# MATHEMATICAL MODEL FOR WALKING ROBOT WITH SHAPE MEMORY ALLOY ANKLE

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Abstract: The paper presents a simultaneous force and length variation mode in shape memory alloy (SMA) robotic application. The robotic ankle contains four SMA actuators and a spherical articulation. In order to assure a high efficient robotic architecture, the mechanical and the control structure have to assure a real-time response to the work environment changes. The load variations or the difference between the moment of full contact step and the non contact moment for a waking robot are the standard situations for a SMA robotic ankle. The paper is divided in five sections. First section makes a short introduction in the physical description and conventional applications of shape memory alloy materials. Then, are presented the mathematical model for robotic ankle, the walking robot geometrical structure and the causality ordering of the active pair of legs, in this case with one free joint. In the last section some experimental results are presented. These results were obtained by using MATLAB programs, conceived by authors, for design and simulation of walking robots control algorithms.

## 1 INTRODUCTION

The shape memory effect was first noted over 50 years ago; it was not until 1962, however, with the discovery of a nickel titanium shape memory alloy by Buehler, that serious investigations were undertaken to understand the mechanism of the shape memory effect (Bîzdoacă and Pană, 2004), (Funakubo, 1987), (Graesser and Cozarelli, 1994), (Schroeder and Boller, 1993).

The shape memory alloys possess the ability to undergo shape change at low temperature and retain this deformation until they are heated, at which point they return to their original shape.

The nickel titanium alloys, used in the present research, generally refereed to as Nitinol, have compositions of approximately 50 atomic % Ni/ 50 atomic % Ti, with small additions of copper, iron, cobalt or chromium. The alloys are four times the cost of Cu-Zn-Al alloys, but it possesses several advantages as greater ductility, more recoverable motion, excellent corrosion resistance, stable transformation temperatures, high biocompatability and the ability to be electrically heated for shape recovery.

Shape memory actuators are considered to be low power actuators and such as compete with solenoids, bimetals and to some degree was motors. It is estimated that shape memory springs can provide over 100 times the work output of thermal bimetals.

The use of shape memory alloy can sometimes simplify a mechanism or device, reducing the overall number of parts, increasing reliability and therefore reducing associated quality costs.

Because of its high resistivity of 80 – 89 micro ohm-cm, nickel titanium can be self heated by passing an electrical current through it. The basic rule for electrical actuation is that the temperature of complete transformation to martensite $M_f$ , of the actuator, must be well above the maximum ambient temperature expected (Delay and Chandrasekaran, 1987).



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## 2 MATHEMATICAL MODEL OF SHAPE MEMORY ALLOY ANKLE

The robotic researches develop up to the present a various mechanical architecture for ankle structure. All projects use the human ankle as model.

The problem in developing efficient ankle structure, concern the dimension and the efficiency of actuators.

The proposed robot ankle structure contains units with SMA actuators. The unit has 4 SMA actuators and a spherical articulation.

Figure 1: The proposed SMA robotic ankle.

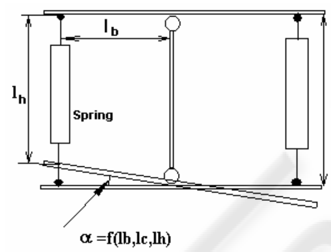


Figure 2: Schematically representation of SMA ankle.

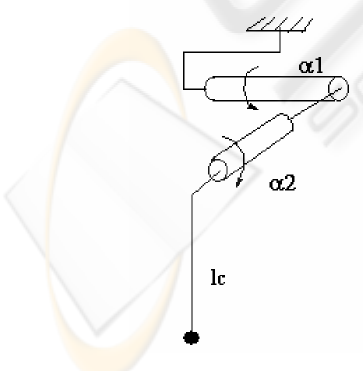


Figure 3: Kinematics representation of SMA ankle.

The mathematical model of the ankle is very simple:

$$H_{ankle}(\alpha_1,\alpha_2,l_c) = \begin{matrix} \cos\alpha_2 & 0 & \sin\alpha_2 & l_c\sin\alpha_2 \\ \sin\alpha_1\sin\alpha_2 & \cos\alpha_1 & -\sin\alpha_1\cos\alpha_2 & -l_c\sin\alpha_1\cos\alpha_2 \\ -\cos\alpha_1\sin\alpha_2 & \sin\alpha_1 & \cos\alpha_1\cos\alpha_2 & l_c\cos\alpha_1\cos\alpha_2 \\ 0 & 0 & 0 & 1 \end{matrix} \quad (1)$$

Analysing the angle dependence versus the SMA spring variation, a highly nonlinear function results:

$$\alpha = \arcsin\left[\cos\left(\mathrm{arctg}\left(l_h\right)\right)\frac{2l_b^2+l_c^2}{2l_b}\right] + \mathrm{arctg}\left(l_h\right) \quad (2)$$

where $l_h$ is the length after the heating process; $l_c$ is the spring length after cooling and $l_b$ the base length.

