

# NONPLANARITY AND EFFICIENT MULTIPLE FEATURE EXTRACTION

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Abstract: A stripe-based image evaluation scheme for real-time vision has been developed allowing efficient detection of the following classes of features: 1. 'Nonplanarity' feature for separating image regions treatable by planar shading models from the rest containing textured regions and corners; 2. edges and 3. smoothly shaded regions between edges, and 4. corners for stable 2-D feature tracking. All these features are detected by evaluating receptive fields (masks) with four mask elements shifted through stripes, both in row and column direction. Efficiency stems from re-use of intermediate results in mask elements in neighboring stripes and from coordinated use of these results in different feature extractors. Corner detection with compute-intensive algorithms can be confined to a small (but highly likely) fraction of the images exploiting the efficient nonplanarity feature. Application to road scenes is discussed.

## 1 INTRODUCTION

Computing power per microprocessors keeps increasing at an almost constant rate of one order of magnitude every 4 to 5 years. This allows combining algorithms even for real-time vision which have been developed for separate use some time ago. The goal of the combined image evaluation method presented here is: 1. to start from as few assumptions on intensity distributions in image sequences as possible, and 2. to re-use as many intermediate results as possible. A rich feature set allows better real-time understanding of dynamic scenes. Since pixel-noise is an important factor in outdoor environments, some kind of smoothing has to be taken into account. This is done by fitting a planar intensity distribution model to a local image region if it exhibits some smoothness conditions; otherwise the region will be characterized as non-homogeneous. Surprisingly, it has turned out that the planarity check for local intensity distribution itself constitutes a nice feature for region segmentation.

Processing images in sequences of stripes allows systematic re-use of intermediate results and provides a nice scheme for navigation in feature arrays for object recognition on higher system levels (not detailed here). Most of the elementary methods

for feature extraction are not new; the reader not acquainted with these methods can find an extensive bibliography including text books in (Price K, USC, Vision – Notes, bibliography, especially chapters 6 to 8). Exploiting the new "nonplanarity feature", they are combined in a very efficient manner. For the same reason of efficiency, an image scaling stage has been put upfront in which pixel intensities are averaged over rectangular regions called 'cells' of size  $m_c \cdot n_c$ . These form the base for image interpretation; image pyramid levels are subsumed by the special case  $m_c = n_c = 2$ .

## 2 STRIPE SELECTION AND DECOMPOSITION INTO ELEMENTARY BLOCKS

The field size for the least-squares fit of a planar pixel-intensity model is  $(2 \cdot m)$  by  $(2 \cdot n)$ , and is called the 'model support region' or mask region. For improving re-use of intermediate computational results, this support region is subdivided into basic (elementary) image regions (called **mask elements** or briefly 'mels') that can be defined by two numbers: The number of cells in stripe direction  $m$ , and normal to it (width of half-stripe)  $n$ . In figure 1,

$m$  has been selected as 4 and  $n$  as 2; the total stripe width thus is 4, and the total mask region is  $8 \cdot 4$  cells. For  $m = n = m_c = n_c = 1$  the highest possible image resolution will be obtained, however, a rather strong influence of noise on the pixel level may show up in the results in this case.

When working with video fields (sub-images with only odd or even row-indices as is often done in practical applications) it makes sense for horizontal stripes to choose  $m = 2 \cdot n$ ; this yields pixel averaging at least in row direction for  $n = m_c = n_c = 1$ . Rendering these mels as squares, finally yields the original rectangular image shape with half the resolution of the original full-frame. Shifting stripe evaluation by half the stripe width only, all intermediate mel results in one half-stripe can be re-used directly in the next stripe by just changing sign. The price to be paid for this convenience is that the results obtained have to be represented at the center point of the support region (mask) which is exactly at cell (pixel) boundaries. However, since sub-pixel accuracy is looked for anyway, this is of no concern. Still open is the question of how to proceed within a stripe. Figure 1 suggests taking steps equal to the length of a mel; this covers all pixels in stripe

