_2)$ is not quadratic and cannot therefore be minimised using linear least-squares. Fig. 3 shows that the cross-relation cost function is highly non-quadratic away from the minimum corresponding to the value of the true time constants.

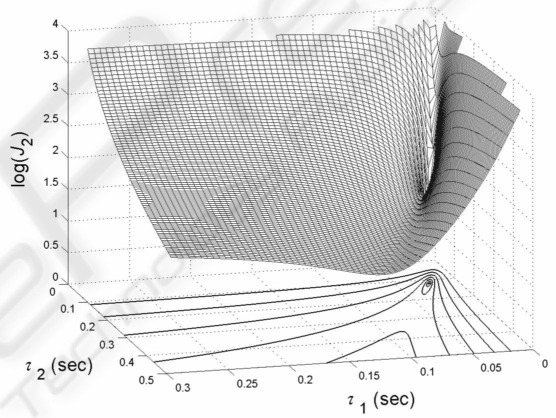


Figure 2: Three-dimensional plot of $\log(J_2)$.

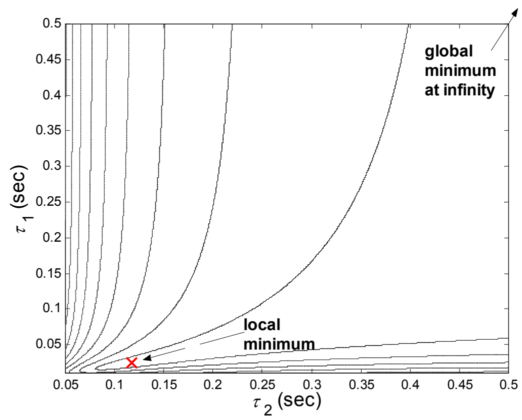


Figure 3: Contour plot of $J_2$ (cross: local minimum).

More importantly, the cost function has a second minimum when both time constant values approach infinity. Under these conditions, both low-pass filters (10) take infinite amounts of time to respond.

In other words, they are effectively open-circuited and their differences will always be zero. The existence of this minimum applies regardless of the noise conditions or any violations of the modelling assumptions. The minimum at infinity is thus in fact the global minimum, while the true time constant value is located at a local minimum. In the absence of noise, $J_2 = 0$ at both the global and local minima.

In addition, the narrow basin of attraction of the desired local minimum coupled with the global minimum at infinity has serious implications for optimisation complexity since search bounds have to be carefully selected to avoid divergence of gradient search algorithms to the global minimum. Consequently, in this study a robust, but inefficient, grid based search has been adopted to avoid these issues. To reduce the associated computational load different step sizes are used for each time constant. Noting from Fig. 3 that, at least locally,

$$\frac{\partial J_2}{\partial \tau_1} > \frac{\partial J_2}{\partial \tau_2}, \tag{13}$$

it can be concluded that the cost function is more sensitive to changes in the smaller thermocouple time constant; hence greater accuracy is required in estimating this value.

## 4 SIMULATION RESULTS

A MATLAB® simulation of a two-thermocouple probe system (Fig. 4) was used to evaluate the performance of the proposed cross-relation (CR) blind sensor characterisation. Thermocouples 1 and 2 were modelled as first-order low-pass filters according to (1) with time constants $\tau_1 = 23.8$ and $\tau_2 = 116.8$ ms respectively. The simulated fluid temperature was varied sinusoidally according to

$$T_{\text{fluid}}(t) = 16.5\sin(20\pi t) + 50.5\ , \tag{14}$$

and the resulting temperature measurements sampled every 2 ms. Each simulation ran for 5 s.

The level of zero-mean white Gaussian measurement noise added to the thermocouple signals is described by the noise level $L_e$, defined as

$$L_e = \frac{\text{var}(n_i)}{\text{var}(T_{\text{fluid}})} \cdot 100\%, \qquad i = 1, 2, \tag{15}$$

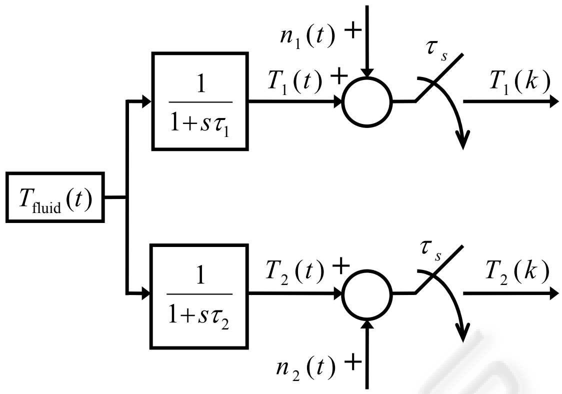


Figure 4: Simulated two-thermocouple measurement system.

where $n_1$ and $n_2$ are the noises added to the thermocouples. For a given $L_e$ , the algorithm performance was assessed in terms of percentage estimation errors:

$$e = \frac{\tau - \hat{\tau}}{\tau} \cdot 100\%\ . \tag{16}$$

To reduce the time required for completing the simulation, the following search ranges and intervals (13) were chosen for the cross-relation (CR) algorithm:

$$\begin{aligned} &10 < \hat{\tau}_1 < 30 \text{ ms; at every 0.5 ms,} \\ &100 < \hat{\tau}_2 < 130 \text{ ms; at every 2.5 ms.} \end{aligned} \tag{17}$$

Of particular importance was the removal of the first 1000 data samples before computing $J_2(\hat{\tau}_1, \hat{\tau}_2)$ , using the remaining 1500 sets of CR outputs $T_{12}$ and $T_{21}$. This was required to eliminate the effect of transients on parameter estimation accuracy during each iteration of CR simulation (Fig. 1). The number of samples removed was estimated to exceed the 98% settling time for the system (i.e. five times the largest time constant $\tau_2$ ) which equated to about 0.6 s or 300 samples.

