

# SPEEDING UP SNAKES

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**Abstract:** In this paper we summarize new and existing approaches for the semiautomatic image segmentation based on active contour models. In order to replace the manual segmentation of images of the medical research of the Center of Anatomy at the Georg August University of Göttingen we developed a user interface based on snakes. Due to the huge images (sometimes bigger than 100 megapixels) the research deals with, an efficient implementation is essential.

We use a multiresolution model to achieve a fast convergence in coarse scales. The subdivision of an active contour into multiple segments and their treatment as open snakes allows us to exclude those parts of the contour from the calculation, which have already aligned with the desired curve. In addition, the band structure of the iteration matrices can be used to set up a linear algorithm for the computation of one single deformation step. Finally, we gained an acceleration of the initial computation of the Edge Map and the Gradient Vector Flow by the use of contemporary CPU architectures.

Furthermore, the storage of huge images next to additional data structures, such as the Gradient Vector Flow, requires lots of memory. We show a possibility to save memory by a lossy scaling of the traditional potential image forces.

## 1 INTRODUCTION

Imaging techniques have achieved a great importance in many scientific domains, particularly in the medical science. Whole series of digital photos are consulted for the analysis of three-dimensional structures. In order to reconstruct these three-dimensional models, it is necessary to perform an image segmentation.

In the framework of studying cellular events in the embryonic development at the Georg August University of Göttingen, the segmentation of huge images of serial histological sections is done manually in a tedious process. We developed a user interface that allows a semiautomatic segmentation of these images based on active contours.

The snake model, first introduced by Kass (Kass et al., 1987), offers a practical tool for the semiautomatic image segmentation. It combines many advantages but features also some drawbacks which gave reason for the publication of several improvements.

One major disadvantage is the necessity of initializing a contour nearby the object's edges due to the narrow attraction range of the classical image en-

ergy. Kass proposed a convolution of the image with a Gaussian kernel which slightly increases the capture range by blurring the original image. Rueckert's (Rueckert and Burger, 1996) multiscale approach faces that problem by using different scales of the kernel's standard deviation. A balloon model was proposed by Cohen (Cohen, 1991). Pressure forces along the contour's normals allow the inflation or the accelerated deflation of the snake. In order to increase the attraction range of image edges Xu and Prince (Xu and Prince, 1997) introduced an alternative image energy called Gradient Vector Flow. They calculate a diffusion of the gradient vectors of a gray-level Edge Map which additionally enables the active contour to move into concave boundary regions. This is another problem associated with the classical snake model.

The snake's tendency of shrinking due to the formulation of the internal energy terms discouraging the stretching and bending behavior is sometimes stated as a disadvantage, too. By applying Cohen's balloon model (Cohen, 1991) a snake is able to counteract the contraction. Perrin (Perrin and Smith, 2001) proposed alternative formulations of the internal energy which

also produce locally smooth curvatures and an evenly spacing of the control points. In contrast to the traditional method this approach avoids shrinking and pushing the model toward a straight line.

In (Mayer et al., 1997) ribbon snakes are proposed for the road extraction in cartographic images, i.e. the detection of thick lines with a significant width. A ribbon snake is represented by a set of control points on the centerline and a width at each point forming two ribbon sides that are forced to be parallel. Kerschner (Kerschner, 2003) modifies this approach and treats the two ribbon sides as separate snakes, called twin snakes, with an additional attraction force among each other which are thus relatively independent of their initial position. This idea is similar to the dual active contour model, published by Gunn (Gunn and Nixon, 1997). In order to relax the sensitive choice of parameters and the snake's initialization nearby the desired edges, they use two interlinked contours. One contour expands from inside the target object while the other one contracts from the outside. By the means of an alternating deformation process the coupled snakes try to reach the same equilibrium. That way, even weak local minima can be rejected.

Ziplock snakes were introduced by Neuenschwander (Neuenschwander et al., 1997). Only the two end points of the open snake must be specified by the operator, but exactly on the desired curve. The snake is first approximated with a straight line between these points. Beginning at the end points, gradually the vertices are activated to optimize their position, i.e. take part in the deformation process which is finished when all vertices are movable and the snake stabilizes.

