DUAL CONTROLLERS FOR DISCRETE-TIME STOCHASTIC AMPLITUDE-CONSTRAINED SYSTEMS

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Abstract: The paper considers a suboptimal solution to the dual control problem for discrete-time stochastic systems in the case of amplitude constraint imposed on the control signal. The objective of the control is to minimize the variance of the output around the given reference sequence. The presented approaches are based on: an MIDC (Modified Innovation Dual Controller) derived from an IDC (Innovation Dual Controller), a TSDSC (Two-stage Dual Suboptimal Control, and a PP (Pole Placement) controller. Finally, the certainty equivalence (CE) control method is included for comparative analysis. In all algorithms, the standard Kalman filter equations are applied for estimation of the unknown system parameters. Example of second order system is simulated in order to compare the performance of control methods. Conclusions yielded from simulation study are given.

1 INTRODUCTION

Much work has been done on the optimal control of stochastic systems which contain parametric uncertainty. The problem is inherently related with the dual control problem originally presented by Fel'dbaum who suggested that in the dual control, the problems of learning and control should be considered simultaneously in order to minimize the cost function. In general, learning and controlling have contradictory goals, particularly for the finite horizon control problems. The concept of duality has inspired the development of many control techniques which involve the dual effect of the control signal. They can be separated in two classes: explicit dual and implicit dual (Bayard and Eslami, 1985). Unfortunately, the dual approach does not result in computationally feasible optimal algorithms. A variety of suboptimal solutions has been proposed and many of them were heuristic identifier-controller structures. Other controllers like minimax controllers (Sebald, 1979), Bayes controllers (Sworder, 1966) or MRAC (Model Reference Adaptive Controller) (Åström and Wittenmark, 1989) are available.

The objective of this paper is to present and compare different approaches to suboptimal solution of the minimum variance control problem of discrete-time stochastic systems with unknown parameters. In this paper, an amplitude-constrained control input is considered which is an important practical case. A majority of proposed solutions in the literature does not include the input constraint into the design of control system. The saturation imposed on control signal deteriorates the probability density function (pdf) of the state from the Gaussian which makes finding an optimal control difficult even when system parameters are known. The dual methods described here are: the MIDC method which is the modification of the IDC (R. Milito and Cadorin, 1982) approach, the method based on the two-stage dual suboptimal control (TSDSC) approach (Maitelliand and Yoneyama, 1994) and the method based on the pole placement approach (Filatov and Unbehauen, 2004).

The Iteration in Policy Space (IPS) algorithm and its reduced complexity version were proposed by Bayard (Bayard, 1991) for a general nonlinear system. In this algorithm the stochastic dynamic programming equations are solved forward in time ,using a nested stochastic approximation technique. The method is based on a specific computational architecture denoted as a H block. The method needs a filter propagating the state and parameter estimates with as-

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sociated covariance matrices.

In (Królikowski, 2000), some modifications including input constraint have been introduced into the original version of the IPS algorithm and its performance has been compared with MIDC algorithm.

This paper has a tutorial nature, and the possibility of incorporating the input constraint into the control algorithms was the motivation for a selection of the overviewed approaches.

Performance of the considered algorithms is illustrated by simulation study of second-order system with control signal constrained in amplitude.

2 CONTROL PROBLEM FORMULATION

Consider a discrete-time linear single-input single-output system described by ARX model

$$A(q^{-1})y_k = B(q^{-1})u_k + w_k, \tag{1}$$

where $A(q^{-1}) = 1 + a_{1,k}q^{-1} + \cdots + a_{na,k}q^{-na}$, $B(q^{-1}) = b_{1,k}q^{-1} + \cdots + b_{nb,k}q^{-nb}$, y_k is the output available for measurement, u_k is the control signal, $\{w_k\}$ is a sequence of independent identically distributed gaussian variables with zero mean and variance σ_w^2. Process noise w_k is statistically independent of the initial condition y_0. The system (1) is parametrized by a vector θ_k containing $na+nb$ unknown parameters $\{a_{i,k}\}$ and $\{b_{i,k}\}$ which in general can be assumed to vary according to the equation

$$\underline{\theta}_{k+1} = \Phi\underline{\theta}_k + e_k \tag{2}$$

where Φ is a known matrix and $\{e_k\}$ is a sequence of independent identically distributed gaussian variables with zero mean and variance matrix R_e. Particularly, for the constant parameters we have

$$\underline{\theta}_{k+1} = \underline{\theta}_k = \underline{\theta} = (b_1, \cdots, b_{nb}, a_1, \cdots a_{na})^T, \tag{3}$$

and then $\Phi = I$, $e_k = 0$ in (2).