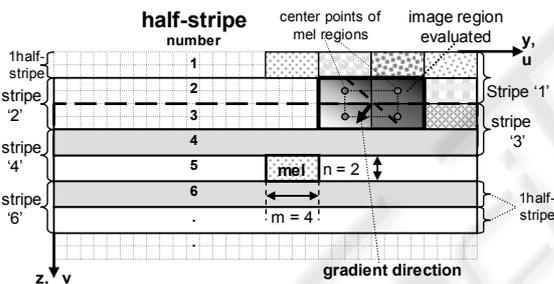


Figure 1: Stripe definition (rows, horizontal) in the operator 'UBM2' (cell-grid); mask elements (mels) are defined as basic units (Hofmann 2004).

direction once and is very efficient. However, shifting mels by just one cell in stripe direction yields smoother (low-pass-filtered) results. For larger mel-lengths, intermediate computational results can be used as shown in figure 2, lower part. The new summed value for the next mel can be obtained by subtracting the value of the last column ( $j-2$ ) and adding the one of the next column ( $j+2$ ) in the example shown. Image evaluation progresses top-down and from left to right.

The goal of the approach selected was to obtain an algorithm allowing easy adaptation to limited computing power onboard vehicles; since high resolution is required in a relatively small part of images only, in general in outdoor scenes, this region can be treated with more finely tuned parameters (foveal – peripheral differentiation).

### 3 REDUCTION OF A STRIPE TO A VECTOR WITH ATTRIBUTES

The first step in mel-computation is to sum up all  $n$  cell values in direction of the width of the half-stripe (lower part in figure 2). This reduces the half-stripe for search to a vector, irrespective of stripe width specified. It is represented in figure 2 by the bottom row (note the reduction in size at the boundaries). All further computations are based on these values which represent the average cell intensity at the location in the stripe when divided by the number of cells summed. However, these individual divisions are superfluous computations and can be spared; only the final results have to be scaled properly.

The operations to be performed for gradient computation in horizontal and vertical direction are shown in the upper left and center part of figure 2. Summing two mel values (vertically in the left and horizontally in the center sub-figure) and subtracting the corresponding other two sums yields the difference in (average) intensities in horizontal and vertical direction of the support region. Dividing these numbers by the distances between the centers

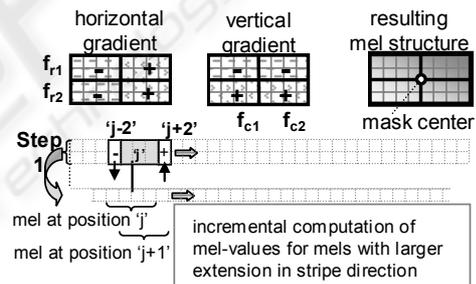


Figure 2: Mask elements (mels) for efficient computation of gradients and average.

of the mels yields a measure of the (averaged) horizontal and vertical image intensity gradient at that location. Combining both results allows computing absolute gradient direction and magnitude. This corresponds to determining a local tangent plane to the image intensity distribution for each support region (mask) selected.

However, it may not be meaningful to enforce a planar approximation if the intensities vary irregularly by a large amount. For example, in the mask of figure 3a) planar approximation does not make sense. It shows the situation with intensities as vectors above the center of each mel. For simplicity the vectors have been chosen of equal magnitude on the diagonals. The interpolating plane is indicated by the dotted lines; its origin is located at the top of the central vector representing the average intensity  $I_M$  in the mask region. From the dots at the center of each mel in this plane it can be recognized that two

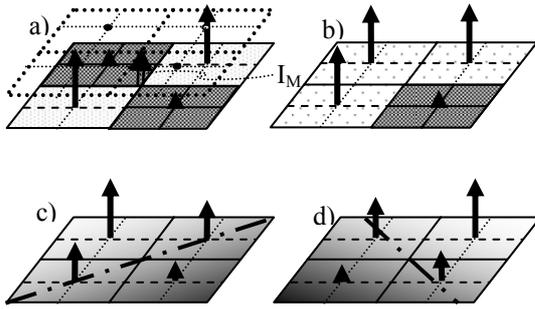


Figure 3: Feature types detectable by the method 'UBM2' in stripe analysis.

diagonally adjacent vectors of average cell intensity are well above, respectively below the interpolating plane. This is typical for two corners (checkerboard) or a textured area (e.g. a saddle point).