The resulting means and standard deviations of the parameter estimation error (16), for both $\beta$−GTLS (Section 2.2) and CR (Section 3.1) algorithms are shown in Fig. 5. Note results for $\hat{\tau}_2$ are similar to those illustrated for $\hat{\tau}_1$ and are thus omitted.

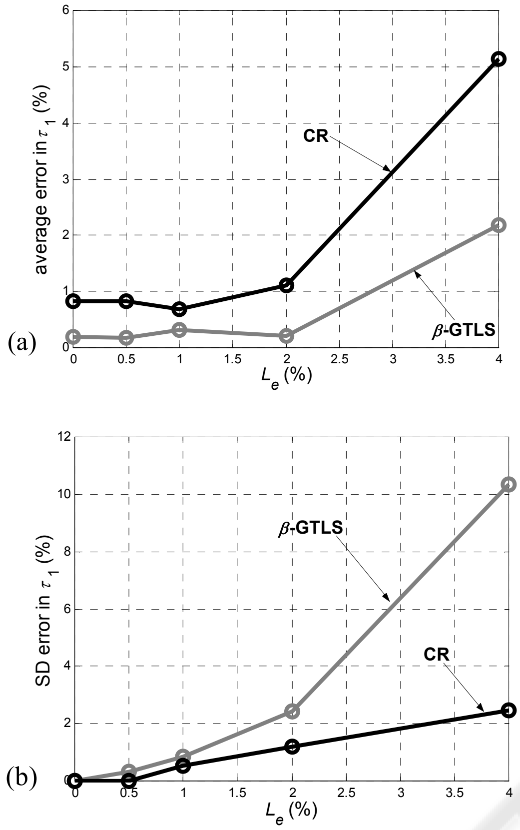


Figure 5: (a) Means and (b) standard deviations of $e$ of $\hat{\tau}_1$ averaged over 100 Monte-Carlo runs.

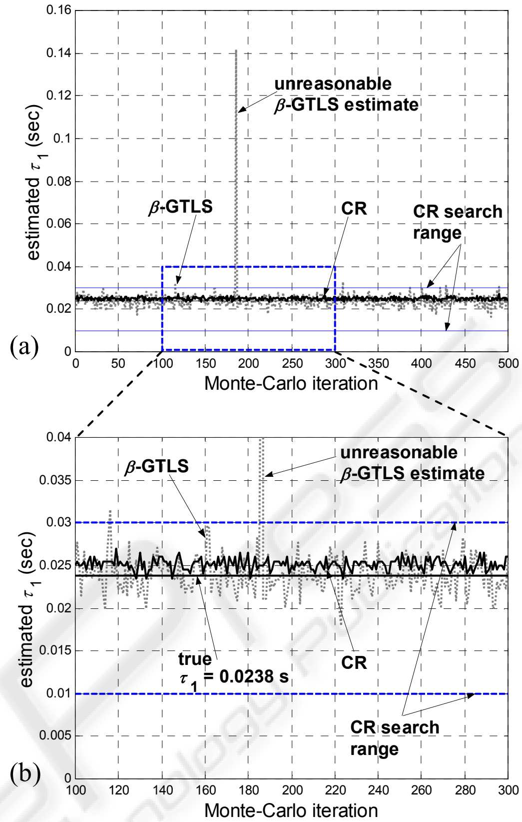


Figure 6: 500 Monte-Carlo runs of $\hat{\tau}_1$ of $\beta$−GTLS and CR, where (b) is a magnified version of (a).

These results suggset that CR produces biased parameter estimates since their expected mean errors are greater than that of $\beta$−GTLS. However, the estimation standard deviations of CR are less than that of $\beta$−GTLS.

With regard to the search intervals taken for CR, two issues need to be considered when looking at the graphs. Firstly, a major contribution to the CR bias comes from the low resolution of the search grid used. Since, when $\tau_1 = 23.8$ ms, an interval of 0.5 ms represents an 'artificial' estimation bias of up to 2.1%. This can be reduced if a finer search grid is employed, at the expense of increasing the already heavy computation load. Similarly, the CR standard deviation errors may be 2.1% larger than the reported values because of the finite resolution employed, although this is unlikely due to the intrinsic noise-filtering capability of CR.

The noise-resilient property of CR compared to GTLS is further highlighted in Fig. 6, where 500 Monte-Carlo simulations were performed. It can be seen that one unreasonable $\hat{\tau}_1$ value was returned by $\beta$−GTLS while the CR approach is well-behaved, although its estimate is asymptotically biased. Hence, CR can be used to verify whether a GTLS estimate is genuine or corrupted by signal outliers, improving the overall reliability of sensor characterisation.

# 5 CONCLUSIONS

A novel cross-relation (CR) sensor characterisation method has been presented. It does not require *a priori* knowledge of the thermocouple time constant ratio $\alpha$, as required in many other characterisation algorithms. CR is more noise-tolerant in the sense of reduced parameter estimation variance when compared to the alternatives such as $\beta$−GTLS. The robustness arises because the CR process involves passing each thermocouple output through a first-order block, which removes, at least partially, measurement noise during identification. As a result, CR can be employed to verify estimation validity, thereby increasing the overall reliability of other characterisation methods.

The computational complexity of CR, due to the inefficient grid based search used in this study, means that it is most appropriate for offline sensor characterisation. Further investigations include ways to speed up the computation and reduce the estimation bias.

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