OPTIMAL POWER ALLOCATION IN A MIMO-OFDM TWISTED PAIR TRANSMISSION SYSTEM WITH FAR-END CROSSTALK

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Abstract: Crosstalk between neighbouring wire pairs is one of the major impairments in digital transmission via multi-pair copper cables, which essentially limits the transmission quality and the throughput of such cables. For high-rate transmission, often the strong near-end crosstalk (NEXT) disturbance is avoided or suppressed and only the far-end crosstalk (FEXT) remains as crosstalk influence. If FEXT is present, signal parts are transmitted via the FEXT paths from the transmitter to the receiver in addition to the direct transmission paths. Therefore transmission schemes are of great practical interest, which take advantage of the signal parts transmitted via the FEXT paths. Here a SVD (singular-value decomposition) equalized MIMO-OFDM system is investigated, which is able to take advantage of the FEXT signal path. Based on the Lagrange multiplier method an optimal power allocation schema is considered in order to reduce the overall bit-error rate at a fixed data rate and fixed QAM constellation sizes. Thereby an interesting combination of SVD equalization and power allocation is considered, where the transmit power is not only adapted to the subchannels but rather to the symbol amplitudes of the SVD equalized data block. As a result it can be seen that the exploitation of FEXT is important for wireline transmission systems in particular with high couplings between neighbouring wire pairs and the power allocation is possible taking the different subcarriers into account.

1 INTRODUCTION

OFDM (orthogonal frequency division multiplex) is a widely accepted transmission schema in both, wireline and wireless transmission. Examples include digital subscriber line (DSL) (Bingham, 2000), European digital video broadcast (DVB), digital audio broadcast (DAB) and wireless local area networks (WLAN) such as 802.11a and HIPERLAN/2. A lot of publications have been published in the literature where the resilience of multicarrier transmission systems against the delay spread was highlighted, providing a sufficient guard interval length (Bingham, 2000; van Nee and Prasad, 2000).

In a multiuser scenario considered here, a resilience against intersymbol interference (ISI) isn't sufficient to fulfill given quality criteria. In local cable networks, crosstalk is one of the most limiting disturbances (Valenti, 2002). Since the NEXT is a very strong disturbance several techniques have been developed in order to avoid or suppress it (Honig et al., 1990). In this case only the FEXT remains as crosstalk influence. Often optical fibre transmission is used up to a building's entrance and the last few hundred metres within the building are bridged by copper cables. For such short cables used in high-data rate systems in the local cable area, the FEXT is particularly strong (Valenti, 2002) and as a result heavy multiuser interference arise. This has lead to a great interest in transmission systems which are capable to take such disturbances into account.

In different publications, e. g, in (Lange and Ahrens, 2005), it was theoretically shown, that gains are possible by FEXT exploitation. In this contribution an interesting approach for the practical exploitation of the FEXT signal parts is presented: On each wire pair the multicarrier technique OFDM (Hanzo et al., 2000) is used and in addition the mutual impact of the wire pairs in a cable binder via far-end crosstalk is taken into account. Therefore the n-pair cable is modelled as a (n, n) MIMO transmission system and the combination of singular-value decomposition (SVD) and optimal power allocation using the Lagrange Multiplier method is considered with the aim

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of a bit-error minimization at a given data rate. Contrary to other publications considering a similar topic, here the focus lies on the combination of singular-value decomposition and power allocation. Thereby the optimal power allocation solution is presented for given boundary conditions (fixed QAM constellation size and limited total transmit power).

The remaining part of this contribution is organized as follows: Section 2 introduces the cable characteristics and the considered system model including the MIMO-OFDM transmission systems with SVD-based equalization. In section 3 possible optimization objectives for MIMO transmission systems are discussed and the underlying optimization criteria are briefly reviewed. In section 4 the transmit power allocation scheme is explained and in section 5 the obtained results are presented and discussed. Finally, section 6 provides some concluding remarks.