Figure 3b) represents a perfect (gray value) corner. Of course, the quadrant with the differing gray value may be located anywhere in the mask. In general, all gray values will differ from each other. The challenge is to find algorithms allowing reasonable separation of these feature types from regions fit for interpolation with planar shading models (lower part of figure 3) at low computational costs. The goal is to segment image stripes into regions with smooth shading, corner points, and extended non-homogeneous regions (textured areas). It will turn out that **'nonplanarity'** is a new, easily computable feature on its own (see section 5). Corner points are of special value in tracking since they often allow determining optical feature flow in image sequences if robustly recognizable.

Stripe regions fit for approximation by sequences of shading models are characterized by their average intensities and their intensity gradients. By interpolation of results from neighboring masks, extreme values of gradients including their orientation are determined to sub-pixel accuracy. Note that, contrary to the previous standard method *KRONOS* (Mysliwetz, 1990; Dickmanns Dirk, 1992) referred to here in the sequel as '*UBMI*', no direction has to be specified in advance; the direction of the maximal gradient is a result of the interpolation process. For this reason the method *UBM2* is called 'direction-sensitive' (instead of 'direction selective' in the case of *UBMI*). It is therefore well suited for initial (strictly 'bottom-up') image analysis with the 'Hofmann-operator' (Hofmann, 2004), while *UBMI* is very efficient once predominant edge directions in the image are known and their changes can be estimated by the 4-D approach (Dickmanns, Wuensche, 1999).

#### 4 INTERPOLATION OF AN INTENSITY PLANE IN A MASK

Average image intensities  $I_{c,ij}$  within 'cells' of size  $m_c \cdot n_c$  are assumed to have been computed beforehand. Cells are used to generate multiple scale images, like e.g. (2\*2) pyramid images of reduced size and resolution for efficient search of larger-scale features. When working with video-fields, cells of size 2 in row and 1 in column direction will bring some smoothing in row direction and lead to much shorter image evaluation times. When coarse-scale results are sufficient, as for example with high-resolution images for regions nearby, cells of size 4 by 2 efficiently yield scene characteristics for these regions, while for regions further away full resolution  $m_c = n_c = 1$  can be applied in much reduced image areas; this focal - peripheral differentiation contributes to efficiency in image sequence evaluation. The region of evaluation at high-resolution may be directed by an attention focusing process on a higher system level based on results from a first coarse analysis (in the present or in previous images). Define  $I_{mel,sum,ij}$  as sum of  $m \cdot n$  (average) cell intensities at location  $i, j$  in the mask and  $I_M$  as average of these values

$$I_M = (I_{mel,sum,11} + I_{mel,sum,12} + I_{mel,sum,21} + I_{mel,sum,22}) / 4 \quad (1)$$

there follows for the four normalized mel-intensities in figure 2, top right:

$$I_{ij} = I_{mel,sum,ij} / I_M \quad (2)$$

Their sum adds up to 1. From sequences of these four numbers of order of magnitude '1' the following features as symbolic descriptors for transition from image data to objects perceived is derived: **1. 'Planar shading' models, 2. 'edges', 3. 'textured areas' (nonplanar elements) and 4. 'corners'.**

Figure 4 shows the local gradients in row (index  $r$ ) and column direction (index  $c$ ) which play a central role in determining these features. The (normalized) gradients in a mask then are:

