PERCEPTUAL ORGANIZATION OF DIRECTIONAL PRIMITIVES USING A PSEUDOCOLOR FUZZY HOUGH TRANSFORM FOR ARC DETECTION

Marta Penas, Manuel G. Penedo, Noelia Barreira
Dpt. Computer Science. Universidade da Coruña
Campus de Elviña S/N. 15071, A Coruña. Spain

María José Carreira
Dpt. Electronics and Computer Science. Universidade de Santiago de Compostela
Campus Sur. 15782, Santiago de Compostela. Spain

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Abstract: This paper describes a computational framework for extracting the low-level directional primitives present in an image and organizing them into circular arcs. The system is divided into three stages: extraction of the directional features through an efficient implementation of the Gabor wavelet decomposition, reduction of the high dimensional Gabor results by means of growing cell structures and detection of the circular arcs by means of a pseudo-color Fuzzy Hough Transform.

1 INTRODUCTION

The boundaries of objects in an image often lead to oriented and localised changes in intensity called edges. Edge, segment and arc detection are the first steps in many image analysis applications and they are of great importance as they constitute the basis for higher level analysis. It has always been a fundamental problem in computer vision that the higher level processing stages suffer due to either too little or too much data from the lower levels of the processing. Thus, the quality of data available for further analysis is very critical.

This paper describes a framework for the extraction of the directional properties present in an image through Gabor wavelet decomposition (Gabor, 1946) and the detection of the circular arcs that approximate these properties through a circle detector based on the fuzzy Hough transform (Duda and Hart, 1972).

The Gabor wavelet decomposition framework presented here is a computationally expensive process, but provides precise information about the image pixel orientations and is independent of the image type. Moreover, we have implemented (M. Penas and Penedo, 2003a) an approximation to Gabor wavelets that reduces the computational time and memory requirements through the use of a Gabor decomposition in the spatial domain, which is faster than conventional frequency domain implementations.

A previous paper (M. Penas and Mariño, 2005) describes a line segment detector based in the combination of a pseudo-colour Hough transform and the Burns segment detector. The fuzzy Hough transform works properly in simple images that only contain the objects to detect, but has some limitations in complex images due to its global nature. Due to this limitations, we have proposed a refinement of the pseudo-colour Hough transform through the introduction of some principles of the Burns segment detector.

This paper introduces the pseudo-colour fuzzy Hough transform for the detection of circular arcs. These circular arcs could be combined with the line segments previously detected in a perceptual grouping framework that integrated the relationships offered by the Gestalt psychology (parallelism, continuity, similarity, symmetry, common region and closure) among input tokens to form large groups that could be useful for object detection or image classification.

2 EXTRACTION OF DIRECTIONAL PRIMITIVES

This section contains a brief introduction of the first stages of the process, the extraction of the directional primitives present in the image through Gabor wavelet decomposition and the organisation of these results through a growing cell structure. These stages construct the pseudo-colour images that the circular

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arc detector will receive as input.

Gabor wavelets (Gabor, 1946) are complex exponential signals modulated by Gaussians with two important properties that make them good edge detectors: the optimisation of edge localisation (Deemter and Buf, 2000) and the absence of image-dependent parameter tuning. Their most important drawback is their greedy demand for both memory and computation time. In a previous paper (M. Penas and Penedo, 2003b), we developed a more efficient, multiresolution spatial domain implementation of the Gabor wavelet decomposition based on the convolution of 11 1D-component masks obtained through the decomposition of the 2D masks that define the wavelets. This implementation uses the good edge localisation property of Gabor wavelets, with the exact position of an edge determined as a conjunction between a maximum in the modulus and a zero crossing in the even or the odd part of the Gabor results.

In our Gabor decomposition, the input image is filtered with a bank of 8 filters centred at frequency $\frac{1}{4}$ and 8 orientations ($\frac{k\pi}{8}, k = 0..7$) leading to 8 resulting images. A reduction of this output space dimensionality is necessary in the interest of efficiency. Auto-organised structures are a suitable instrument to achieve this dimensionality reduction as they allow the reduction of the input space and the projection of the topological order in the input space to the output structure, simultaneously.