2 SYSTEM MODEL

The distorting influence of the cable on the wanted signal is modelled by the cable transfer function

$$G_{\mathrm{k}}(f) = \mathrm{e}^{-l\sqrt{\mathrm{j}\frac{f}{f_0}}} \quad , \tag{1}$$

where l denotes the cable length (in km) and f_0 represents the characteristic cable frequency (in $\mathrm{MHz}\cdot\mathrm{km}^2$) (Kreß and Krieghoff, 1973).

The far-end crosstalk coupling is covered by the transfer function $G_{\mathrm{F}}(f)$ with

$$|G_{\mathrm{F}}(f)|^2 = K_{\mathrm{F}} \cdot l \cdot f^2 \quad , \tag{2}$$

whereby K_{F} is a coupling constant of the far-end crosstalk, which depends on the cable properties such as the type of isolation, the number of wire pairs and the kind of combination of the wire pairs within the binders (Valenti, 2002).

The considered cable binder consists of n wire pairs and therefore a (n, n) MIMO transmission system arises. The mapping of the transmit signals $u_{\mathrm{s}\,\mu}(t)$ onto the received signals $u_{\mathrm{k}\,\mu}(t)$ (with $\mu = 1,\ldots,n$) can be described accordingly to Fig. 1.

On each wire pair of the cable binder OFDM (orthogonal frequency division multiplexing) is used as transmission technique to combat the effects of the frequency-selective channel (Bahai and Saltzberg, 1999; Bingham, 2000). In such a (n,n)-MIMO-OFDM system, an N-point IFFT (N subchannels) has to be performed on every wire pair. By inserting a guard interval (GI) in front of the transmit signal and removing it at the receiver side an interchannel interference (ICI) and intersymbol interference (ISI) free transmission can be established, assuming that the length of the GI is longer than the temporal extension of the channel impulse response (van Nee and Prasad, 2000). After removing the GI at the receiver an N-point FFT has to be performed. The arising system model is depicted in Fig. 2.

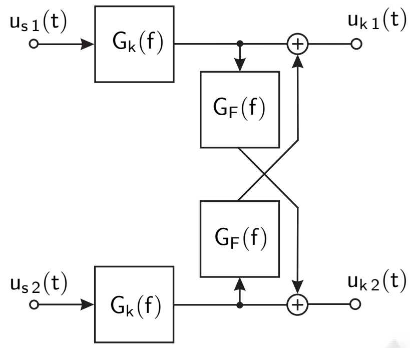

Figure 1: MIMO cable transmission model system with FEXT ($n = 2$).

The kth data block $\boldsymbol{a}_\mu[k]$ of the length $n_{\mathrm{b}} = N$ transmitted by the wire pair μ (with $\mu = 1,\ldots,n$) is denoted by

$$\boldsymbol{a}_\mu[k] = (a_{1\,\mu}[k], a_{2\,\mu}[k], \ldots, a_{N\,\mu}[k])^{\mathrm{T}} \tag{3}$$

and results in a received vector ($\mu = 1,\ldots,n$)

$$\boldsymbol{u}_\mu[k] = (u_{1\,\mu}[k], u_{2\,\mu}[k], \ldots, u_{N\,\mu}[k])^{\mathrm{T}} \quad . \tag{4}$$

To get an adequate system model, it is necessary to rearrange the symbols of the data vector $\boldsymbol{a}_\mu[k]$ (with $\mu = 1,\ldots,n$) defined in (3). Combining all symbols which are transmitted via the same subcarrier in one vector results in

$$\widetilde{\boldsymbol{a}}_\kappa[k] = (a_{\kappa\,1}[k], \ldots, a_{\kappa\,\mu}[k], \ldots, a_{\kappa\,n}[k])^{\mathrm{T}} \quad . \tag{5}$$

