THE USE OF MODULATING FUNCTIONS FOR IDENTIFICATION OF CONTINUOUS SYSTEMS WITH TIME-VARYING PARAMETERS

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Abstract: In the paper the use of modulating functions for the optimal identification of the structure and parameters of continuous linear systems is presented. The modulating functions with compact support [0, h] are used in convolution filter for transformation of input/output signal derivatives. Based on pre-filtered functions continuous moving window [t-T, t] is used for on-line identification of piecewise constant parameters Θ changes of linear system. Optimal quadratic method for identification is presented – with the use of quadratic constraints on parameters Θ. The numerical results of some examples are shown.

1 INTRODUCTION

One of the continuous model identification method is based on the use of modulating functions. In 1957 an application of integral transformation with the use of compact support functions for parameter identification of differential equation based on input and output continuous measurement signals given on [0,T] interval was proposed (Shinbrot, 1957). Such FIR type filter may be used directly for identification of continuous systems because of special properties of convolution.

During primary stage of identification each term in differential equation is convoluted (multiplying and integrating) with known modulating functions on assumed interval [0,T]. Modulating functions and their derivatives have compact support of length h, where h<<T, i.e. vanish at the ends of the interval h, hence initial condition terms of the model also vanish and finally one can have the new algebraic (non differential) model with the same unknown parameters. In the second stage of identification algorithm, optimal parameter identification problem is formulated as a problem of minimization of norm of the equation error in function space L^2 (Byrski W., 1995, 1996). For nontrivial optimal parameters solution in optimisation task the constraints for parameters should be assumed. Assumption of quadratic constrains of parameters, leads to Gram matrix G and calculation of their eigenvectors.

Above described methodology one can use for identification of changes in time-varying parameters, which are piecewise constant. To this aim the on-line version of the above method with convolution filter was prepared and tested. For precise identification of parameters changes the short interval T in on-line moving observation window should be assumed.

Linear time-varying systems were investigated in many monographs (D'Angelo H., 1970), Niedzwiecki M., 2000). Publications and references on identification methods for time-varying parameters one can find e.g. in survey (Kleiman E., 2000). However the use of modulating function was not proposed and tested.

2 MODULATING FUNCTIONS

The idea of using modulating functions and integral transformation follows from the fact that convolution of unknown signal derivative y' and some known function φ is equal to convolution of original measured signal y and known derivative φ'. Integration by parts shows that the proper choice of modulating function φ with special properties enables omitting the initial conditions problem (which are also unknown). Moreover the use of integral transformation to signals reduces the level of noise. For identification procedure different

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modulating function can be chosen. Convolution filter with different modulation functions has different filtration properties. The required properties of modulating functions follow from below presented identification problem.

Given LTI model of the continuous SISO system which has to be identified

$$\sum_{i=0}^{n} a_i \, y^{(i)}(t) = \sum_{i=0}^{m} b_i \, u^{(i)}(t) \qquad (1)$$

where $y^{(i)}(t)$, $u^{(i)}(t)$ are the i-th derivatives of the output and input, respectively, $m \le n$ and the $n+m+2$ unknown parameters a_i , b_i are constant. It can be assumed that measurements of y and u on the interval $[t_0, T]$ are given, where T can be also considered as the current time.

In order to avoid the difficulties caused by the presence of derivatives $y^{(i)}$, $u^{(i)}$ in the model (1) this model can be transformed into a more convenient form by means of convolution. Choosing some special filtering function φ with known derivatives $\varphi^{(i)}$ one can calculate the convolution of the both sides of model (1) and the function φ. This function φ is supposed to be nonzero in interval [0,h] and zero outside this interval (function with compact support). The convolution represents continuous shifting window h along time axis.

$$y_i(t) \overset{df}{=} [y * \varphi]^{(i)}(t) = [y^{(i)} * \varphi](t) = [y * \varphi^{(i)}](t) =$$
$$= \int_{t-h}^{t} y(\tau)\varphi^{(i)}(t-\tau)d\tau = \int_{0}^{h} y(t-\tau)\,\varphi^{(i)}(\tau)\,d\tau \qquad (2)$$

This operation generates the new functions $y_i(t)$, $u_i(t)$, for $i=0,..,n(m)$ for $t \in [t_0+h, T]$. Hence the differential model (1) becomes an algebraic one (3). Different modulating functions were proposed in literature (Loeb J., 1965), (Maletinsky V., 1979), (Preisig H., 1993). In our approach for numerical tests we will use Loeb-Cahen functions

$$\varphi(t) = t^N (h-t)^M, \quad t \in [0,h],$$

with $\min(M,N) \ge n$, $N \ne M$, and n is order of system.

