PREEMPTIVE SCHEDULING IN A TWO-STAGE MULTIPROCESSOR FLOWSHOP WITH RESOURCE CONSTRAINTS

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Abstract: A heuristic combining the column generation technique and a genetic algorithm is proposed for solving the problem of preemptive scheduling in a two-stage flowshop with parallel unrelated machines and renewable resources at the first stage and a single machine at the second stage. The objective is to minimize the makespan. The lower bound on the optimal makespan is derived to be used in the performance analysis of the heuristic. The performance of the heuristic is analyzed by a computational experiment. The results show that the heuristic is able to find near-optimal solutions in reasonable computation time.

1 INTRODUCTION

This paper proposes a heuristic combining the column generation (CG) technique with a genetic algorithm (GA) for solving the multiprocessor flowshop scheduling problem which can be briefly described as follows: there are a number of preemptive jobs to be processed at two stages, each job being processed first at stage 1 then at stage 2. Stage 1 consists of a number of parallel unrelated machines, at stage 2 there is a single machine. Upon completion at stage 1 a job is ready to be processed at stage 2: it may be processed at stage 2 when the machine is available there, or it may reside in a buffer space of unlimited capacity following stage 1 until the machine at stage 2 becomes available. At stage 1, a job can be processed on any of the parallel machines, and its processing times may be different on different machines. The processing of a job on a machine of stage 1 may be interrupted at any moment and resumed later on the same or another machine. A job during its processing at stage 1 requires some amounts of additional renewable resources. The total amounts of these resources available at any moment are limited.

The objective is to find a feasible schedule which minimizes the maximum job completion time in the two-stage flowshop, C_{max}, referred to as makespan.

This problem is NP-hard in the strong sense since the problem of preemptive scheduling in the two-stage flowshop with two identical parallel machines at one stage and one machine at another is NP-hard in the strong sense (Hoogeveen et al., 1996).

During the last decade the flowshops with multiple machines (FSMP), also called hybrid flowshops, received considerable attention from researchers. Most literature in this area addresses the minimum makespan problems under the assumption that preemptions of jobs are not allowed and the parallel machines at each stage are identical, e.g. (Gupta, 1988; Chen, 1995; Haouari and M'Hallah, 1997; Brah and Loo, 1999; Linn and Zhang, 1999; Oguz et al., 2003). Only few papers concern the flowshop with parallel machines that are not identical (Suresh, 1997; Ruiz and Maroto, 2006).

To the best of our knowledge the multiprocessor flowshop scheduling problem with additional resource constraints has not been considered in the literature so far.

In this study, two-stage multiprocessor flowshop scheduling research is extended by considering the preemptive scheduling of unrelated parallel machines with additional resource constraints. Such a problem may arise in real-life systems that are encountered in a variety of industries, e.g. in chemical, food and cosmetics industries. These systems are often subjected to some additional resource constraints for example on the availability of the additional resources such as skilled labor, tools, power. Preemption of jobs usu-

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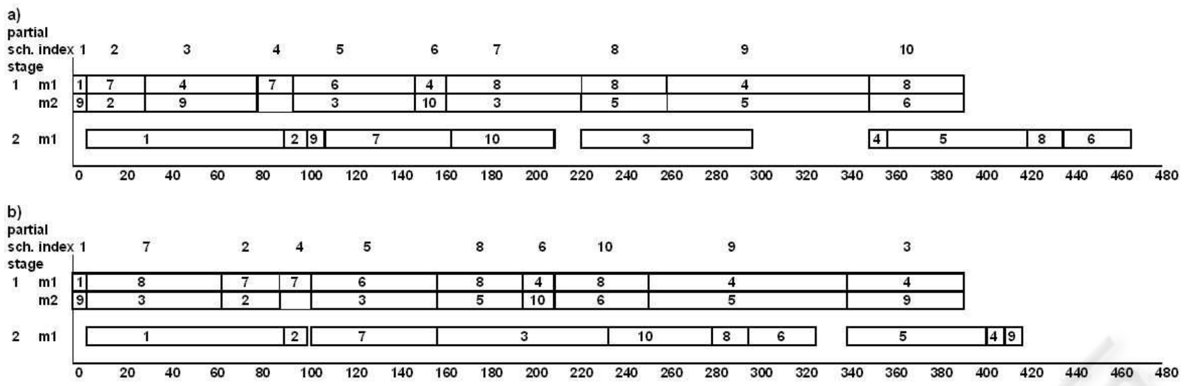
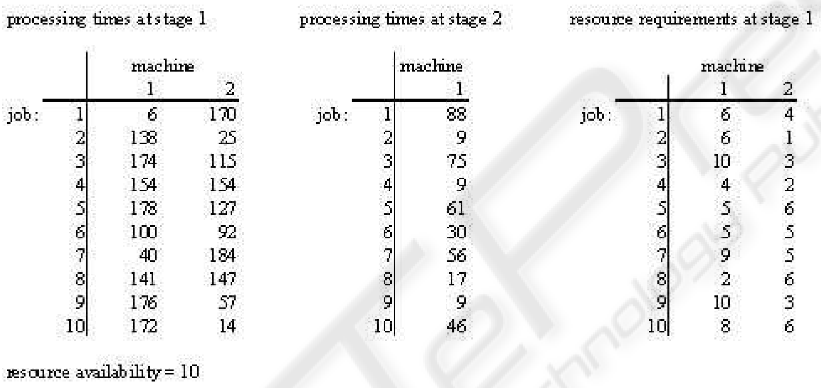