Stacking all subcarriers in one vector leads to

$$\widetilde{\boldsymbol{a}}[k] = \left(\widetilde{\boldsymbol{a}}_1^{\mathrm{T}}[k], \ldots, \widetilde{\boldsymbol{a}}_\kappa^{\mathrm{T}}[k], \ldots, \widetilde{\boldsymbol{a}}_N^{\mathrm{T}}[k]\right)^{\mathrm{T}} \quad , \tag{6}$$

where $\widetilde{\boldsymbol{a}}_\kappa[k]$, defined in (5), contains all symbols which are transmitted via the subcarrier κ. Therefore (6) contains all symbols which are transmitted in one time slot simultaneously since n_{b} equals N.

After removing the GI at the receiver side a N-point FFT has to be performed and leads to the received vector similarly to (6)

$$\widetilde{\boldsymbol{u}}[k] = \left(\widetilde{\boldsymbol{u}}_1^{\mathrm{T}}[k], \ldots, \widetilde{\boldsymbol{u}}_\kappa^{\mathrm{T}}[k], \ldots, \widetilde{\boldsymbol{u}}_N^{\mathrm{T}}[k]\right)^{\mathrm{T}} \quad , \tag{7}$$

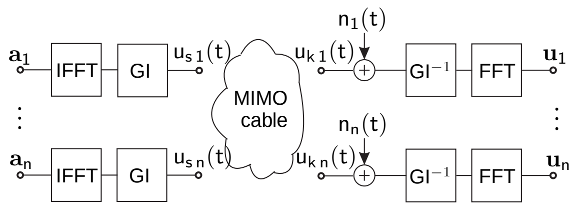

Figure 2: MIMO-OFDM cable transmission model.

where $\widetilde{\boldsymbol{u}}_\kappa[k]$ contains the ISI- and ICI-free receive symbols of the subcarrier κ. However $\widetilde{\boldsymbol{u}}_\kappa[k]$ still contains the crosstalk between neighboring wire pairs on each subcarrier. Due to the GI this vector has the same length as the data vector defined in (6).

Additionally a white Gaussian noise $n_\mu(t)$ (with $\mu = 1, 2, \ldots, n$) with power spectral density Ψ_0 is assumed, which results after receive filtering in the vector $\boldsymbol{n}$ and can be defined similar to (7) as

$$\boldsymbol{n}[k] = \left(\boldsymbol{n}_1^{\mathrm{T}}[k], \ldots, \boldsymbol{n}_\kappa^{\mathrm{T}}[k], \ldots, \boldsymbol{n}_N^{\mathrm{T}}[k]\right)^{\mathrm{T}} \quad . \qquad (8)$$

Thereby it is assumed that the noise components are independently from each other, which can be justified by the rectangular shape of the receive filter functions.

The block oriented transmission system description is given by

$$\widetilde{\boldsymbol{u}} = \mathbf{R} \cdot \widetilde{\boldsymbol{a}} + \boldsymbol{n} \quad . \qquad (9)$$

The matrix $\mathbf{R}$ has a block diagonal structure

$$\mathbf{R} = \begin{bmatrix} \mathbf{R}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{R}_N \end{bmatrix} \quad . \qquad (10)$$

In equation (10) zero-matrices are denoted by $\mathbf{0}$ and for the matrices $\mathbf{R}_\kappa$ (with $\kappa = 1, \ldots, N$) the following syntax is used

$$\mathbf{R}_\kappa = \begin{bmatrix} r_{1\,1}^{(\kappa)} & \cdots & r_{1\,n}^{(\kappa)} \\ \vdots & \ddots & \vdots \\ r_{n\,1}^{(\kappa)} & \cdots & r_{n\,n}^{(\kappa)} \end{bmatrix} \quad , \qquad (11)$$