$$\sum_{i=0}^{n} a_i \, y_i(t) = \sum_{i=0}^{m} b_i \, u_i(t) \qquad (3)$$

3 OPTIMAL IDENTIFICATION

For $t_0=0$ continuous measurements of the input u and output y on interval [0,T] are given. We assume that $u, y \in L^2(0,T)$. After the convolution of both sides of (1) with an arbitrary assumed function φ and with their derivatives $\varphi^{(i)}$ one can obtain new functions $y_i \in L^2(h,T)$, $u_i \in L^2(h,T)$ according to (2). The term $\varepsilon \in L^2(h,T)$ added to algebraic equation (3) denotes the combined effects of immeasurable noise or general equation error EE

$$\sum_{i=0}^{n} a_i \, y_i(t) = \sum_{i=0}^{m} b_i \, u_i(t) + \varepsilon(t)$$

The norm of difference ε(t) of both sides of model (3) can represent the performance index of identification.

Denoting by c(t), the vector of convolutions and by Θ the vector of parameters a_i , b_i one can have

$$\varepsilon(t) = [y_0(t), \ldots, y_n(t), -u_0(t), \ldots, -u_m(t)] \begin{bmatrix} a \\ b \end{bmatrix} = c^T(t)\theta$$

The statement of the minimization problem is:

$$\min_{\Theta} J^2 = \min_{\Theta} \|\varepsilon(t)\|^2_{L^2[h,T]} = \min_{\Theta} \|c^T(t)\Theta\|^2_{L^2}$$

$$J^2 = \left\langle c^T(t)\theta, \, c^T(t)\theta \right\rangle_{L^2} = \theta^T \left\langle c(t), c^T(t) \right\rangle \theta = \theta^T \mathbf{G} \, \theta \qquad (4)$$

The squared norm (4) has a form of inner product in space L^2. The real symmetric matrix **G** is the Gram matrix of inner products of functions which are elements of vector c(t)

$$\mathbf{G} = \begin{bmatrix} \langle y_0, y_0 \rangle & \ldots, & \langle y_0, y_n \rangle & -\langle y_0, u_0 \rangle & \ldots, & -\langle y_0, u_m \rangle \\ \ldots, & \ldots, & \ldots, & \ldots, & \ldots, & \ldots, \\ \langle y_n, y_0 \rangle & \ldots, & \langle y_n, y_n \rangle & -\langle y_n, u_0 \rangle & \ldots, & -\langle y_n, u_m \rangle \\ -\langle u_0, y_0 \rangle & \ldots, & -\langle u_0, y_n \rangle & \langle u_0, u_0 \rangle & \ldots, & \langle u_0, u_m \rangle \\ \ldots, & \ldots, & \ldots, & \ldots, & \ldots, & \ldots, \\ -\langle u_m, y_0 \rangle & \ldots, & -\langle u_m, y_n \rangle & \langle u_m, u_0 \rangle & \ldots, & \langle u_m, u_m \rangle \end{bmatrix},$$

where inner products are given by formulae

$$\langle y_i, u_j \rangle = \int_{h}^{T} y_i(\tau)\, u_j(\tau) d\tau =$$
$$\int_{h}^{T} \left[\int_{0}^{h} \varphi^{(i)}(s) y(\tau - s) ds \right] \left[\int_{0}^{h} \varphi^{(j)}(s) u(\tau - s) ds \right] d\tau \qquad (5)$$

For avoiding trivial solution $\Theta = 0$ one must chose the constraints for parameters. The common constraint used by all researchers is for instance a_n=1 or a_0=1 what is interpreted as normalization. However quadratic constraints are more general.

3.1 Optimal Solution Under Quadratic Constraints

One can formulate the optimization problem (4) with unit ball $\Theta \in \mathbf{B}(0,1)$ constraint for the parameters

$$\mathbf{B} = \{\Theta \in \mathbf{R}^{n+m+2} : \|\Theta\|^2 = \Theta^T\Theta = 1\} \tag{6}$$

The Lagrangian functional L for the above problem has a form

$$L = \Theta^T G\, \Theta + \lambda(1 - \Theta^T\Theta)$$

From the necessary condition of minimum it follows directly that

$$\frac{\partial L}{\partial \Theta} = 2\mathbf{G}\Theta - 2\lambda\Theta = 0 \cdots \Rightarrow \mathbf{G}\Theta^o = \lambda\Theta^o \tag{7}$$

Hence the optimal element Θ^o is an eigenvector of the Gram matrix **G** and the Lagrange multiplier λ is its eigenvalue. From the definition of spectral norm it follows that the minimum of J^2 on unit ball $\mathbf{B}(0,1)$ is equal to the minimum eigenvalue

$$\min_B J^2 = \min_B \left[\Theta^T \mathbf{G}\Theta\right] = \lambda_{min} \tag{8}$$

The optimal eigenvector $\Theta^o = w_{min}$ should thus be chosen, as that which correspond to the minimum eigenvalue λ_{min} of **G.** Gram matrix **G** is real and non-negatively definite. From the formulae (8) one can find the optimal parameters of (1) and (3) based on the calculations on interval [0,T]. Such a parameters represent average value for overall interval [0,T], (Byrski W., 1999, 2000).