As the real variation is restricted (between 100% and 92 %), the linearization can occur because of linear behaviour for the specified evolution (Bîzdoacă and Pană, 2003).

## 3 WALKING ROBOT GEOMETRICAL STRUCTURE

It is considered the walking robot structure as depicted in Fig.4, having three normal legs $L^i$, $L^j$, $L^p$ and a head equivalent to another leg, $L_0$, containing the robot centre of gravity, G, placed in its foot. The robot body RB is characterized by two position vectors $O^0$, $O^1$ and the leg joining points denoted $R^i$, $R^j$, $R^p$. The joining point of the head, $L^0$, is the central point $O^0$, $R^0 = O^0$, so the robot body RB is univocally characterized by the set,

$$RB = \{O^0, O^1, \lambda^i, \lambda^j, \lambda^p, \lambda^0\} \quad (3)$$

where $\lambda^0 = 0$.

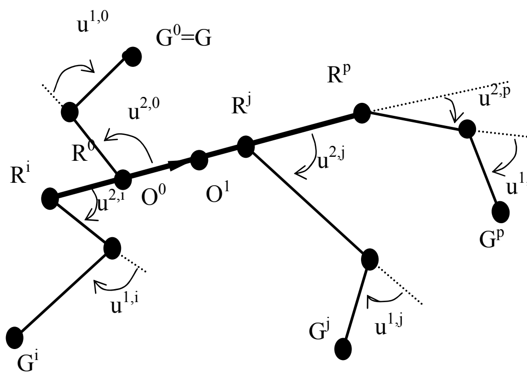


Figure 4: Geometrical structure of the robot.

The robot position in the vertical plane is defined by the pair of the position vectors $O^0$, $O^1$ where $|O^1 - O^0| = 1$, or by the vector $O^0$ and the scalar θ, the angular direction of the robot body.

Each of the four robot legs $L^i, L^j, L^p, L^0$ is characterized by a so-called Existence Relation ER(L) depending on specific variables as it is presented in (Petrişor, 2005), (Petrişor, Marin, Bîzdoacă and Cazacu, 2006).

The mathematical model of this object is a Variable Causality Dynamic Systems VCDS and it is analiyzed from this point of view.

A pair of legs $\{L_i, L_j\}$ constitutes the so called Active Pair of Legs (APL) if the robot body position is the same irrespective of the feet position of all the other legs different of $L_i$ and $L_j$. A label is assigned to each possible APL. The APL label is expressed by a variable $q$ called Index of Activity (IA), which can take $N_a$ values, numbers or strings of characters. All the other legs that at a time instant do not belong to APL are called Passive Legs (PL). The leg in APL, having a free joining point is called slave leg, the opposite of the motor leg whose both joining points are external controlled.

# 4 CAUSALITY ORDERING OF AN ACTIVE PAIR OF LEGS WITH ONE FREE JOINT

In this structure, only one angle is free so three joints are external controlled (EC).It is denoted this by

$$q = 'ij',\ c = [\text{motor12},\ \text{motor01},\ c^p] \tag{4}$$

This causality ordering is corresponded to the state having the leg $L^i$ as a motor leg which controls two degree of freedom and the leg $L^j$, a slave leg, which can control only one scalar component , so the angle $u^{2,j}$ is free and the angle $u^{1,j}$ is EC.

In this structure $u^{2,j}$ is obtained from the following relations:

$$\tilde{u}^{2,j}_{(t)} = \begin{cases} \psi_{2j} & \text{if } \ u^{2,j}(t-\varepsilon) > 0 \Leftrightarrow \hat{s}^j = \text{up} \\ -\psi_{2j} & \text{if } \ u^{2,j}(t-\varepsilon) \le 0 \Leftrightarrow \hat{s}^j = \text{down} \end{cases} \tag{5}$$

$$\tilde{u}^{2,j}_{(t)} = \begin{cases} -\varphi_{2j} + \psi_{2j} & \text{dacă } \ -\varphi_{2j} + \psi_{2j} \le 0 \\ -\varphi_{2j} + \psi_{2j} - \pi & \text{dacă } \ -\varphi_{2j} + \psi_{2j} > 0 \end{cases} \tag{6}$$

$$\tilde{u}^{2,j}_{(t)} = \begin{cases} -\varphi_{2j} + \psi_{2j} + \pi & \text{dacă } \ -\varphi_{2j} + \psi_{2j} \le 0 \\ -\varphi_{2j} + \psi_{2j} & \text{dacă } \ -\varphi_{2j} + \psi_{2j} > 0 \end{cases} \tag{7}$$

Therefore, in this causality structure the kinematics restriction $|R^j - R^i| = r_{ij}$ is accomplished by changing the value of $u^{2,j}$ at $\tilde{u}^{2,j}$ giving by the equation (5), (6), (7).