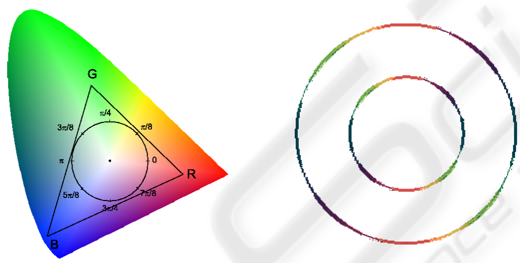

Figure 1: Left: Colourmap circle inside the RGB triangle. Right: All orientations after the GCS analysis.

Self-organised maps, growing cell structures and growing neural gas structures were investigated and compared for their power of dimensionality reduction of Gabor decomposition results (M. Penas and Penedo, 2001). Growing cell structures (GCS) (Fritzke, 1994) provided significantly better results. They are artificial neural networks based on self-organised maps that eliminate the restrictions of the *a priori* network size definition, incorporating a mechanism to add new processing elements when needed, while maintaining the network topology.

To represent the different directionalities provided by the auto-organised structure, each processing element was assigned a colour from a colourmap to indicate its orientation. The colourmap was obtained from 8 equidistant points on the perimeter of the maximum circle inside the RGB triangle in the chromaticity diagram (Wyszecki and Stiles, 1982), centred at white (see fig. 1-left). These 8 points represent the colours assigned to the 8 main orientations considered. Points in the perimeter of the circle show the colour graduation assigned to the processing elements in the GCS. Fig.1 right shows the GCS output from a ring demonstrating the colours of the entire direction space, i.e. $0 - 2\pi$.

Fig. 2 shows the results from three input images that contain multiple circular arcs. The first column shows the original images and the second column the results from GCS analysis.

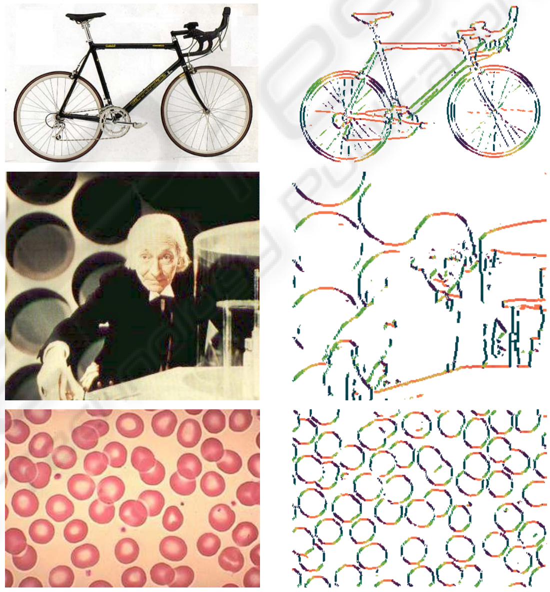

Figure 2: Left: images 'bicycle', 'face' and 'cells'. Right: results of GCS analysis.

3 CIRCULAR ARC DETECTION THROUGH THE HOUGH TRANSFORM

The Hough transform is widely used in Computer Vision and Pattern Recognition for the detection of geometrical shapes that can be defined through parametric equations. Traditional Hough transform implementations are based on the results obtained by classical edge detectors, like Canny or Sobel. This paper describes the design and implementation of a fuzzy Hough transform based on the pseudo-colour images obtained from previous processes, this is, colour images where each colour represents an orientation.

The Hough transform for circle detection is based on the parametric equation of the circle, defined as:

$$(x_i - a)^2 + (y_i - b)^2 = r^2 \qquad (1)$$

where (a, b) are the coordinates of the circle centre and r is the radius. The implementation of the Hough transform in a digital computer requires the quantisation of the continuous $a - b - r$ space into suitable sized $(\triangle a \times \triangle b \times \triangle r)$ cubes and the association of each of these cubes with a cell of a 3D accumulator array A of $(N_a \times N_b \times N_r)$ size.