with the elements describing the couplings of the data symbols on the subchannel κ. Based on the symmetry of the considered transmission system $r_{\nu\,\mu}^{(\kappa)}$ (for $\nu = \mu$) can be determined taking the FFT of $g_{\mathrm{k}}(t) = \mathcal{F}^{-1}\{G_{\mathrm{k}}(f)\}$ into account. The elements $r_{\nu\,\mu}^{(\kappa)}$ (for $\nu \neq \mu$) consider the coupling between neighbouring wire pairs and can be ascertained calculating the FFT of $g_{\mathrm{k\,fn}}(t) = \mathcal{F}^{-1}\{G_{\mathrm{F}}(f) \cdot G_{\mathrm{k}}(f)\}$. The κth value of this vector represents $r_{\nu\,\mu}^{(\kappa)}$. The elements $r_{\nu\,\mu}^{(\kappa)}$ (for $\nu \neq \mu$) are assumed to be identical for each κ, although in practical systems the coupling between the wire pairs is slightly different and it depends on their arrangement in the binder (Valenti, 2002).

The remaining interferences on each subcarrier can now be eliminated by an efficient equalization strategy. A popular strategy is represented by the singular value decomposition (SVD), which can be done on each subcarrier separately. The SVD of the matrix $\mathbf{R}_\kappa$ can be written as

$$\mathbf{R}_\kappa = \widetilde{\mathbf{U}}_\kappa \cdot \widetilde{\mathbf{V}}_\kappa \cdot \widetilde{\mathbf{W}}_\kappa^{\mathrm{H}} \quad , \qquad (12)$$

where $\widetilde{\mathbf{U}}_\kappa$ and $\widetilde{\mathbf{W}}_\kappa^{\mathrm{H}}$ are unitary matrices and $\widetilde{\mathbf{V}}_\kappa$ is a real diagonal matrix (Kovalyov, 2004).

The rearranged data vector $\widetilde{\boldsymbol{a}}_\kappa$ (5) is multiplied by the matrix $\widetilde{\mathbf{W}}_\kappa$ and results in the transmit data vector $\widetilde{\boldsymbol{b}}_\kappa$. The received vector $\widetilde{\boldsymbol{u}}_\kappa = \mathbf{R}_\kappa \cdot \widetilde{\boldsymbol{b}}_\kappa + \boldsymbol{n}_\kappa$ is multiplied by the matrix $\widetilde{\mathbf{U}}_\kappa^{\mathrm{H}}$. Thereby neither the transmit power nor the noise power is enhanced. The overall transmission relationship for the subcarrier κ (with $\kappa = 1, \ldots, N$) is defined as

$$\begin{aligned} \widetilde{\boldsymbol{y}}_\kappa &= \widetilde{\mathbf{U}}_\kappa^{\mathrm{H}} \cdot \widetilde{\boldsymbol{u}}_\kappa = \widetilde{\mathbf{U}}_\kappa^{\mathrm{H}} \cdot \left(\mathbf{R}_\kappa \cdot \widetilde{\mathbf{W}}_\kappa \cdot \widetilde{\boldsymbol{a}}_\kappa + \boldsymbol{n}_\kappa\right) \\ &= \widetilde{\mathbf{V}}_\kappa \cdot \widetilde{\boldsymbol{a}}_\kappa + \widetilde{\mathbf{U}}_\kappa^{\mathrm{H}} \cdot \boldsymbol{n}_\kappa \quad . \end{aligned} \qquad (13)$$

3 OPTIMIZATION OBJECTIVES AND QUALITY CRITERIA

Current signal processing strategies for MIMO systems typically fall into two categories: data throughput maximization at a given transmission quality or bit-error rate minimization at a fixed data rate. In this contribution we have restricted ourselves to the BER minimization at a fixed data rate. Thereby optimal but highly complex or suboptimal solutions with reduced complexity can be found, e. g. (Krongold et al., 2000; Jang and Lee, 2003; Park and Lee, 2004; Ahrens and Lange, 2006).