The control signal is subjected to an amplitude constraint

$$|\, u_k \,| \leq \alpha \tag{4}$$

and the information state I_k at time k is defined by

$$I_k = [y_k, ..., y_1, u_{k-1}, ..., u_0, I_0] \tag{5}$$

where I_0 denotes the initial conditions.

An admissible control policy Π is defined by a sequence of controls $\Pi = [u_0, ..., u_{N-1}]$ where each control u_k is a function of I_k and satisfies the constraint (4). The control objective is to find a control policy Π which minimizes the following expected cost function

$$J = E[\sum_{k=0}^{N-1}(y_{k+1} - r_{k+1})^2] \tag{6}$$

where $\{r_k\}$ is a given reference sequence. An admissible control policy minimizing (6) can be labelled by CCLO (Constrained Closed-Loop Optimal) in keeping with the standard nomenclature, i.e. $\Pi^{\text{CCLO}} = [u_0^{\text{CCLO}}, ..., u_{N-1}^{\text{CCLO}}]$. This control policy has no closed form, and control policies presented in the following section can be viewed as a suboptimal approach to the Π^{CCLO}.

3 SUBOPTIMAL DUAL CONTROL METHODS

In this section, we shall briefly describe three methods giving an approximate solution to the problem formulated in Section 2. The first one is the MIDC algorithm based on the IDC approach (R. Milito and Cadorin, 1982) which is an explicit dual control approach.

3.1 Method based on the Innovation Dual Control (IDC) Approach: Derivation of Π^{MIDC}

The IDC has been derived for system (1) with unconstrained control and constant parameters (3). The following cost function was considered

$$J = \frac{1}{2}E[(y_{k+1} - r_{k+1})^2 - \lambda_{k+1}\varepsilon_{k+1}^2 | I_k] \tag{7}$$

where $\lambda_{k+1} \geq 0$ is the learning weight, and ε_{k+1} is the innovation, see (16).

The modified IDC, u_k^{MIDC}, takes the constraint into account which results in the following closed-form expression

$$u_k^{\text{MIDC}} = = -\text{sat}\left(\frac{[(1-\lambda_{k+1})\underline{p}^T_{b_1\underline{\theta}^*,k} + \hat{\underline{\theta}}_k^{*T}\hat{b}_{1,k}]\underline{s}_k^* - \hat{b}_{1,k}r_{k+1}}{(1-\lambda_{k+1})p_{b_1,k} + \hat{b}_{1,k}^2}; \alpha\right) \tag{8}$$

where

$$\begin{aligned} \underline{s}_k &= (u_k, u_{k-1}, \ldots, u_{k-nb+1}, -y_k, \ldots, -y_{k-na+1})^T = \\ &= (u_k, \underline{s}_k^{*T})^T, \end{aligned} \tag{9}$$

and following partitioning is introduced for parameter covariance matrix P_k

$$P_k = \begin{bmatrix} p_{b_1,k} & \underline{p}^T_{b_1\underline{\theta}^*,k} \\ \underline{p}_{b_1\underline{\theta}^*,k} & P_{\underline{\theta}^*,k} \end{bmatrix} \tag{10}$$

corresponding to the partition of $\underline{\theta}$

$$\underline{\theta} = (b_1, \underline{\theta}^{*T})^T \tag{11}$$

with

$$\underline{\theta}^* = (b_2, \ldots, b_{nb}, a_1, \ldots, a_{na})^T. \tag{12}$$

The estimates $\hat{\underline{\theta}}_k$ needed to calculate u_k^{MIDC} can be obtained in many ways. A common way is to use the standard Kalman filter in a form of suitable recursive procedure for parameter estimation, i.e.

$$\hat{\underline{\theta}}_{k+1} = \Phi \hat{\underline{\theta}}_k + \underline{k}_{k+1} \varepsilon_{k+1} \tag{13}$$

$$\underline{k}_{k+1} = \Phi P_k \underline{s}_k [\underline{s}_k^T P_k \underline{s}_k + \sigma_w^2]^{-1} \tag{14}$$

$$P_{k+1} = [\Phi - \underline{k}_{k+1} \underline{s}_k^T] P_k \Phi^T + R_e, \tag{15}$$

$$\varepsilon_{k+1} = y_{k+1} - \underline{s}_k^T \hat{\underline{\theta}}_k. \tag{16}$$