$$f_{r_1} = (I_{12} - I_{11}) / m \quad (\text{upper row}) \quad (3a)$$

$$f_{r_2} = (I_{22} - I_{21}) / m \quad (\text{lower row}) \quad (3b)$$

$$f_{c_1} = (I_{21} - I_{11}) / n \quad (\text{left column}) \quad (3c)$$

$$f_{c_2} = (I_{22} - I_{12}) / n \quad (\text{right column}) \quad (3d)$$

The global gradient components of the mask in row and column direction then are

$$f_r = (f_{r_1} + f_{r_2}) / 2 \quad (\text{global row}); \quad (4a)$$

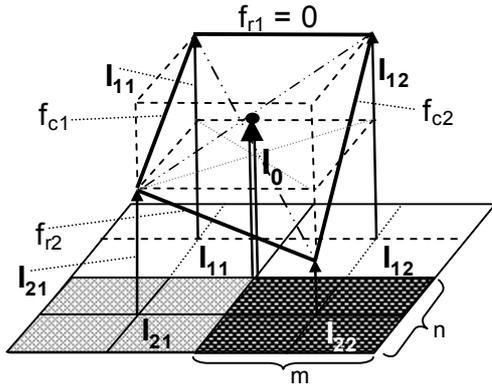


Figure 4: Intensity representations in mask region with four mask elements  $I_{ij}$ ; local gradients  $f_{kl}$ .

$$f_c = (f_{c_1} + f_{c_2})/2 \quad (\text{global column}). \quad (4b)$$

The normalized global gradient and its angular orientation are obtained as

$$g = \sqrt{f_r^2 + f_c^2} \quad \text{and} \quad \alpha = \arctan(f_c / f_r). \quad (5)$$

**Adaptation of a planar shading model in mask area:** The origin for the planar approximation function to the discrete intensity values ( $\sim$  tangent plane) is chosen at the center of the mask area where all four mels meet. The model of the planar intensity approximation with the least sum of errors squared in the four mel-centers has the yet unknown values  $I_c$ ,  $g_u$  and  $g_v$  (intensity at the origin and gradients in u- and v-direction). According to this two-dimensional linear model, the intensities at the mel-centers are computed as functions of the unknown optimal parameters:

$$\begin{aligned} I_{11p} &= I_0 - g_u m/2 - g_v n/2 \\ I_{12p} &= I_0 + g_u m/2 - g_v n/2 \\ I_{21p} &= I_0 - g_u m/2 + g_v n/2 \\ I_{22p} &= I_0 + g_u m/2 + g_v n/2 \end{aligned} \quad (6)$$

Let the measured values from the image be

$I_{11\mu}$ ,  $I_{12\mu}$ ,  $I_{21\mu}$  and  $I_{22\mu}$ . Then the errors

$$e_{ij} = I_{ijp} - I_{ij\mu} \quad \text{can be written:} \quad (7)$$

$$\begin{bmatrix} e_{11} \\ e_{12} \\ e_{21} \\ e_{22} \end{bmatrix} = \begin{bmatrix} 1 & -m/2 & -n/2 \\ 1 & +m/2 & -n/2 \\ 1 & -m/2 & +n/2 \\ 1 & +m/2 & +n/2 \end{bmatrix} \begin{bmatrix} I_0 \\ g_u \\ g_v \end{bmatrix} - \begin{bmatrix} I_{11\mu} \\ I_{12\mu} \\ I_{21\mu} \\ I_{22\mu} \end{bmatrix}$$

In order to minimize the sum of the errors squared, this is written in matrix form

$$\underline{e} = \underline{A} \underline{p} - \underline{I}_\mu. \quad (8)$$

The sum of the errors squared is  $e^T e$  and shall be minimized by proper selection of

$$[I_0 \ g_u \ g_v] = \underline{p}^T. \quad (9)$$

The well known solution via pseudo-inverse is

$$\underline{p} = (\underline{A}^T \underline{A})^{-1} \underline{A}^T \underline{I}_\mu, \quad (10)$$

which finally yields

$$\underline{p}^T = [I_0 \ g_u \ g_v]^T = [1 \ f_r \ f_c]^T. \quad (11)$$

According to eqs. (1) and (2) the  $\mathbf{1}$  as first component of  $\underline{p}$  means that the origin of the interpolating plane with least squares error sum has to be chosen as the average intensity of the mask  $I_M$ .