The signal-to-noise ratio (SNR) is a reasonable performance criterion for noise-dominated scenarios. A signal-to-noise ratio

$$\varrho = \frac{(\text{Half vertical eye opening})^2}{\text{Disturbance Power}} = \frac{(U_{\mathrm{A}})^2}{(U_{\mathrm{R}})^2} \qquad (14)$$

is often defined as a quality parameter (Kreß et al., 1975) with the half vertical eye opening U_{A} and the noise disturbance power U_{R}^2 per quadrature component. Between the signal-to-noise ratio $\varrho = U_{\mathrm{A}}^2/U_{\mathrm{R}}^2$ and the bit-error probability in the general case of M-ary quadrature amplitude modulation (QAM) the interrelationship

$$P_{\mathrm{f}} = \frac{2}{\mathrm{ld}(M)}\left(1 - \frac{1}{\sqrt{M}}\right)\mathrm{erfc}\left(\sqrt{\frac{\varrho}{2}}\right) \qquad (15)$$

holds (Kalet, 1987; Proakis, 2000). The SVD-based equalization on each subcarrier leads to a different half vertical eye opening

$$U_{\mathrm{A}}^{(\varepsilon)} = \sqrt{\xi_\varepsilon} \cdot U_{\mathrm{s}} \qquad (16)$$

for each data symbol. Here, U_{s} denotes the half-level transmit amplitude and $\sqrt{\xi_\varepsilon}$ are the positive square roots of the eigenvalues of the matrix $\mathbf{R}_\kappa^{\mathrm{H}}\,\mathbf{R}_\kappa$, describing the distortions on each subcarrier. Furthermore, each symbol of the data vector $\boldsymbol{a}$ is disturbed by a noise with identical disturbance power in the quadrature components, which is assumed to be uncorrelated with power U_R^2 each. The bit-error probability per symbol for QAM is defined as

$$P_{\mathrm{f}\,\mu} = \frac{2}{\mathrm{ld}(M)}\left(1-\frac{1}{\sqrt{M}}\right)\,\mathrm{erfc}\left(\frac{U_\mathrm{A}^{(\mu)}}{\sqrt{2}\,U_\mathrm{R}}\right) \quad . \tag{17}$$

The aggregate bit-error probability

$$P_\mathrm{f} = \frac{1}{N_\mathrm{b}}\sum_{\mu=0}^{N_\mathrm{b}-1} P_{\mathrm{f}\,\mu} \tag{18}$$

is derived by averaging over the error probabilities of all $N_\mathrm{b} = n \cdot n_\mathrm{b}$ symbols of the data block, since all eye openings occur with the same probability.

4 POWER ALLOCATION

The half vertical eye opening of each symbol position is weighted by the factor $\sqrt{p_\mu}$ and therefore all eye openings of the data block are in general different from each other. Assuming an identical noise power for all symbol positions, the symbol positions with the smallest half vertical eye openings dominate the bit-error rate. Here a transmit power partitioning scheme would be necessary in order to minimize the overall bit-error rate under the constraint of a limited total transmit power.

In a power allocation scheme each symbol of the data block is weighted by a real factor $\sqrt{p_\mu}$. This setup leads to the half vertical eye opening

$$U_\mathrm{A,PA}^{(\mu)} = \sqrt{p_\mu}\cdot\sqrt{\xi_\mu}\cdot U_\mathrm{s} \tag{19}$$

per symbol. The power allocation evaluates the half-level amplitude U_s of the μth symbol by the factor $\sqrt{p_\mu}$. This causes in general a modified transmit amplitude $U_\mathrm{s}\sqrt{p_\mu}$ for each symbol of the transmit data vector and the signal constellation changes. Together with the noise disturbance per quadrature component a BER per symbol and block can be calculated:

$$P_{\mathrm{f}\,\mu} = \frac{2\left(1-\frac{1}{\sqrt{M}}\right)}{\mathrm{ld}(M)}\,\mathrm{erfc}\left(\sqrt{\frac{p_\mu\,\xi_\mu}{2}}\cdot\frac{U_\mathrm{s}}{U_\mathrm{R}}\right) \quad . \tag{20}$$