# 5 EXPERIMENTAL RESULTS

The causal ordering c=[motor12, motor 01, $c^p$] is implemented together with other causal orderings, in the RoPa platform for design and simulation of walking robots control algorithms. The RoPa platform is a complex of Matlab programs to analyze and design walking robots, evolving in uncertain environments according to a new control approach called Stable State Transition Approach (SSTA), also conceived by the authors.

The causal ordering developed in this paper is activated by selecting the causal variable cz=[12 1 0]. The RoPa platform allows animation, recording of the evolutions and playback them.

In the following there are presented some experimental results of walking robot behaviour considering this causal ordering.

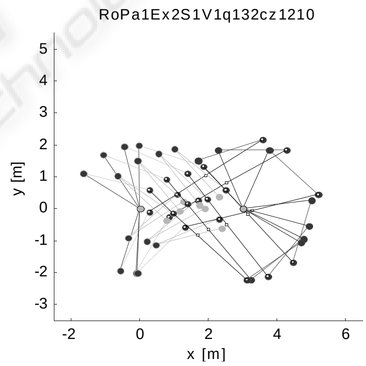


Figure 5: The robot kinematics evolution.

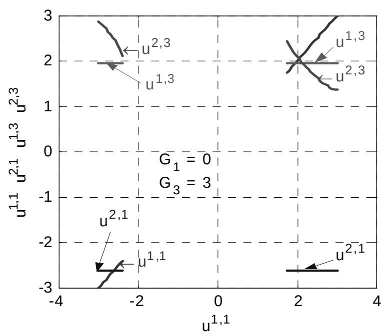


Figure 6: Controlled angles with respect to the input angle.

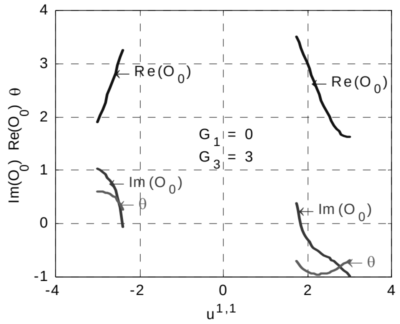


Figure 7: Robot body position with respect to the input angle.

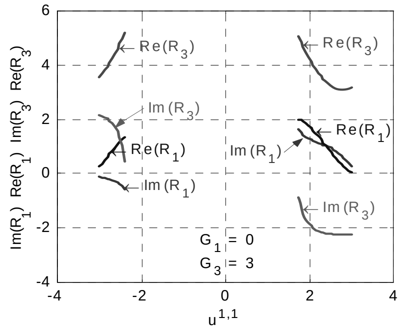


Figure 8: Joints positions with respect to the input angle.

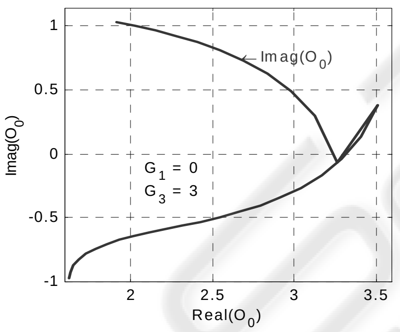


Figure 9: Reference point $O^0$ locus in evolution scene.

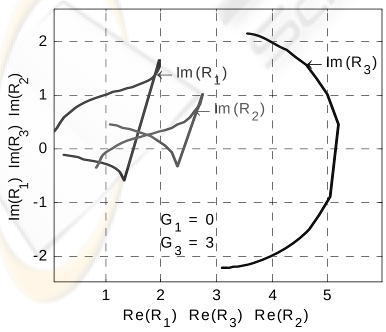


Figure 10: Joints points' locus in evolution scene.

# 6 CONCLUSIONS

Shape memory alloy ankle add to the walking robot architecture additional control possibility for adapting the structure to various environment.

The causal ordering developed in the paper is useful when it must control the shoulder position $R_i$ irrespective of the other variables. It can be controlled by the angle $u^{1,j}$, the shoulder angle of the second active leg $L_j$.

The approach of the kinematics structure by complex number representation of the variables allowed to solve the equations system that describe the position of the variables that are involved, between witch there are kinematics correlations. In this way, it is obtained an explicit representation of the input-output dependence, in this causality structure.

As it can be seen from the above experimental results, this causal ordering is perfectly integrated in the RoPa structure proving the correctness of the theoretical results.

The mathematical model developed in the paper becomes an element of the VCDS walking robot model. The simulations used MATLAB environment show the robustness of the mathematical model.

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