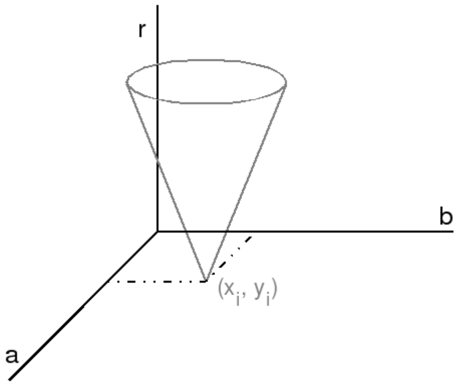

Figure 3: 3D voting space of the standard Hough transform for circle detection.

In the standard Hough transform, each edge pixel (x_i, y_i) votes for all the possible circles that pass through it. This voting process generates a 3D cone as depicted in fig. 3. This approach needs to be enhanced to deal with the edges in our colour-labelled images, where the colour of a pixel represents the orientation of its normal line an determines where the centre of the circle must be located. Therefore, the voting space of a pixel (x_i, y_i) is reduced to the line depicted in fig. 4. Thus, the use of the orientational information highly reduces the computational complexity of the process.

Due to the multiplicity of the edges in the pseudo-colour images and the inaccuracies in the edge location and orientation, we have implemented a fuzzy Hough transform where each pixel votes, not only for the orientation defined by its colour, but also for the adjacent orientations.

3.1 Fuzzy Hough Transform for Circle Detection

The process begins with the quantisation of the Hough space into $(N_a \times N_b \times N_r)$ cells, where $N_a = I_x$, $N_b = I_y$, $N_r = min(I_x, I_y)$ and (I_x, I_y) are the dimensions of the input image. As previously mentioned, each cell is assigned to a position in a 3D accumulator array A. In this quantisation, $\triangle a = \triangle b = \triangle r = 1$ in order to achieve the maximum precision possible.

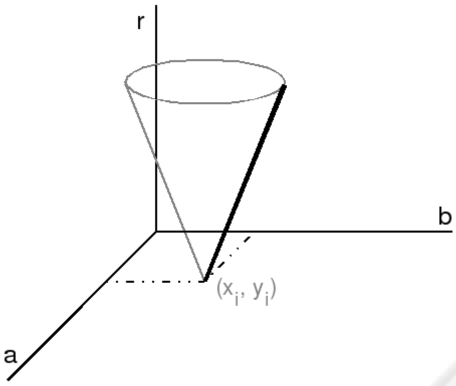

Figure 4: The line on the cone's surface is the voting space of the pseudo-colour Hough transform.

Then, the contribution of each pixel $P = (x_i, y_i)$ to the accumulator array is computed. First, the angle θ_P of the pixel is determined from its colour. Then, the voting space of the pixel is constructed. In the fuzzy Hough transform, each pixel votes for the set of centres and the corresponding radios contained in the grey area depicted in fig. 5. This area is delimited by the two lines that pass through P with slopes $\theta_P + \frac{\pi}{12}$ and $\theta_P - \frac{\pi}{12}$ and by the maximum radius stipulated.

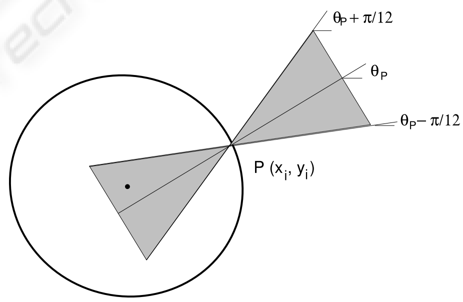

Figure 5: Voting space of pixel P in the pseudo-colour fuzzy Hough transform for circle detection.