3.2 Method based on the Two-stage Dual Suboptimal Control (TSDSC) Approach: Derivation of Π^{TSDSC}

The TSDSC proposed in (Maitelliand and Yoneyama, 1994) has been derived for system (1) with stochastic parameters (2). Below this method is extended for the input-constrained case. The cost function considered for TSDSC is given by

$$J = \frac{1}{2} E[(y_{k+1} - r)^2 + (y_{k+2} - r)^2 | I_k] \tag{17}$$

and according to (Maitelliand and Yoneyama, 1994) can be obtained as a quadratic form in u_k and u_{k+1}, i.e.

$$J = \frac{1}{2}[a u_k + b u_{k+1} + c u_k u_{k+1} + d u_k^2 + e u_{k+1}^2] \tag{18}$$

where a, b, c, d, e are expressions depending on current data $\underline{s}_k^*$, reference signal r and parameter estimates $\hat{\underline{\theta}}_k$ (Maitelliand and Yoneyama, 1994). Solving a necessary optimality condition the unconstrained control signal is

$$u_k^{\text{TSDSC,un}} = \frac{bc - 2ae}{4de - c^2}. \tag{19}$$

This control law has been taken for simulation analysis in (Maitelliand and Yoneyama, 1994). Imposing the cutoff the constrained control signal is

$$u_k^{\text{TSDSC,co}} = sat(u_k^{\text{TSDSC,un}}; \alpha). \tag{20}$$

The cost function (18) can be represented as a quadratic form

$$J = \frac{1}{2}[\underline{u}_k^T A \underline{u}_k + \underline{b}^T \underline{u}_k] \tag{21}$$

where $\underline{u}_k = (u_k, u_{k+1})^T$, and

$$A = \begin{bmatrix} d & \frac{1}{2}c \\ \frac{1}{2}c & e \end{bmatrix}, \underline{b} = \begin{bmatrix} a \\ b \end{bmatrix}. \tag{22}$$

The condition $4de - c^2 > 0$ together with $d > 0$ implies positive definitness and guarantees convexity. Minimization of (21) under constraint (4) is a standard QP problem resulting in $\underline{u}_k^{\text{TSDSC,qp}}$. The constrained control $u_k^{\text{TSDSC,qp}}$ is then applied to the system in receding horizon framework.

3.3 Method based on the Pole Placement (PP) Approach: Derivation of Π^{PP}

Let the desired stable closed-loop polynomial be described by $A^*(q^{-1}) = 1 + a_1^* q^{-1} + \cdots + a_{n^*} q^{-n^*}$. A dual version of a direct adaptive PP controller proposed in (N.M. Filatov and Keuchel, 1993; Filatov and Unbehauen, 2004) has been derived for system (1) where integral actions can be included. To this end, a bicriterial approach has been used to solve the synthesis problem. The two criteria correspond to the two goals of the dual adaptive control, namely to control the system output close to the reference signal, and to accelerate the parameter estimation process for future control improvment. Incorporating the amplitude constraint of the control input yields

$$u_k^{\text{PP}} = sat\Big(u_k^{\text{CAUT}} + \\ + \eta\, tr P_k sign(p_{d_0,k} \bar{u}_k^{\text{CAUT}} + \underline{p}_{d_0\underline{p}_1,k}^T \underline{m}_{1,k})\, ;\, \alpha\Big) \tag{23}$$

where u_k^{CAUT} is the cautious action given by

$$u_k^{\text{CAUT}} = -\frac{(\underline{p}_{r_0\underline{p}_0,k}^T + \hat{\underline{p}}_{0,k}^T \hat{r}_{0,k})\underline{m}_{0,k} - \hat{r}_{0,k} r_k}{p_{r_0,k} + \hat{r}_{0,k}^2}, \tag{24}$$