Figure 1: An illustrative example. The resulting schedules: a) the feasible schedule with a random sequence of the partial schedules, b) the final schedule with the sequence of the partial schedules minimizing the makespan.



processing times at stage 1

job	machine 1	machine 2
1	6	170
2	138	25
3	174	115
4	154	154
5	178	127
6	100	92
7	40	184
8	141	147
9	176	57
10	172	14

processing times at stage 2

job	machine 1
1	88
2	9
3	75
4	9
5	61
6	30
7	56
8	17
9	9
10	46

resource requirements at stage 1

job	machine 1	machine 2
1	6	4
2	6	1
3	10	3
4	4	2
5	5	6
6	5	5
7	9	5
8	2	6
9	10	3
10	8	6

resource availability = 10

Figure 2: The data for an illustrative example.

ally results in shortening a schedule. The problem with parallel unrelated machines at the first stage and a single machine at the second stage may arise in a manufacturing environment in which products are initially processed on any of parallel machines and then each product must go through a final testing operation, which is to be carried out on a common testing machine.

2 THE FRAMEWORK OF THE HEURISTIC

The proposed heuristic proceeds in two steps.

Step 1. A column generation algorithm solves the minimum makespan problem of unrelated parallel machines scheduling with additional resource constraints which occurs at stage 1. The solution is composed of a number of partial schedules and its makespan does not depend on the ordering of the partial schedules. A partial schedule assigns some jobs (or parts of jobs) to machines for simultaneous processing during a certain period of time, so that resource constraints are fulfilled at every moment.

Step 2. A genetic algorithm finds the sequence of the partial schedules that minimizes the makespan in the two-stage flowshop which is equal to the maximum job completion time at stage 2. For each sequence of the partial schedules generated in the search process the completion times of jobs at stage 1 are calculated. Then, a schedule for the two-stage flowshop is constructed and its makespan is calculated taking into account the job ready times (equal to the job completion times at stage 1) and processing times of jobs at stage 2.

To illustrate the performance of the heuristic we present the following example. Consider an instance of 10 jobs with the job processing times, resource requirements and resource availability as shown in Figure 2. There are two stages: stage 1 contains two parallel unrelated machines, stage 2 consists of one machine. Figure 1 presents two schedules for this instance. Each schedule in the two-stage flowshop may be treated as composed of a schedule of the first stage and a schedule of the second stage. The schedules of the first stage parallel machines are composed of 10 partial schedules. Each partial schedule satisfies resource constraints at a time. The availability of the resource at any moment is 10 units. For example, in the partial schedule of index 1, S_1, job 1 and job 9 require, respectively, 6 and 3 units (see Figure 2) of the resource at a time, so the total usage of the resource at any moment in this partial schedule is equal to 9 and is no greater than 10. Similarly, all remaining partial schedules satisfy resource constraints at any moment.

In Figure 1a the sequence of the partial schedules is S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}. We can see that for this sequence of the partial schedules, the first job completed at stage 1 is job 1 (it is finished in S_1). After completing at stage 1, job 1 starts on the machine at stage 2 because this machine is free. The next job completed at stage 1 (in S_2) is the job of index 2. After its completion job 2 is stored in the buffer space between the stages until the machine of stage 2 is freed up. The indices of successive jobs completed at stage 1 are: 1, 2, 9, 7, 10, 3, 4, 5, 8, 6. In Figure 1a, we observe that the machine at stage 2 remains idle after finishing processing jobs 10 and 3 when it waits for the completion of jobs 3 and 4, respectively, at stage 1.