One further drawback of classical snakes is their topological inflexibility. Geometric active contours, developed independently by Caselles (Caselles et al., 1993) and Malladi (Malladi et al., 1995), are based on the theory of curve evolution and geometric flows. Using a level set based implementation (Osher and Sethian, 1988), the contours can easily split and merge in the deformation process. A parametric active contour model, that solves this problem, is the topologically adaptable snake model, called T-Snake which was presented by McInerney (McInerney and Terzopoulos, 1995). There, a simplicial grid is superposed over the image domain which is used to iteratively reparameterize the deforming active contour, also allowing a dynamical change in topology.

Due to the different problems associated with snakes, an adaption of the model is required for each special case. This paper summarizes some existing approaches and some new aspects of speeding up the snake deforming algorithm that are applied in our implementation. It comprises a multiresolution strategy, the use of multithreading techniques, subdividing the snake into segments and a detailed description how to compute one iteration step in  $\mathcal{O}(n)$  instead of  $\mathcal{O}(n^3)$ .

## 2 ACTIVE CONTOUR MODELS

A snake is a unit speed curve  $\vec{v} : [0, 1]_{\mathbb{R}} \rightarrow \mathbb{R}^2$  with  $\vec{v}(s) = (x(s), y(s))^T$  that deforms on the spatial domain of an image by minimizing the following energy functional:

$$E = \int_0^1 \frac{1}{2} \left( \alpha \left| \frac{\partial \vec{v}}{\partial s} \right|^2 + \beta \left| \frac{\partial^2 \vec{v}}{\partial s^2} \right|^2 \right) + E_{ext}(\vec{v}) ds \quad (1)$$

where  $\alpha$  and  $\beta$  are positive weighting parameters. The first two terms belong to the internal energy  $E_{int}$  controlling the smoothness of the snake while the external energy  $E_{ext} = E_{img} + E_{con}$  consists of a potential energy  $E_{img}$  providing information about the image and possibly a constraint energy  $E_{con}$ .

The image energy  $E_{img}$  defines a potential field with low values at the desired image features. For the edge extraction of an image  $I$  the following term is often used (Xu and Prince, 1997):

$$E_{img}(x, y) = -\kappa |\nabla(G_{\sigma}(x, y) * I(x, y))|^2 \quad (2)$$

where  $G_{\sigma}$  is a two-dimensional Gaussian function with the standard deviation  $\sigma$ ,  $\nabla$  is the gradient operator and  $\kappa$  is a positive weighting parameter.

User interactions are often desirable to influence the deformation, e.g. if an active contour is trapped in a local energy minimum. Formulations of the constraint energy  $E_{con}$  can be used to integrate an individual behavior. Kass proposed user-defined points, called springs and volcanos, that attract and push away snakes. These points are useful instruments to control a snake in the deforming process.

Using the calculus of variations a curve that minimizes the integral (1) has to satisfy the Euler equation:

$$\alpha \frac{\partial^2 \vec{v}}{\partial s^2} - \beta \frac{\partial^4 \vec{v}}{\partial s^4} - \nabla E_{ext} = \vec{0} \quad (3)$$

This is often taken as a force balance equation:

$$\vec{F}_{int} + \vec{F}_{ext} = \vec{0} \quad (4)$$

with  $\vec{F}_{int} = \alpha \frac{\partial^2 \vec{v}}{\partial s^2} - \beta \frac{\partial^4 \vec{v}}{\partial s^4}$  and  $\vec{F}_{ext} = \vec{F}_{img} + \vec{F}_{con} = -\nabla E_{ext}$ . Treating  $\vec{v}$  as function of time and discretization of the equation (3) by approximating the snake as a polygon and the partial derivatives as finite differences, a solution can be found iteratively as follows (Kass et al., 1987):