The contribution of the pixel P to the accumulator array is not homogeneous for all the $(a - b - r)$ in fig. 5. It is maximum over the line that passes through P with slope θ_P and decrease with the orientation difference. Specifically, the contribution is defined through the following Gaussian function:

$$A(a_j, b_j, r_j) = e^{-\beta \cdot d(\theta_P, \theta_j)} \qquad (2)$$

where the parameter that defines the decay of the Gaussian function β is set to 10, as the contribution to the accumulator array must rapidly decrease with the orientation difference, falling to the minimum when $d(\theta_P, \theta_j) = \frac{\pi}{12}$.

Once the voting process has finished, the following step is the detection of maxima. Each maximum detected in the accumulator array corresponds to a circle in the image that can contain one or more arcs. For each maximum over a predefined threshold, an inverse Hough transform takes place removing the contributions to the accumulator array of the pixels belonging, not only to the circle detected, but also to the concentric neighbouring ones. The detection of arcs takes place simultaneously to the inverse Hough transform. When a pixel is removed from the accumulator array, its orthogonal projection to the circle it belongs to is determined. Once all the pixels involved have been analysed, the circle is sequentially searched determining which pixels have been coloured and thus belong to an arc.

Fig. 6 right shows the circle detection results over the images depicted in fig. 6 left.

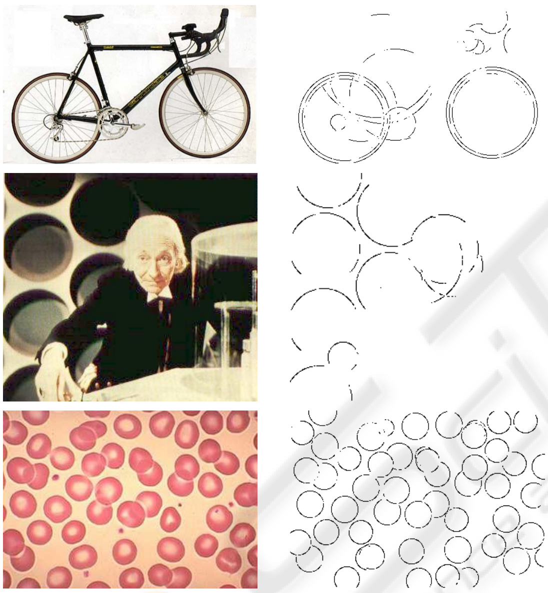

Figure 6: Left: input images. Right: results of the arc detection process with the fuzzy Hough transform.

The implementation just described accurately detects the centres and radius of the circles but has two important drawbacks, the computational complexity and the memory elapsed. Next section describes a probabilistic implementation of the fuzzy Hough transform that reduces the computational complexity at the expense of reducing the quality of the results.

3.2 Probabilistic Fuzzy Hough Transform for Circle Detection

The probabilistic Hough transform (Goulermas and Liatsis, 1999) is based on the possibility of detecting the circles in an image through the analysis of a limited subset of the edge points. It is also based in the following property depicted in fig. 7: if A and B are two points belonging to a circle, the normal lines that connect them with the centre of the circle and the perpendicular bisector of the segment that joins them intercept in the centre of the circle.

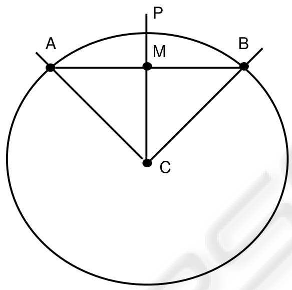

Figure 7: Property that relates two points that belong to a circle.

Due to the innacuracies in the orientation estimation and the effect of the discretization necessary in the digital images, each point votes not only for the radius and centre determined by the intersection of the normals and the perpendicular bisector, but for those in the voting space depicted in fig. 8. This voting space consists of a line segment that belongs to the perpendicular bisector and is delimited by the maximum radios allowed and/or the voting spaces of the fuzzy Hough transform for circle detection in points A and B, respectively.