The aggregate bit-error probability per block yields

$$P_\mathrm{f} = \frac{2\left(1-\frac{1}{\sqrt{M}}\right)}{\mathrm{ld}(M)\,N_\mathrm{b}}\sum_{\mu=0}^{N_\mathrm{b}-1}\mathrm{erfc}\left(\sqrt{\frac{p_\mu\,\xi_\mu}{2}}\cdot\frac{U_\mathrm{s}}{U_\mathrm{R}}\right) \quad . \tag{21}$$

In the subchannels of the multicarrier system investigated in this contribution M-ary square QAM with transmit power (Proakis, 2000)

$$P_{\mathrm{s\,QAM}} = \frac{2}{3}\,U_\mathrm{s}^2\,(M-1) \tag{22}$$

is used (Proakis, 2000). Using a parallel transmission over N subchannels the overall mean transmit power per wire yields to

$$P_\mathrm{s} = N\cdot P_{\mathrm{s\,QAM}} = N\,\frac{2}{3}U_\mathrm{s}^2(M-1) \quad , \tag{23}$$

and results in a total transmit power of $n\,P_\mathrm{s}$ by taking n wire-pairs into account.

Considering now generally different half-level amplitudes $U_\mathrm{s}\sqrt{p_\mu}$ after power allocation on the symbol layers, it follows

$$P_{\mathrm{s}\,\mu} = \left(\sqrt{p_\mu}\right)^2\cdot\frac{2}{3}U_\mathrm{s}^2(M-1) = p_\mu\,\frac{2}{3}U_\mathrm{s}^2(M-1) \tag{24}$$

for the μth symbol position.

If now a block of N_b data symbols, transmitted over N parallel subchannels per wire pair, is analyzed with these generally different half-level amplitudes $U_\mathrm{s}\sqrt{p_\mu}$ after power allocation, the mean transmit power of the block becomes

$$P_\mathrm{s,PA} = n\,N\,\frac{2}{3}U_\mathrm{s}^2(M-1)\,\frac{1}{N_\mathrm{b}}\sum_{\mu=0}^{N_\mathrm{b}-1}\left(\sqrt{p_\mu}\right)^2 \quad . \tag{25}$$

From the requirement

$$P_\mathrm{s,PA} - n\,P_\mathrm{s} = 0 \tag{26}$$

that the overall mean transmit power for the whole binder consisting of n wire pairs is limited to $n\,P_\mathrm{s}$ it follows, that the auxiliary condition

$$\begin{aligned} \frac{n\,N}{N_\mathrm{b}}\,P_{\mathrm{s\,QAM}}\sum_{\mu=0}^{N_\mathrm{b}-1}\left(\sqrt{p_\mu}\right)^2 - n\,N\,P_{\mathrm{s\,QAM}} &= 0 \\ \sum_{\mu=0}^{N_\mathrm{b}-1} p_\mu - N_\mathrm{b} &= 0 \end{aligned} \tag{27}$$

has to be maintained.

In order to find the optimal $\sqrt{p_\mu}$ the Lagrange multiplier method is used (Park and Lee, 2004; Ahrens and Lange, 2006). Contrary to other publications, here the power allocation has not been carried out on each subcarrier independently from each other, although this might be possible. To consider the subcarrier specific distortions (e. g. increasing cable attenuation with increasing subcarrier indices) in a best possible way, here all subcarrier singular values are combined in one vector, in order to smooth out the distortions. The Lagrangian cost function $J(p_0,\cdots,p_{N_\mathrm{b}-1})$ may be expressed as

$$J(\cdots) = \frac{A}{N_\mathrm{b}}\sum_{\mu=0}^{N_\mathrm{b}-1}\mathrm{erfc}\left(\sqrt{\frac{p_\mu\,\xi_\mu}{2}}\cdot\frac{U_\mathrm{s}}{U_\mathrm{R}}\right)+\lambda\cdot B_{N_\mathrm{b}} \quad , \tag{28}$$

with the Lagrange multiplier λ (Park and Lee, 2004) and

$$A = \frac{2}{\mathrm{ld}(M)} \left(1 - \frac{1}{\sqrt{M}}\right) \quad . \tag{29}$$