In Figure 1b the sequence of the partial schedules is $S_1, S_7, S_2, S_4, S_5, S_8, S_6, S_{10}, S_9, S_3$. This sequence has been found by the GA so as to minimize the maximum job completion time at stage 2 (the makespan of the whole schedule). For this sequence of the partial schedules, jobs are completed at stage 1 in the following order: 1, 2, 7, 3, 10, 8, 6, 5, 4, 9, and their completion times are different than those in Figure 1a, which results in a much smaller idle time of the second stage machine and a much shorter schedule then those in Figure 1a.

3 NOTATION

In this paper jobs are indexed by j, parallel machines at stage 1 by i, resource types by r. The parameters of the problem considered are as follows:

n	the number of jobs,
m	the number of machines at stage 1,
l	the number of types of renewable resources,
p_{ij}	the processing time of job j on machine i at stage 1,
s_j	the processing time of job j on the machine at stage 2,
W_r	the number of units of resource r available at a time,
α_{ijr}	the number of units of resource r required at every moment during processing job j on machine i at stage 1.

4 HEURISTIC DESCRIPTION

4.1 Solving the Problem of Stage 1

As stated in Section 2, first, the problem of resource constrained preemptive scheduling of parallel unrelated machines so as to minimize the makespan is solved. The solution to this problem is represented by a set S of partial schedules S_β, $\beta \in B$, where B is the set of indices of all feasible partial schedules. Partial schedule is determined by its duration Δ_β and the values of v_{ij}^{β} ($i = 1 \ldots m$,$j = 1 \ldots n$) representing an assignment of jobs to machines, where $v_{ij}^{\beta} = 1$ if job j is processed on machine i in partial schedule S_β and $v_{ij}^{\beta} = 0$, otherwise. The problem is formally defined as follows:

$$\min \sum_{\beta \in B} \Delta_\beta \tag{1}$$

subject to:

$$\sum_{\beta \in B} \Delta_\beta \sum_{i=1}^{m} \frac{v_{ij}^{\beta}}{p_{ij}} = 1, \quad j = 1, \ldots, n \tag{2}$$

$$\sum_{j=1}^{n} v_{ij}^{\beta} \leq 1, \quad i = 1, \ldots, m, \beta \in B \tag{3}$$

$$\sum_{i=1}^{m} v_{ij}^{\beta} \leq 1, \quad j = 1, \ldots, n, \beta \in B \tag{4}$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \alpha_{ijr} v_{ij}^{\beta} \leq W_r, \quad r = 1, \ldots, l, \beta \in B \tag{5}$$

$$v_{ij}^{\beta} \in \{0, 1\}, \quad i = 1, \ldots, m, j = 1, \ldots, n, \beta \in B \tag{6}$$

$$\Delta_\beta \geq 0, \quad \beta \in B \tag{7}$$

where Δ_β ($\beta \in B$) and v_{ij}^{β} ($i = 1, \ldots, m$, $j = 1, \ldots, n$, $\beta \in B$) are decision variables. Constraints (2) ensure that all jobs are completed at stage 1 of the two-stage flowshop. Constraints (3) and (4) ensure that, respectively, each machine works on at most one job at a time and each job is processed on no more than one machine at a time. Due to constraints (5) the usage of each resource at every moment does not exceed its availability.

In the general case, the above problem is known to be NP-complete (Slowinski, 1980). It can be optimally solved by means of a CG algorithm. The theoretical basis of the CG technique has been provided by

Dantzig and Wolfe in (Dantzig and Wolfe, 1960) (for applications of the CG technique see e.g. (Gilmore and Gomory, 1961; Barnhart et al., 1998; Figielska, 1999; Chen and Lee, 2002)).

A CG algorithm does not generate explicitly all columns of the problem, which correspond to all partial schedules. It works only with a subset of columns and adds a new column which improves the solution. At each iteration of the CG algorithm, the schedule length is minimized by solving the LP problem of the form:

$$\min \sum_{\beta \in \tilde{B}} \Delta_\beta \tag{8}$$

subject to:

$$\sum_{\beta \in \tilde{B}} \Delta_\beta \sum_{i=1}^{m} \frac{v_{ij}^{\beta}}{p_{ij}} = 1, \quad j = 1, \ldots, n \tag{9}$$