$$\vec{v}^{(t+1)} = (A + \gamma I)^{-1} (\gamma \vec{v}^{(t)} - \vec{F}_{img}(\vec{v}^{(t)}) - \vec{F}_{con}(\vec{v}^{(t)})) \quad (5)$$

where  $\vec{v}^{(t)}$  is the vector of control points of the snake at time step  $t$ ,  $\vec{v}^{(t+1)}$  at time step  $t + 1$  respectively,  $A$  is a quadratic iteration matrix according to the dimension of the snake vector containing elements that are linear combinations of  $\alpha$  and  $\beta$ ,  $I$  is the identity matrix and  $\gamma$  is a positive weighting parameter controlling the step size of each iteration which can be considered as viscosity parameter.

## 2.1 Extending Abilities

Due to the limited capture range of the image forces and the sensitivity of local minima an initial contour must be placed very close to the desired object boundaries. This is one of the most important disadvantages of classical snakes.

One approach diminishing that problem was published by Xu and Prince (Xu and Prince, 1997). The replacement of the classical image force formulation by a vector field, they called Gradient Vector Flow (GVF), provides a larger attraction range by the presence of boundary information in homogeneous regions. They define an Edge Map  $f(x, y)$  derived from the image  $I(x, y)$  having large values at the features of interest, e.g. image edges:

$$f(x, y) = |\nabla(G_\sigma(x, y) * I(x, y))|^2 \quad (6)$$

Note, the field  $\nabla f$  has vectors pointing to the image edges, but still with a narrow capture range. The GVF field  $\vec{g}(x, y) = (u(x, y), v(x, y))^T$  is calculated in the next step by minimizing the energy functional:

$$\mathcal{E} = \iint \mu(u_x^2 + u_y^2 + v_x^2 + v_y^2) + |\nabla f|^2 |\vec{g} - \nabla f|^2 dx dy \quad (7)$$

The energy is dominated by the partial derivatives of the vector field in homogeneous regions where  $|\nabla f|$  is small, yielding a smooth field.  $|\nabla f|$  is large at image edges and there  $\mathcal{E}$  is minimal when  $\vec{g} = \nabla f$ .  $\mu$  is a positive weighting parameter which should be increased the more noisy the image is. A possible way to find a solution of equation (7) was described in (Xu and Prince, 1998). Finally the potential force  $\vec{F}_{img}$  in equation (5) is replaced by the GVF term  $-\vec{g}$ .

Figure 1 illustrates the diffusion of the image gradient considering a simple U-shaped object as example. The enlarged attraction range of image edges simplifies the active contour's convergence into concave boundary regions. Further on, the immediate closeness of initial contours to the desired object boundaries is not necessary any longer. But the advantages of this method are bought dearly by the expensive computation of the GVF field, especially with huge images. We observed, GVF snakes produced more qualitative results in our tests if a Gaussian filter was applied to the image.

Cohen (Cohen, 1991) proposed a balloon model to solve the problems encountered with the limited capture range of image edges while using the traditional image energy. To allow an expansion of active contours they introduced an inflation force:

$$\vec{F}_p = w_p \vec{n}(s) \quad (8)$$

where  $\vec{n}(s)$  is the unit normal vector of the oriented contour at the point  $\vec{v}(s)$  and  $w_p$  is a weighting parameter that controls the influence of that force. Applying this pressure force, as an additional part of the

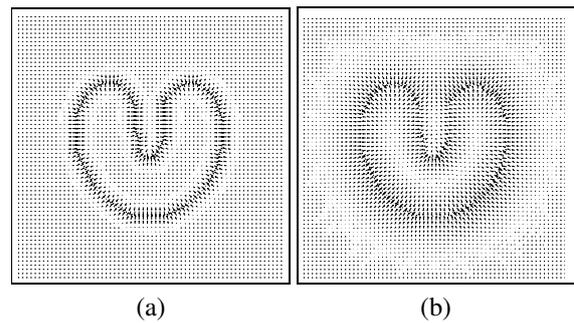


Figure 1: Image forces. (a) Traditional potential force field. (b) Gradient Vector Flow field.

constraint forces  $\vec{F}_{con}$ , the curve behaves like a balloon, that is inflated. So it counteracts the snake's tendency to shrink under the predominant influence of the elasticity force. Inflating active contours are also able to detect closed object contours not only from outside with the implicit aid of the shrinking behavior but also from inside. Using this pressure force the snake can easier move into concave boundary regions. Changing the sign of  $w_p$  makes the snake deflating instead of inflating. An adequate choice of  $w_p$  can speed up the convergence process, particularly when strong edges should be detected.