## 5 RECOGNIZING TEXTURED REGIONS

By substituting eq. (11) into (7), forming  $(e_{12} - e_{11})$  and  $(e_{22} - e_{21})$  as well as  $(e_{21} - e_{11})$  and  $(e_{22} - e_{12})$ , and by summing and differencing the results, one finally obtains

$$e_{21} = e_{12} \quad \text{and} \quad e_{11} = e_{22}; \quad (12)$$

this means that the errors on each diagonal are equal. With eqs. (1 and 7) the sum of all errors  $e_{ij}$  is zero. This means that the errors on the two diagonals have opposite signs, but their magnitudes are equal! These results allow an efficient combination of feature extraction algorithms by forming the four local gradients after eq. (3) and the two components of the gradient within the mask after eq. (4). All four errors of a planar shading model can thus be determined by just one of the four Eqs. (7), that is by 2 multiplications and 4 additions/subtractions. The planar shading model is used when the residues are

$$|e_{ij}| < \varepsilon_{pl,max} \quad (\text{dubbed 'MaxErr'}). \quad (13)$$

From typical road scene images 96 to 99 % of all masks yield errors  $|e_{ij}| < 5\%$  and more than 99 % of all masks yield errors  $|e_{ij}| < 10\%$  (see Table 1 for detailed results). For the rest of the cases, different feature classes have to be applied; they cannot reasonably be approximated by planes. The threshold level MaxErr can be chosen from experience in the task domain and should be selected according to the amount of smoothing desired by the parameters  $m$ ,  $n$ ,  $m_c$  and  $n_c$ .

It can be noticed from **Table 1** that the number of non-planarity features in column search is usually much higher than in row search. Releasing the threshold MaxErr from 5 % to 7.5 % reduces the number of remaining nonplanarity features to less

Table 1: Statistical results of ‘nonplanarity’ features in a typical highway scene.

m, n, m <sub>c</sub> , n <sub>c</sub>	thresh. MaxErr %	row search		column search	
		feature number	% of mels	feature number	% of mels
1 1 1 1	5	1 291	0.59	2 859	1.3
1 1 1 1	7.5	470	0.21	1 113	0.51
3 3 2 1	5	1 553	1.42	3 136	2.86
3 3 2 1	7.5	655	0.60	1 232	1.13
2 2 2 2	5	881	1.62	2 132	3.92
2 2 2 2	7.5	389	0.72	920	1.69
2 2 2 2	10	216	0.40	455	0.84

than one half, in general. Note that 2½ % intensity corresponds to about 6 gray levels out of 256 (8 bit). **Figure 5** shows results for the finest resolution possible, where a mask element is identical to a pixel (rows 1 and 2 in Table 1, images afterwards 2:1 horizontally compressed). Comparing this case (1111) to the following one (3321) shows that the absolute number of non-planarity features is higher even though the number of mels is cut in half by the cell size  $m_c = 2$ ,  $n_c = 1$ . Due to the averaging process



Figure 5 ‘Nonplanarity’ features in original video-field as function of threshold MaxErr (cell size  $m_c = 1$ ,  $n_c = 1$ , mel size 1·1, finest possible resolution ~ 250 x 740 pixel). Top MaxErr = 5 %; 1291 features in row-, 2859 in column search (0.59 % resp. 1.3 % of pixels). Bottom MaxErr = 7.5 %; 470 features in row-, 1113 in column search (0.214 % resp. 0.51 %).

over a larger area, apparently the local nonlinearity in the mask area has been increased (more pixels averaged).

**Figure 6** shows two ‘nonplanarity’ feature sets from both row- and column- search with error thresholds set to 5 and 7.5 % (cell size  $m_c = 2$ ,  $n_c = 1$ , mel size 3·3, corresponding to rows 3 and 4 in Table 1). The reduction in number of features is immediately recognized; the locations of occurrence remain almost the same, however. These are the regions where stable features for tracking, avoiding the aperture problem (sliding along edges) are more likely to be found.

Since computing time decreases with the number of cells as basis for forming the mask elements, this means that nonplanarity features may be an efficient

means for tracking points of interest even in reduced images. All significant corners for tracking are among the nonplanarity features. They can now be searched for with more involved methods, which



Figure 6: Distribution of ‘nonplanarity’ features in a typical highway scene. Left: threshold ErrMax = 5 %; right: ErrMax = 7.5 %. Results from row- and column-search are super-imposed (horizontal and vertical white line elements); cell size  $m_c = 2$ ,  $n_c = 1$ , mel size 3·3.

however, have to be applied to a much reduced set of candidate image regions (a few percent only, see table 1 and section 8).