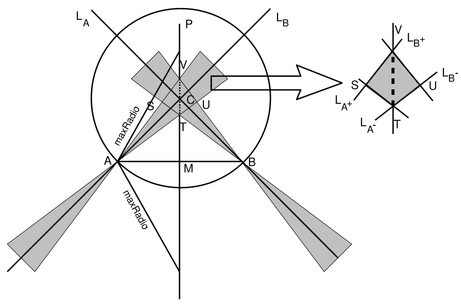

Figure 8: Probabilistic fuzzy Hough transform. The voting space is the dotted line segment enlarged on the right of the figure.

The operation of the algorithm is divided into two phases, the voting and the search for maxima in the accumulator array. The voting begins with the random selection and storage in a list L of a small percentage n ($\approx 5\%$ of N) of the N edge points. Then, for each of the $K = \frac{n \cdot (n-1)}{2}$ combinations of point pairs, the voting space is computed and the corresponding cells in the accumulator array are incremented.

As in the fuzzy Hough transform, the contribution to the accumulator array is not homogeneous and depends on the orientation distance (see eq. 2).

Once the voting process has finished, the search for maxima in the accumulator array begins. Each maximum detected represents a candidate circle in the image but, as only a small subset of points has voted, in order to check the existence of the circle, the inverse Hough transform must calculate the percentage of edge points that belong to the circle detected. Only if this percentage is over a predefined threshold, the candidate is considered a circle.

If the candidate is considered a circle, all the points that belong to it are deleted from the edge image and the accumulator array, the list L is updated up to its original size n with points chosen at random and the next maximum is analysed. If the circle is not valid, a predefined number of points t ($t \approx 20\%$ of n) are erased and the same number of new points are inserted into L. The points erased are those that less contributed to the accumulator array. The algorithm stops when three non-valid circles are consecutively detected.

Fig. 9 left shows the results obtained from the three input images depicted fig. 9 right.

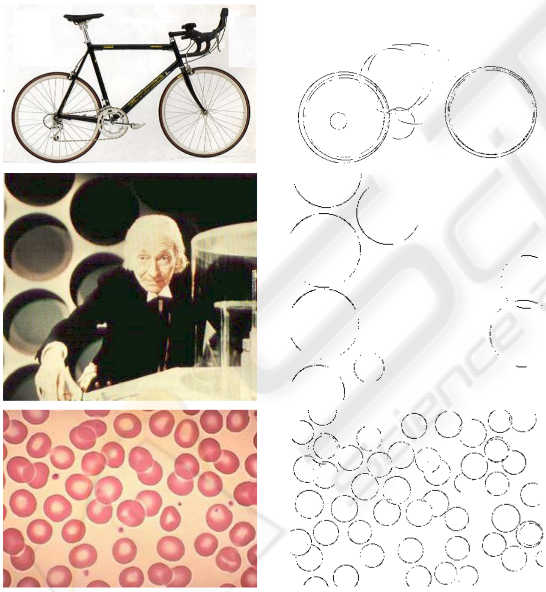

Figure 9: Left: input images. Right: results of the arc detection process with the probabilistic Hough transform.

The probabilistic fuzzy Hough transform has an important advantage with respect to the non-probabilistic fuzzy Hough transform, the high computational complexity reduction. But it also has some important drawbacks. The first one is due to the use of a small subset of points, which leads to inaccuracies in the results specially when the image contains circles with very different sizes. Also, as the points are randomly chosen, the results can vary among the executions. Finally, the probabilistic Hough transform works with combinations of pairs of points, if the sample of points analysed has to be renewed many times during the execution, the computational time can exponentially grow.

In order to improve the results of the probabilistic fuzzy Hough transform and to reduce the computational complexity of the fuzzy Hough transform, some basic principles from both implementations have been combined in a new probabilistic fuzzy Hough transform.

This implementation of the Hough transform is very similar to the fuzzy Hough transform introduced in sec. 3.1 but only a determined percentage N of the input edge points is analysed. The sample points are not randomly chosen, but homogeneously selected from the input image. Empirically, we have checked that a percentage of $N \geq 33\%$ of the edge points produces good results.