## 3 SPEEDING UP METHODS

This section describes the different methods for a fast semiautomatic segmentation using snakes. The research at the Georg August University of Göttingen often deals with image sizes of more than 100 megapixels. Therefore an efficient implementation is essential for a profitable practical use.

### 3.1 Acceleration of Initial Computations

Once the potential image forces are determined, they can be used for the entire segmentation process without recalculation. In order to speed up the computation of the Edge Map and the GVF we take advantage of current processor architectures that support the parallel execution of multiple arithmetical floating point operations. However properly memory aligned data is necessary for fast loading and storing instructions in order to gain a noticeable acceleration. In fact loading and storing time could slow down the computation in spite of parallel execution due to unaligned data.

Further on, we use separate background threads for their computation, particularly the GVF, which is very

time expensive for huge images like those from the research in Göttingen. This saves a little time because in the meantime the user is able to perform independent tasks like setting up one or more initial contours or adjusting parameter values.

### 3.2 Multiresolution with an Automation Approach

We use a multiresolution approach which is very similar to Leroy (Leroy et al., 1996). After a huge image has been loaded several smaller scales of the image are computed. Each scale has half the width and height of the above scale. This computation is repeated until the coarsest image size is about one megapixel.

Starting with a rough initial contour with a few points in the coarsest scale, we can set up an iterative process. By solving small systems of linear equations our approach determines a suitable solution curve very rapidly. Furthermore, the tendency to shrink is stronger if the active contour has only a few points, due to the structure of the internal forces, i.e. the snake needs less iterations. A simple polygonal initialized snake is filled with many collinear points on the edges of the polygon. If these points are located in regions without a potential image force they have a minimal energy. So they will not move unless a discrete curvature is present which can be found, of course, at the polygon's vertices. That means that these vertices are the first deforming parts of the snake spreading a smaller curvature to their neighbors. One can imagine that the snake is shrinking faster the less points it has. In fact, this behavior can be desired, though it is commonly seen as a disadvantage. Often, a snake has to move across regions without any edge information when detecting object boundaries from outside by contraction. Therefore, it has to deform while being exposed only to the internal forces.

In order to keep an approximate point distance, new points are inserted when switching to a higher resolution next to scaling the contour to an appropriate size. That means the result of the last lower scale is used as the initial curve for the current scale. When the original resolution is reached the snake should be already very close to the desired curve and therefore only a few iterations are needed for a more detailed extraction. This approach leads to an acceleration, because the biggest distance of the contour is covered in coarse scales with only a few points and in higher scales just minimal refinements are needed.

User interaction can be further reduced if acceptable results can be achieved without the need of parameter adjustments during the deforming process. Based on that multiresolution strategy, we have designed an automation technique, that allows an auto-

matic image segmentation for images with a sufficient quality. We start with a given initial contour in the coarsest scale with the traditional image forces (no GVF). Besides we use the opposite image forces for the beginning convergence process, i.e. using  $\vec{F}_{img}$  instead of  $-\vec{F}_{img}$  in equation (5). That way, we can keep the snake slightly distant from the desired object boundaries. Otherwise we observed the contour often moving across weak edges in lower scales. The deformation procedure automatically switches to the next scale after the snake has stabilized its position. It is scaled to double size and additional points are inserted. For each reached scale the potential image forces are recomputed, according to the new size. These steps are repeated until we reached the original size of the image. Then the classical image energy (with the correct sign) or the GVF field is used for the last time the deforming process gets started. Converged close to the desired edges in the meantime the snake can now fit to them very quickly.