The parameter B_{N_b} in (28) describes the boundary condition

$$B_{N_\mathrm{b}} = \sum_{\mu=0}^{N_\mathrm{b}-1} p_\mu - N_\mathrm{b} = 0 \tag{30}$$

following from (27). Differentiating the Lagrangian cost function $J(p_0, \cdots, p_{N_\mathrm{b}-1})$ with respect to the p_μ and setting it to zero, leads to the optimal set of power allocation coefficients. As solution for the p_μ we get (a computer algebra system such as MAPLE or MATLAB can come in handy)

$$p_\mu = \frac{1}{\xi_\mu} \frac{U_\mathrm{R}^2}{U_\mathrm{s}^2} \, \mathrm{W}\left(\frac{A^2\,\xi_\mu^2}{2\,\pi\,N_\mathrm{b}^2\,\lambda^2} \frac{U_\mathrm{s}^4}{U_\mathrm{R}^4}\right) \quad , \tag{31}$$

where $\mathrm{W}(x)$ describes the Lambert W function (Corless et al., 1996). The parameter λ can be calculated by insertion of (31) in (30) and numeric analysis. With calculated λ the optimal p_μ can be determined using (31).

Power allocation with lower complexity can be achieved by suboptimal methods, which can on the one hand rely on an approximation for the $\mathrm{erfc}(x)$ function or which ensure on the other hand equal signal-to-noise ratios per symbol (Ahrens and Lange, 2006).

5 RESULTS

The FEXT impact is in particular strong for short cables (Valenti, 2002). Therefore for numerical analysis an exemplary cable of length $l = 0.4\,\mathrm{km}$ with $n = 10$ wire pairs is chosen. The wire diameter is $0.6\,\mathrm{mm}$ and hence a characteristic cable frequency of $f_0 = 0.178\,\mathrm{MHz}\cdot\mathrm{km}^2$ is assumed. On each of the wire pairs a multicarrier system with $N = 10$ subcarriers was considered. The actual crosstalk circumstances are difficult to acquire and they vary from cable to cable. Therefore a mean FEXT coupling constant of $K_\mathrm{F} = 10^{-13}\,(\mathrm{Hz}^2\cdot\mathrm{km})^{-1}$ is exemplarily employed (Valenti, 2002; Aslanis and Cioffi, 1992). The average transmit power on each wire pair is supposed to be $P_\mathrm{s} = 1\,\mathrm{V}^2$ and as an external disturbance a white Gaussian noise with power spectral density Ψ_0 is assumed. Identical systems on all wire pairs were presumed (multicarrier symbol duration $T_\mathrm{s} = 2\,\mu\mathrm{s}$, M-ary QAM, a block length of $n_\mathrm{b} = 10$ and a guard interval length of $T_\mathrm{g} = T_\mathrm{s}/2$). Furthermore, the baseband channel of the multicarrier system is excluded from the transmission in order to provide this frequency range for analogue telephone transmission.