$$\Delta_\beta \geq 0, \quad \beta \in \tilde{B} \tag{10}$$

where $\beta \subset \tilde{B}$ denotes a subset of indices of columns, Δ_β ($\beta \in \tilde{B}$) are decision variables, and the values of v_{ij}^{β} ($i = 1, \ldots, m$, $j = 1, \ldots, n$, $\beta \in \tilde{B}$) are fixed (determined in the previous iterations) or (before the first iteration) given in advance. Let π_j^* ($j = 1, \ldots, n$) be the optimal solution of the dual problem to the problem (8)-(10) (π_j^* are dual variables corresponding to constraints (9)). If there exists a column $\beta \in B \setminus \tilde{B}$ such that $\sum_{j=1}^{n} \sum_{i=1}^{m} \frac{\pi_j^* v_{ij}^{\beta}}{pij} - 1 > 0$, then the current set $\tilde{B}$ can be extended by this new index β and a new iteration of the CG algorithm is started. Otherwise, the optimal solution is found and the algorithm stops.

4.2 Finding the Schedule of Minimal Makespan

The aim is the minimization of the makespan in the two-stage flowshop. A GA finds the ordering of the partial schedules which provides the schedule for the two-stage flowshop with minimum makespan.

A GA (Holland, 1975) is a search technique that imitates the natural selection and biological evolutionary process. GAs have been used in a wide variety of applications, particularly in combinatorial optimization problems and they were proved to be able to provide near optimal solutions in reasonable time.

A GA starts with a population of randomly generated candidate solutions (called chromosomes). A chromosome is represented by a string of numbers called genes. Each chromosome in the population is evaluated according to some fitness measure. Certain pairs of chromosomes (parents) are selected on the basis of their fitness. Each of these pairs combines to produce new chromosomes (offspring) and some of the offspring are randomly modified. A new population is then formed replacing some of the original population by an identical number of offspring. The process is repeated until a stopping criterion is met.

Let $P(t)$ denotes the population at iteration t and pop_size is the population size. The GA applied in this paper can be outlined as follows.

```
1. Generate and evaluate the initial population $P(t)$, $t = 0$.
2. Repeat the following steps until stopping condition is satisfied.
   2.1. Repeat the following loop $\frac{pop\_size}{2}$ times ($pop\_size$ is an even number).
        2.1.1. Select two parents from $P(t)$.
        2.1.2. Apply the crossover operator over the parent chromosomes and produce 2 offspring chromosomes.
        2.1.3. Apply the mutation operator over the offspring.
        2.1.4. Copy the offspring to population $P(t+1)$.
   2.2. Evaluate $P(t+1)$.
   2.3. Replace the worst chromosome of $P(t+1)$ by the best chromosome found so far.
   2.4. Set $t = t + 1$.
3. Return the best chromosome found.
```

The factors which characterize the GA applied to the problem considered in this paper are determined as follows.

Solution representation. A solution to the sequencing problem solved by the GA is coded as a single chromosome whose genes represent the indices of partial schedules.

Initial population. An initial population of chromosomes is randomly generated.

Evaluation. The value of an objective function, which is equal to the makespan in the two-stage flowshop, is used to measure the fitness of a chromosome. For each partial schedule sequence (chromosome) generated in the search process, a schedule for the two-stage flowshop is constructed and the makespan is calculated taking into account ready times and processing times of jobs at stage 2.

Parent selection. The binary tournament selection method is used. In a binary tournament selection, two chromosomes are randomly chosen. The more fit (with a smaller objective function value) is then taken as a parent chromosome. Two binary tournaments are held to produce two parents.

Crossover. The two-point crossover operator PMX (Goldberg, 1989) is applied to each pair of parent chromosomes with a probability P_{crs} (crossover probability).

Mutation. The genes of each chromosome in the population are considered one by one, and the gene being considered swaps its value with another randomly generated gene of the same chromosome with a probability P_{mut} (mutation probability).

Stopping condition. The search process terminates when the best objective function value (makespan) found so far is not updated for a predetermined number of iterations.

On the basis of the preliminary computational experiment the following values of the genetic parameters which ensure a good performance of the algorithm were selected: $pop\_size = 30$, $P_{crs} = 0.8$, $P_{mut} = 0.01$, the number of iterations without any improvement of the best solution found so far is set at 250.

5 LOWER BOUND

Since determination of the optimal solution to the considered problem is practically impossible (for larger size problems it is impossible even in the case of flowshop with identical parallel machines and no resource constraints (Santos et al., 1995)), a simple and easily computable lower bound on the optimal makespan is derived to evaluate the quality of the proposed heuristic.