$\bar{u}_k^{\text{CAUT}} = u_k^{\text{CAUT}} + \sum_{i=1}^{n^*} a_i^* u_{k-i}$, $\underline{p}_0 = (s_0, \ldots, s_{ns}, r_1, \ldots, r_{nr})^T$, $\underline{m}_{0,k} = (y_k, \ldots, y_{k-ns}, u_{k-1}, \ldots, u_{k-nr})^T$, and $\eta \geq 0$ is the parameter responsible for probing. In this case the following partitioning is introduced for parameter covariance matrix P_k

$$P_k = \begin{bmatrix} p_{d_0,k} & \underline{p}_{d_0\underline{p}_1,k}^T \\ \underline{p}_{d_0\underline{p}_1,k} & P_{\underline{p}_1,k} \end{bmatrix} \tag{25}$$

corresponding to the partition of parameter vector $\underline{p}$

$$\underline{p} = (-d_0, \underline{p}_1^T)^T \tag{26}$$

where

$$\underline{p}_1 = (-d_1, \ldots, d_{nd}, -f_1, \ldots, -f_{nf}, r_0, \ldots, r_{nr}, s_0, \ldots, s_{ns})^T \tag{27}$$

and

$$\underline{m}_k = (\bar{u}_k, \underline{m}_{1,k}^T)^T \quad (28)$$

with $\underline{m}_{1,k} = (\bar{u}_{k-1}, \ldots, \bar{u}_{k-nd}, \bar{y}_k, \ldots, \bar{y}_{k-nf+1}, u_{k-l+1}, \ldots, u_{k-l-nr+2}, y_{k-l+2}, \ldots, y_{k-l+ns+2})^T$. The filtered output and input signals are obtained as $\bar{y}_k = A^*(q^{-1})y_k$, $\bar{u}_k = A^*(q^{-1})u_k$.

The corresponding diophantine equation and Bezout identity are

$$A(q^{-1})[r_0+q^{-1}R(q^{-1})]+q^{-1}B(q^{-1})S(q^{-1})=r_0A^*(q^{-1}), \quad (29)$$

$$A(q^{-1})D(q^{-1}) + B(q^{-1})F(q^{-1}) = r_0q^{-l+2}, \quad (30)$$

where the polynomial degrees are: $nr = na-1$, $ns = na-\kappa-1$, $l = na+nb$, $nd = nb-2$, $nf = na-1$, and κ is the number of possible integrators in the system.

It can be shown that the filtered output $\bar{y}_k$ can be represented in the following regressor form

$$\bar{y}_k = \underline{p}^T \underline{m}_{k-1} + v_k \quad (31)$$

For estimation of parameters $\underline{p}$ (note that parameters $\underline{p}_0$ are included into $\underline{p}$) the Kalman filter algorithm (13)-(16) can again be used where $\hat{\underline{\theta}}_k$ should be replaced by $\hat{\underline{p}}_k$, $\underline{s}_k$ should be replaced by $\underline{m}_k$, ε_{k+1} should be calculated as $\varepsilon_{k+1} = \bar{y}_{k+1} - \underline{m}_k^T \hat{\underline{p}}_k$, and the variance σ_w^2 should be replaced by the variance σ_v^2 which can be evaluated from (29), (30), (1).

4 SIMULATION TESTS

Performance of the described control methods is illustrated through the example of a second-order system with the following true values: $a_1 = -1.8$, $a_2 = 0.9$, $b_1 = 1.0$, $b_2 = 0.5$, where the Kalman filter algorithm (13)-(16) was applied for estimation. The initial parameter estimates were taken half their true values with $P_0 = 10I$. The reference signal was a square wave ± 3, and then the minimal value of constraint α ensuring the tracking is $\alpha_{min} = 3\frac{|A(1)|}{|B(1)|} = 0.2$. Fig. 1 shows the reference, output and input signals during tracking process under the constraint $\alpha = 1$ for all control policies.

For the control policy Π^{MIDC} the constant learning weight was $\lambda_k = \lambda = 0.98$. The policy Π^{PP} was simulated for third order polynomial $A^*(q^{-1})$ having poles at $0.2 \pm i0.1$, -0.1, and for the probing weight $\eta = 0.2$. The control policy Π^{CE} can easily be obtained from MIDC by taking $p_{b_1,k} = 0$, $\underline{p}_{b_1\underline{\theta}^*,k}^T = 0$.

Next, the simulated performance index

$$\bar{J} = \sum_{k=0}^{N-1} (y_{k+1} - r_{k+1})^2$$

was considered. The plots of $\bar{J}$ versus the constraint α are shown in Figs. 2, 3 for $\sigma_w^2 = 0.05, 0.1$, respectively, and $N = 1000$. The control $u_k^{\text{TSDSC,qp}}$ was obtained solving the minimization of quadratic form (20) using MATLAB function *quadprog*. The performance of this control is not included in plots of Figs. 5, 6, because it performs surprisingly essentially inferior with respect to $u_k^{\text{TSDSC,co}}$. In the latter case, a short-term behaviour phenomenon (G.P. Chen and Hope, 1993) can be observed in Figs. 2, 3. This means that when the cutoff method is used then the range of constraint α can be found where for increasing α the performance index is also increasing.