**Figure 7** shows results corresponding to row 6 of table 1; here, the image has been reduced first to the next higher (2·2) pyramid level, decreasing the number of cells to one fourth the number of pixels.



Figure 7: Nonplanarity’ features on first pyramid level of original video-field (cell size  $m_c = 2$ ,  $n_c = 2$ , yielding 125 x 370 cells). Mel size = 2·2 cells; even with these parameters, reducing computing time by more than an order of magnitude, the same image regions with stable features for tracking are found (on larger scales only). [Image compressed 2:1 horizontally after feature extraction.]

Then, the image is analyzed with stripe width 2 and mel size of 2·2 cells, corresponding to 4·4 pixels in one mel and 16·16 pixel in the total mask region of the original video field. As can be seen immediately, the regions of corner-type features for tracking without aperture problems are the same as for the fine resolution in figure 5; only the fine-grain corners are gone here.

However, computing time with the large masks is reduced by more than one order of magnitude. At 7.5 % error level the relative frequency of feature occurrence (in percent of mels) has increased. This

gives a hint for efficient corner search: Find regions of interest on a larger scale first; then, for precise localization of these features, look with higher resolution in the regions found. The smaller corner features missed initially are likely to be less stable under varying aspect conditions. More experience with this approach in different real road scenes has to substantiate these suppositions.

## 6 EDGES FROM GRADIENT COMPONENTS IN SEARCH DIRECTION

During the planarity tests discussed above, the gradient values of the least squares fit to the intensity function in a mask region have been determined (eqs. 3, 4, 11). Edges are defined by extreme values of the gradient function in search direction. These can easily be detected by computing the differences of two consecutive values in search direction and by multiplying them. If the sign of the product is negative, an extreme value has been passed. With parabolic interpolation from the last three values, the location of the extreme value can be determined to sub-cell accuracy. This indicates that accuracy is not necessarily lost when cell sizes are larger than single pixels; if the signals are smooth (and they become smoother by averaging over cells) the locations of the extreme values may be determined to better than one tenth the cell size. Mel-sizes of several pixel in length and width (especially in search direction), therefore, are good candidates for efficient and fast determination of edge locations with this gradient method. Compared with other methods for edge extraction as separate algorithm it may not be the most efficient one; in combination with the extraction of the other features no comparable algorithm is known to the author.

In order to eliminate noise effects from data, the absolute value of the maximum gradient found has to be larger than a threshold value; this admits only significant gradients as candidates for edges. The larger the mel-size, the smaller this threshold should be chosen. Proper threshold values for classes of problems have to be determined by experimentation; in the long run, the system should be capable of doing this on its own, given corresponding payoff functions. - Since edges oriented mainly in search direction are prone to larger errors, these can be excluded by limiting the ratio of the gradient components allowed. When both gradient components are equal in size, the edge direction is 45°. Excluding all cases where

$$|g_z| < \text{anglfactor} \cdot |g_y| \text{ in row search} \quad (\text{a}); \quad (14)$$

$$|g_y| < \text{anglfactver} \cdot |g_z| \text{ in column search} \quad (\text{b}),$$

with  $\text{anglfact}$  slightly smaller than 1, allows finding all edges by combined row and column search. (Close to diagonal edges should be detected in both search directions leading to redundancy for cross checking.) Sub-mel localization of edges is only performed when all threshold conditions are satisfied. The extreme value is found at that location where the derivative of the gradient is zero.

**Figure 8** shows one example of edge extraction with this method. By choosing proper parameters for mask size and threshold values for noise suppression good results can be achieved. The road area is almost free of edges; other objects are clearly marked, and the lane markings nicely show up. Even the mirror images of some objects on the motor hood of the test vehicle VaMP (Mercedes 500 SEL) are detected and marked (as well as the Mercedes star).



Figure 8: Edges from extreme values in gradient components in row- (white) and column search (black),  $[m = n = 3, m_c = 2, n_c = 1]$ .