Let C^*_{CG} denotes the minimal makespan (i.e. the minimum time needed to complete all jobs at stage 1) for the problem occurring at stage 1. The first lower bound on the optimal makespan in the two-stage flowshop is obtained from the following equation:

$$LB_1 = C^*_{CG} + \min_{j=1,\ldots,n}\{s_j\} \tag{11}$$

The second term of the above equation represents the minimum unavoidable idleness at stage 1 which is equal to the smallest job processing time at stage 2.

The second lower bound is given by:

$$LB_2 = \min_{i=1,\ldots,m,j=1,\ldots,n}\{p_{ij}\} + \sum_{j=1}^{n} s_j \tag{12}$$

since the machine at stage 2 remains idle for at least the time needed to complete at stage 1 a job with the smallest processing time at stage 1.

LB_1 and LB_2 will be effective for problem instances which are dominated by jobs with large processing times at stage 1 and stage 2, respectively.

Hence, a lower bound on the optimal makespan in the considered two-stage flowshop will be

$$LB = \max\{LB_1, LB_2\} \tag{13}$$

6 COMPUTATIONAL EXPERIMENT

In this section the results of a computational experiment conducted to evaluate the performance of the proposed heuristic are presented. 360 randomly generated instances were created and examined. Instances were generated for the number of jobs n = 20, 40, 60, 80, 100 and 120, the number of machines at stage 1, m = 2, 3 and 4, and one resource type. Resource requirements, α_{ijr}, were generated from $U[1, 10]$ ($U[a, b]$ denotes the discrete uniform distribution in the range of $[a, b]$), whereas the resource availability, W_1, was set at 10. Processing times at stage 1, p_{ij}, were generated from $U[1, 200]$, $U[1, 250]$ and $U[1, 300]$ for instances with 2, 3 and 4 machines, respectively, whereas processing times at stage 2, s_j, were generated from $U[1, 100]$ for all instances.

Table 1: The computational results.

n	m	δ (%)	CPU time (s)	
			CG alg	GA
20	2	1.16	0.67	0.14
	3	2.48	1.46	0.13
	4	1.73	1.83	0.16
40	2	0.50	2.01	0.59
	3	0.45	4.38	0.52
	4	0.61	3.84	0.48
60	2	0.29	4.56	1.00
	3	0.37	8.36	1.23
	4	0.69	7.21	1.33
80	2	0.19	7.66	1.95
	3	0.45	12.96	2.71
	4	0.24	12.66	2.59
100	2	0.07	12.29	3.82
	3	0.15	21.20	4.45
	4	0.22	17.48	5.15
120	2	0.08	18.71	6.20
	3	0.18	31.05	9.14
	4	0.19	27.54	7.05
average:		0.56	10.88	2.70

To evaluate the performance of the proposed heuristic two performance measures are used. The first is the relative deviation of a heuristic solution from the lower bound on the optimal makespan (in other words the maximum relative deviation from the optimal makespan) defined as

$$\delta = \frac{C - LB}{LB} \times 100\%,$$

where C is the best makespan found by the heuristic. The second performance measure is the CPU time (reported in seconds) consumed by the heuristic to find the best solution.

The results of the experiment are presented in Table 1. The first two columns show the size of the instances. The relative deviation is presented in column 3. Columns 4 and 5 contain the CPU time (in seconds) consumed by the CG algorithm and the GA, respectively. The entries in Table 1 are average values over 20 instances.

The results show that the presented heuristic performs very well for the entire collection of instances. We observe that δ decreases with the increase in the number of jobs. On average $\delta = 1.79\%$, 0.52%, 0.45%, 0.29%, 0.15% and 0.15% for problems with 20, 40, 60, 80, 100, and 120 jobs, respectively. The CPU time of the heuristic grows with the number of jobs. This increase is more significant for the CG algorithm than for the GA. The number of machines does not seem to affect significantly the computation time of the heuristic, however, we observe that in the case of 2 machines the CG algorithm consumes less CPU seconds than in the cases of 3 and 4 machines.

7 CONCLUSIONS

In this paper a heuristic combining the column generation algorithm with the genetic algorithm for solving the two-stage flowshop preemptive scheduling problem with parallel unrelated machines and resource constraints at the first stage, and a single machine at the second stage has been developed. The heuristic has been tested as to its effectiveness in finding a minimum makespan schedule and as to its computation time. The obtained results indicate that for problems with a large number of jobs the heuristic usually finds schedules with makespan close to the optimal value in reasonable computation time.

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