5 CONCLUSIONS

This paper presents various approaches toward a suboptimal solution to the discrete-time dual control problem under the amplitude-constrained control signal. A simulation example of second-order system is given and the performance of the presented control policies is compared by means of the simulated performance index.

The MIDC method seems to be a good suboptimal dual control approach, however it has been found that the MIDC control is quite sensitive to the value of the learning weight λ. In (Królikowski, 2000) it has been found that this method often performs very close to the IPS algorithm (Bayard, 1991).

Performance of all control policies except $\Pi^{\text{TSDSC,co}}$ is comparable, however the differences between all methods are less noticeable when the constraint α gets tight, i.e. when $\alpha \rightarrow \alpha_{min}$. In all considered control policies except $u_k^{\text{TSDSC,co}}$, the performance index increases when the input amplitude constraint gets more tight. This means that for $u_k^{\text{TSDSC,co}}$ the effect of the short term behaviour phenomenon discussed in (G.P. Chen and Hope, 1993) could appear.

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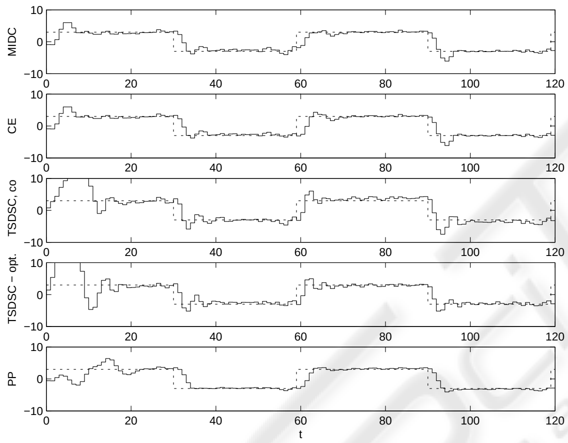

Figure 1: Reference, output and control signals for Π^{MIDC}, Π^{CE}, Π^{TSDSC}, Π^{PP} and $\alpha = 1$.

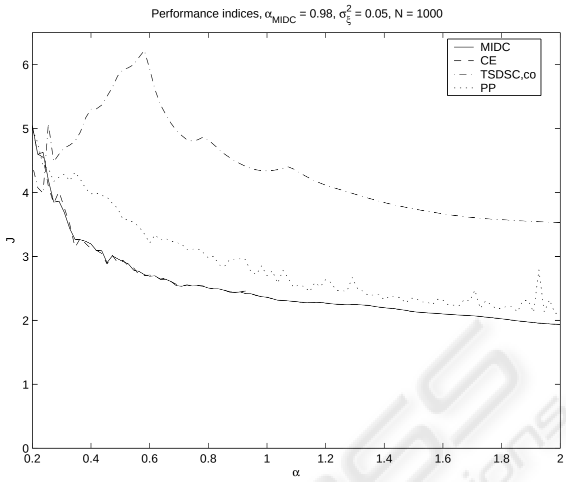

Figure 2: Plots of performance indices for $\sigma_w^2 = 0.05$.

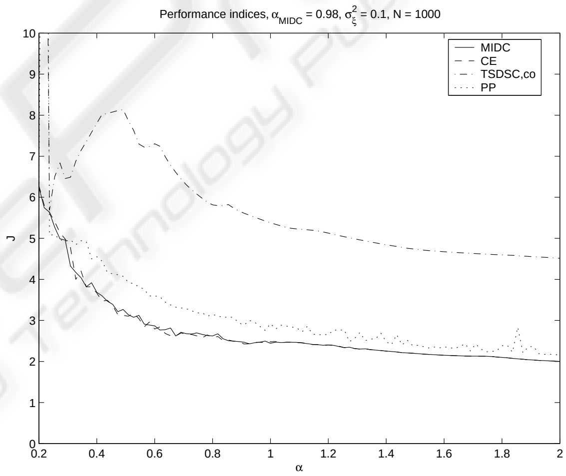

Figure 3: Plots of performance indices for $\sigma_w^2 = 0.1$.