## 7 SHADING MODELS IN STRIPES

Space does not allow going into any detail here; an appreciation of what can be achieved in real time by a single modern PC-type processor may be gained from **figure 9**. The image part to the right shows part of the original video field; neglecting sky and own motor hood (bottom) the rectangular region marked white is analyzed as vertical stripe. In the left part of the figure, segmentation of the stripe is shown as image intensity over image row number (increasing from top to bottom like in video signals). The first large shaded segment around row 100 is part of the sky near the horizon; the right-hand half of the figure represents road area with two lane markings

as brighter regions. The MacAdam-surface of the left lane yields the largest segment with linear intensity shading. The centers and extensions as well as the brightness parameters of each segment are

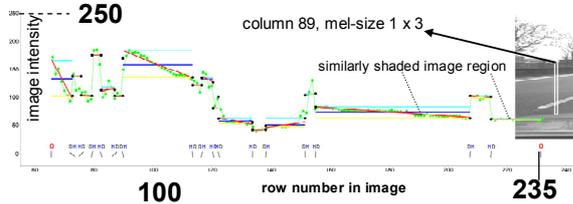


Figure 9: Result from segmentation of a single vertical stripe in a highway scene (see image part on right-hand side) from (Hofmann 2004)

stored.

This is done for each vertical stripe. In this way, intensity values of mels are transformed into lists of segment parameters. In the following step, neighboring areas are checked for merging edges and homogeneous regions into larger 2-D features. This yields extended straight edges and larger homogeneously shaded regions. From these data stored, the symbolically represented image can be reconstructed as real image to show the quality of the representation achieved (Hofmann, 2004).

The challenge in real-time vision is to find the transition from the internal representation as symbols in image space to objects in physical 3-D space and time. Knowing the basic structure of highway scenes with lanes and other objects, hypotheses have to be generated with respect to: where the own vehicle is on the road, where the lane markings are (including the number and widths of lanes actually seen), and where there are other vehicles in the vicinity. A rich set of features alleviates this task. The 4-D approach to dynamic vision has been developed to solve this problem (for a survey see [Dickmanns and Wuensche 1999]; references to detailed descriptions of the approach in many dissertations are given there).

## 8 THE CORNER ALGORITHM

So-called 2-D-features designating image points have been studied since they allow avoiding the ‘aperture problem’; it occurs for features in a plane that are well defined in one of the two degrees of freedom only, like edges. Since general texture analysis requires significantly more computing power not yet available for real-time applications in the general case right now, we will also concentrate on those points of interest which allow reliable recognition, tracking and computation of feature

flow. Starting from (Moravec 1979) well known algorithms for corner detection (among many others) are given by (Harris CG 1988), the KLT-method by (Birchfield S 1994; Lucas BD, Kanade T 1981; Tomasi C, Kanade T 1991; Shi J, Tomasi C 1994) and by (Haralick RM, Shapiro LG 1993), all based on combinations of intensity gradients in more or less extended regions and in several directions. The basic ideas have been adapted and integrated into the present algorithm dubbed ‘UBM2’.

Based on these references the following algorithm for ‘corner detection’ fitting into the mask scheme for planar approximation of the intensity function has been derived and proven efficient. The ‘structural matrix’

$$N = \begin{bmatrix} f_{r1N}^2 + f_{r2N}^2 & 2 \cdot f_{rN} \cdot f_{cN} \\ 2 \cdot f_{rN} \cdot f_{cN} & f_{c1N}^2 + f_{c2N}^2 \end{bmatrix} = \begin{bmatrix} n_{11} & n_{12} \\ n_{12} & n_{22} \end{bmatrix} \quad (15)$$

has been defined with the terms from Eqs (3) and (4). With the equations mentioned the determinant of the matrix N is

$$\det N = n_{11} \cdot n_{22} - n_{12}^2 = 0.75 \cdot n_{11} \cdot n_{22} - 0.5 \cdot (n_{11} \cdot f_{c1} f_{c2} + n_{22} \cdot f_{r1} f_{r2}) - f_{r1} f_{r2} f_{c1} f_{c2} \quad (16)$$