The voting process of the sample points is analogous to the one described in sec. 3.1 for the fuzzy Hough transform. The voting process is followed by the search for maxima in the accumulator array. As in the probabilistic fuzzy Hough transform previously described, each maximum detected represents a possible circle in the image but, as only a small subset of points has voted, in order to check the existence of a circle, the inverse Hough transform must compute the percentage of edge points that belong to the circle detected. Only if this percentage is over a predefined threshold, the candidate is considered a circle.

Fig. 10 right shows the arc detection results from the three input images depicted in fig. 10 left. These results are very similar to those obtained by the fuzzy Hough transform (fig. 6 right) while the computational complexity of this implementation is considerably lower.

4 DISCUSSION AND CONCLUSION

This paper describes a computational framework for the detection of circular arcs in 2D digital images. The framework is divided into three stages: the directional primitive extraction through Gabor wavelets, the organisation of these primitives through auto-organised structures and the circle detection through a probabilistic fuzzy Hough transform.

The pseudo-colour fuzzy Hough transform introduced in sec. 3.1 produces very good results, but it has an important drawback, the high computational complexity. The probabilistic Hough transform introduced in sec. 3.2 overcomes this drawback but

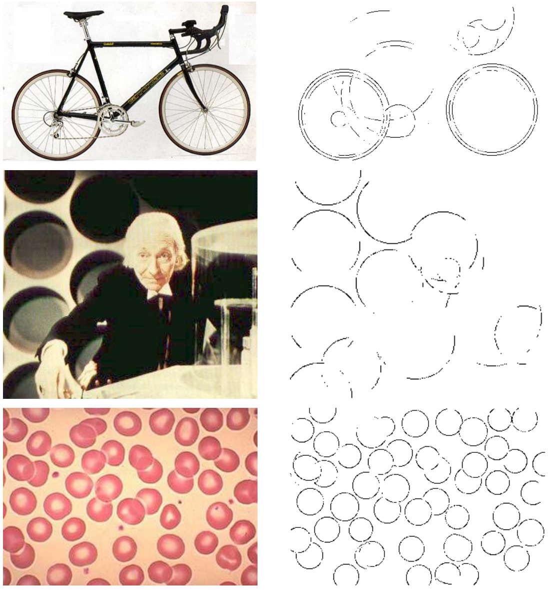

Figure 10: First column: input images. Second column: results of the arc detection process with the new probabilistic Hough transform.

its results are not too accurate. Finally, the probabilistic Hough transform introduced at the end of sec. 3.2 produces good results while keeping the computational complexity low, through the introduction of a new way of selecting the candidate edges.

Fig. 11 shows the computational time (in minutes) of the three versions of the Hough transform described in this paper. The data depicted in fig. 11 have been collected from the analysis of 10 input images containing a different number of edge points ranging from 4026 to 19227, and shows that the computational complexity depends both on the number and the distribution of the edge points.

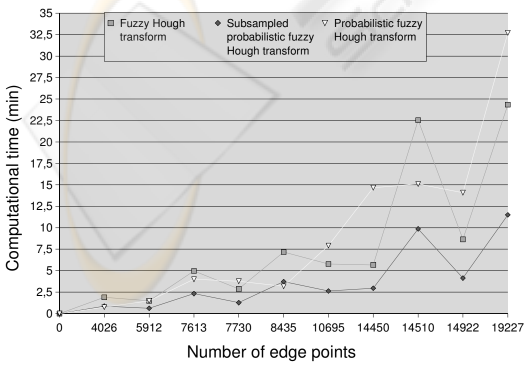

Figure 11: Computational times of the alternative versions of the fuzzy Hough transform.

The three implementations could be improved through the use of directional information. The pseudo-colour images that the fuzzy Hough transform receives as input contain information about the orientation of the edges, but not about their direction. The directional information could be introduced through the application of a Sobel filter to the pseudo-colour edge images, where the noise presence is almost null. The use of directional information could reduce the computational complexity of the three implementations and increment the accuracy of the probabilistic ones.

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