### 3.3 Using Segments

Kerschner (Kerschner, 2003) proposed an approach that uses segments in order to overcome local minima. A traffic light system enables the snake to diagnose the quality of the segmentation result of itself. Thereby segments in different colors (red for rejected results, yellow for unsure results, green for trustworthy results) are presented to the operator who can influence the accepting procedure of the automatic rating system. Typically, yellow and red parts are linked and deform afterward with a modified energy function, e.g. by changing parameter values.

We use segments to gain another acceleration. During the deforming process parts of a snake often reach the desired object boundaries faster than others, e.g. an active contour has not moved into an concave boundary region yet while the rest of the object has been detected already. Points in these parts will not move anymore and thus can be excluded from the iterative minimization procedure. This can be done by subdividing the snake into segments and treating them as open snakes.

The segmentation process is started with one segment – one closed contour. The movement of each point of the snake is traced for the last few iterations. If the position of several adjacent points does not change significantly within that time, two segments are formed. The first segment contains these adjacent points and is locked, i.e. is ignored in the future minimization process. The second segment is filled with the remaining points and further on deforming as open snake by solving a respectively smaller system of equations. Figure 2 shows the use of segments.





Table 1: Comparison between naive and optimized implementation. 1000 deforming steps were calculated in each case. The times neither include the costs of the precomputation of the Edge Map nor the costs for the GVF.

vertices	$\mathcal{O}(n^3)$	$\mathcal{O}(n)$	factor
256	1 min 36 s	4.8 s	20
512	23 min 25 s	8.6 s	166
1024	6 h 28 min	11.2 s	2079
2048	-	22.4 s	-
4096	-	52.8 s	-

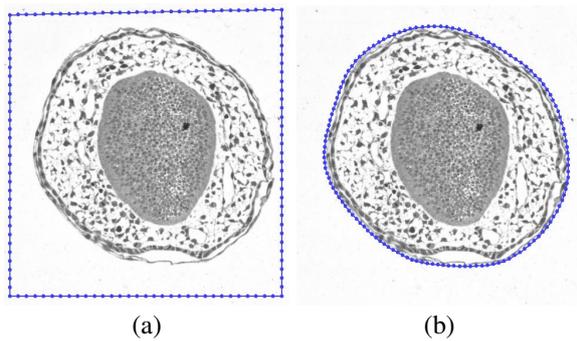


Figure 3: A histological section scaled at  $\frac{1}{8}$  size. (a) Rough initial contour. (b) Deformation to the approximate convex hull of the object.

Finally, the introduced techniques were applied to detect the outer contour of the object of the histological section in figure 4a. The image with a size of  $9100 \times 15965$  pixels was preprocessed with some image filters and scaled to  $3376 \times 5923$  pixels (4b) so that the GVF and the image itself fit into memory. The final contour (4c), has 1555 vertices and was achieved after 2 minutes and 15 seconds and 5660 iterations (without the computational time for the potential forces). Assuming two vertices per second can be set by hand, a manual segmentation would have taken approximately 15 minutes. The deforming process comprises four image scalings and a fine tuning of the result by the insertion of some springs.

## 5 CONCLUSION

In this article, we presented different methods of speeding up the semiautomatic image segmentation process using parametric active contour models. Aiming the replacement of the tedious manual segmentation by a faster semiautomatic segmentation for the medical research at the University of Göttingen, we developed a graphical user interface combining these techniques with other improving approaches

Table 2: Times and deforming steps to achieve the result of figure 3b in different scales.

scale	vertices	iterations	time
12.5%	130	563	1.1 s
25%	240	2113	10.5 s
50%	480	8867	1 min 20 s
100%	950	35269	10 min 5 s