Haralick calls  $\det N$  the ‘Beaudet measure of cornerness’, however, formed with a different term on the cross-diagonal  $n_{12} = \sum f_{ri} f_{ci}$ . With the quadratic enhancement term  $Q = (n_{11} + n_{22}) / 2$  the two eigenvalues  $\lambda_1$  and  $\lambda_2$  of the structural matrix are obtained as

$$\lambda_{1,2} = Q \left[ 1 \pm \sqrt{1 - \det N / Q^2} \right]. \quad (17)$$

Defining  $\lambda_{2N} = \lambda_2 / \lambda_1$ , Haralick’s measure of circularity q becomes

$$q = 1 - \left[ \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2} \right]^2 = \frac{4 \lambda_{2N}^2}{(1 + \lambda_{2N})^2}. \quad (18)$$

It can thus be seen that the normalized second eigenvalue  $\lambda_{2N}$  and circularity q are different expressions for the same property. In both parameters the absolute magnitude of the eigenvalues is lost. As threshold value for corner points a minimal circularity  $q_{min}$  is chosen as lower limit:

$$q > q_{min} \quad (19)$$

$$\text{trace}N = \lambda_1 + \lambda_2 > \text{trace}N_{min}$$

may be selected as additional threshold. In a post-processing step, within a user-defined window D, only the local maximal value  $q^*$  is selected as corner. For larger D the corners tend to move away

from the correct position. With the definitions taken, a double corner (like on a checker board, figure 3a) has  $q = 1$ ; a single ideal corner (figure 3b) has  $q = 0.75$ . For intensity distributions allowing good planar approximations,  $q$  goes towards 0. The threshold value  $q_{\min}$  may be adapted from experience in the domain. Minimal circularity values for stable corners should be set around  $q_{\min} \approx 0.7$ . According to eq. (18) this yields  $\lambda_{2N}$  values smaller than about 0.3. When too many corner candidates are found, it is possible to reduce their number not by lifting  $q_{\min}$  but by adjusting the threshold value 'trace $N_{\min}$ ', which limits the sum of the two eigenvalues. According to the main diagonal of eq. (15) this means prescribing a minimal value for the sum of the squares of all local gradients in the mask. This parameter depends on the absolute magnitude of the gradient components and has thus to be adapted to the actual situation at hand. It is interesting to note that the threshold ErrMax for planarity check (eq. 13) has a similar effect as the boundary for the threshold value trace $N_{\min}$  on corners.

**Figure 10** shows corners (black crosses) found in nonplanarity regions (white bars) in vertical (left) and horizontal search (right) on the first pyramid level ( $m_c = n_c = 2$  yielding a reduced image of about 45,000 cells). Mask size with  $m = n = 2$  thus was 4-4 = 16 pixel. 2001 nonplanarity features (~ 4.4 % of number of cells) with interpolation errors larger than ErrMax = 5 % have been found in vertical search. From these, 108 locations (dark crosses) have been determined satisfying the corner conditions: circularity  $q_{\min} = 0.7$  and trace $N_{\min} = 0.2$  (figure 10, left). The right-hand part of the figure shows result of horizontal search with the same parameters except trace $N_{\min} = 0.15$  (reduced for increasing the number of accepted corners). 865 mask locations (~1.9 %) yield 40 corner candidates (dark crosses). By adjusting threshold levels, the number of corner features obtained can be modified according to the needs in actual applications. Combining corner features obtained with different cell- ( $m_c, n_c$ ) and mel-sizes ( $m, n$ ) yet has to be investigated; it is expected that this will contribute to achieving increased robustness.

The results in row and column search differ mainly because stripes are shifted by half-stripe width  $n$  (here = 2) laterally, while in search direction masks are shifted by just one cell.

## 9 CONCLUSIONS

Checking for the goodness of planarity conditions when fitting local linear intensity models to image segments has led to the new 'nonplanarity'-feature. In typical road scenes, only 1 to 5 % of all mask locations exceed threshold values of 3 to 10 % planarity error (residue values). This yields an efficient pre-selection for checking corner features. The gradient components between the mask elements are used in multiple ways to determine nonplanar intensity regions, corners, edges and segments with linear shading models. Merging of these features over neighboring stripes leads to larger 2-D features. Some applications to road scenes have shown the efficiency achievable.

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