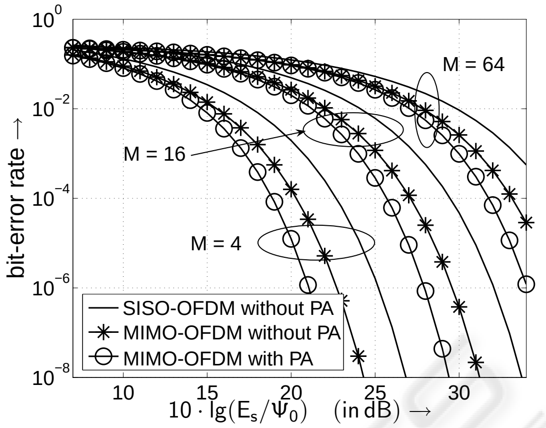

Figure 3: BER comparison analyzing SISO-OFDM ($n = 1$) and MIMO-OFDM ($n = 10, K_\mathrm{F} = 10^{-13}\,(\mathrm{Hz}^2 \cdot \mathrm{km})^{-1}$) with and without PA for different QAM constellation sizes.

For a fair comparison the ratio of symbol energy to noise power spectral density at the cable output is defined for the MIMO case ($n > 1$) according to

$$\frac{E_\mathrm{s}}{\Psi_0} = (T_\mathrm{s} + T_\mathrm{g})\, \frac{P_\mathrm{k} + (n-1) P_\mathrm{k\,fn}}{\Psi_0} \quad , \tag{32}$$

with

$$P_\mathrm{k} = P_\mathrm{s\,QAM} \cdot T_\mathrm{s} \int_{-\infty}^{+\infty} |G_\mathrm{s}(f) \cdot G_\mathrm{k}(f)|^2 \,\mathrm{d}f \tag{33}$$

and

$$P_\mathrm{k\,fn} = P_\mathrm{s\,QAM} \cdot T_\mathrm{s} \int_{-\infty}^{+\infty} |G_\mathrm{s}(f) \cdot G_\mathrm{k}(f) \cdot G_\mathrm{F}(f)|^2 \,\mathrm{d}f \quad . \tag{34}$$

Thereby it is assumed, that the mean transmit power tends to $P_\mathrm{s\,QAM}$ and $G_\mathrm{s}(f)$ is the transmit filter transfer function describing the OFDM pulse shaping. The results are depicted in Fig. 3 with the QAM constellation sizes M as parameter. For purposes of simplicity and in order to obtain meaningful results, the QAM constellation sizes are chosen to be equal in all subchannels of the multicarrier systems. This seems to be reasonable in the example considered here, since short cables do not have very strong frequency-selective characteristics. For general cable transmission the optimization of bit loading with low complexity in the MIMO context remains open for further investigations. Furthermore it seems to be worth mentioning that in case of different QAM constellation sizes M also different overall bit rates can be achieved. This could be used for an adaptation of the bit rate to the user's needs. In case of MIMO-OFDM the signal parts, which are transmitted via the

FEXT paths are no longer disturbance: Now they are exploited as useful signal parts. Therefore the transmission quality is improved compared to the SISO-OFDM case (OFDM transmission over a (fictive) perfectly shielded single wire pair). Similar results are known from MIMO radio transmission with multiple transmit and/or receive antennas, where multiple transmission paths are exploited, too (Raleigh and Cioffi, 1998; Raleigh and Jones, 1999).

The results show that under severe FEXT influence it is worth taking the FEXT signal paths into account (Fig. 3). At small FEXT couplings no significant gains are possible by MIMO-OFDM without PA compared to a perfectly shielded wire pair (SISO-OFDM), because the FEXT coupled signal parts are very small. The results in Fig. 3 show further the potential of appropriate power allocation strategies. The absolut achievable gains depend on the actual cable type and on the isolation of the wire pairs.

6 CONCLUSION

In this contribution, the practical exploitation of the FEXT paths for improving the signal transmission quality was investigated in terms of an exemplary multicarrier transmission system on a symmetric copper cable. It was shown, that the MIMO-OFDM cable transmission enables gains in the BER performance especially under severe FEXT influence. Thereby it could be shown that power allocation is necessary to achieve a minimum bit-error rate. In the exemplary system considered here some restrictions were made, which directly lead to some open points for further investigations: In order to use MIMO-OFDM for cables of any length the most important open point is the optimization of bit loading in combination with the power allocation in the MIMO-OFDM context.

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