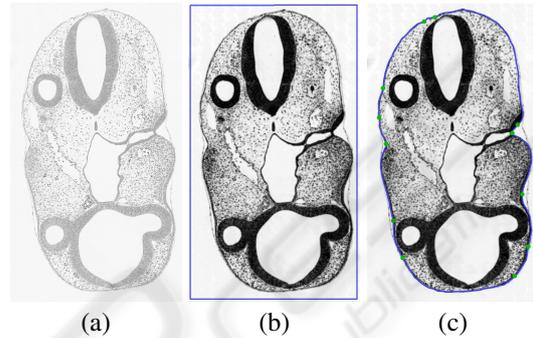


Figure 4: (a) Original image. (b) Filtered and scaled image with initial contour. (c) Final result after interactions.

that were already published, such as the Gradient Vector Flow and the balloon model which are popular extensions of the traditional snake model providing a larger attraction range. This research deals with huge images and thus an efficient and memory saving implementation is desirable to justify the practical application.

It was shown how important it is, to take advantage of the band structure of the underlying matrices. Also the multiresolution method led to a considerable acceleration due to the fast convergence in coarser scales. Using segments is advantageous for the final refinement steps, e.g. in the original scale of the image where many successfully detected parts of the snake can be excluded from the calculation.

Because of dealing with those huge images, memory saving approaches are also very important, like the scaling of the values of the traditional potential energy. Further approaches reducing the memory usage would be useful, especially for the storage of the Gradient Vector Flow.

## APPENDIX: ALGORITHMS

All of the shown algorithms were used to achieve  $\mathcal{O}(n)$  complexity for one single iteration step.

**Algorithm 1:** Efficient Cholesky decomposition

**Input** : Symmetric band matrix  $A = (a_{ij})_{i=1,j=1}^{n,n}$  with the bandwidth  $w$ .

**Output** : Cholesky decomposition of the matrix  $A = LL^T$ .  $A$  will be overwritten with the elements of the matrix  $L$ .

```

 $w_l \leftarrow \lfloor \frac{w}{2} \rfloor$ 
for  $i \leftarrow 1$  to  $n$  do
   $a_{ii} \leftarrow \sqrt{a_{ii} - \sum_{k=\max\{1, i-w_l\}}^{i-1} a_{ik}^2}$ 
  for  $j \leftarrow i+1$  to  $\min\{i+w_l, n\}$  do
     $a_{ji} \leftarrow \frac{1}{a_{ii}} \left( a_{ij} - \sum_{k=\max\{1, j-w_l\}}^{i-1} a_{ik} a_{jk} \right)$ 
  end
end

```

**Algorithm 2:** Efficient forward substitution

**Input** : Lower triangular matrix  $L = (l_{ij})_{i=1,j=1}^{n,n}$  with the lower bandwidth  $w_l$ , result vector  $\vec{b} = (b_1, \dots, b_n)^T$ .

**Output** : Solution vector  $\vec{x} = (x_1, \dots, x_n)^T$  of the system  $L\vec{x} = \vec{b}$ .

```

for  $i \leftarrow 1$  to  $n$  do
   $x_i \leftarrow \frac{1}{l_{ii}} \left( b_i - \sum_{j=\max\{1, i-w_l\}}^{i-1} l_{ij} x_j \right)$ 
end

```

**Algorithm 3:** Efficient LU decomposition

**Input** : Band matrix  $A = (a_{ij})_{i=1,j=1}^{n,n}$  with the lower and upper bandwidth  $w_l$  and  $w_u$ .

**Output** : LU decomposition of the matrix  $A = LU$ .  $A$  will be overwritten with elements of the lower triangular matrix  $L$  below and with elements of the upper triangular matrix  $U$  above of and upon the main diagonal.

```

for  $k \leftarrow 1$  to  $n$  do
  for  $i \leftarrow k+1$  to  $\min\{k+w_l, n\}$  do  $a_{ik} \leftarrow \frac{a_{ik}}{a_{kk}}$ 
  for  $j \leftarrow k+1$  to  $\min\{k+w_u, n\}$  do
    for  $i \leftarrow k+1$  to  $\min\{k+w_l, n\}$  do
       $a_{ij} \leftarrow a_{ij} - a_{ik} a_{kj}$ 
    end
  end
end

```

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