For the current time t>T (in on-line applications) one should repeated calculations in moving window [t-T, t]. Then elements of Gram matrix start to be functions of t and also w(t) and λ_{min}(t). Hence calculations of the minimal λ_{min}(t) and eigenvectors of G(t) should be repeated for every t in interval [t-T ,t]. The value of Θ(t) represent average value of parameters for whole interval [t-T ,t]. If the interval T is short enough it is possible to detect the changes in system parameters.

4 NUMERICAL TESTS

We present application results of above described methodology for third order system with time-varying parameters a_1, a_2 (piecewise constant parameters as in Fig.2 and Fig.3 - solid line), b_0=2.

$$\dddot{y}(t) + a_2(t)\ddot{y}(t) + a_1(t)\dot{y}(t) + y(t) = b_0 u(t)$$

The input and output signals were measured as in Figure1 and Figure 4. For given input/output vectors (10000 samples each) program written in C++ automatically searches for the best continuous model within presumed possibilities.

For starting the program one should prepare the data file with samples. In this file also, many possible orders m_i of input derivative (numerator degrees of Transfer Function) and many possible orders n_i of output derivative (denominator degrees of TF) should be placed. Also many possible values of supports h_i for convolution filter and many different M, N in φ can be proposed. Parameters of each proper model structure ($m_i \le n_i$) are identified by the use of different filters based on (2) and (8). Next each identified model is simulated (by Runge-Kutta method) and the Output Errors are calculated. The best model is automatically chosen. Sometimes different structures give similar small Output Errors – then special procedure search for the structure, which is less sensitive to changes of support h_i in convoluting filter.

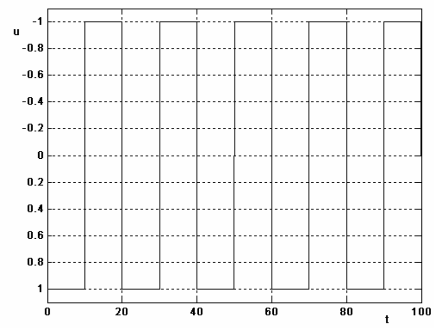

Figure 1: The control signal.

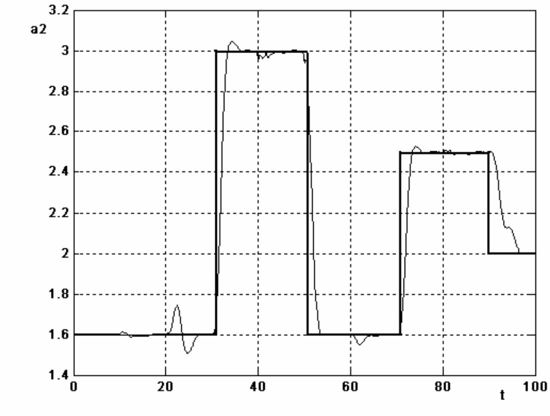

Figure 2: Time-varying parameter a_2.

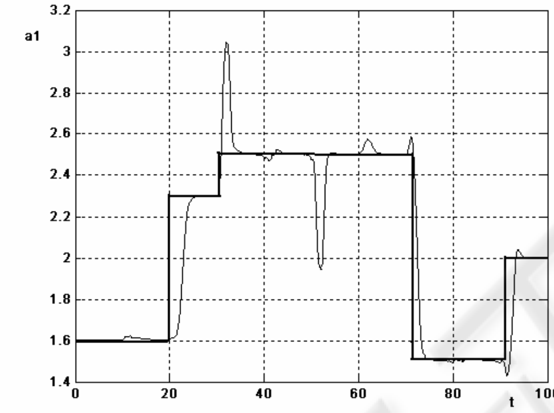

Figure 3: Time-varying parameter a_1.

In Figure 2 and Figure 3 high quality of detection of rapid parameter changes is observed. Exemplary result was obtained for T=5, h=1, M=5, N=4. The differences between measured output and simulated output presented in Figure 4 are almost invisible.

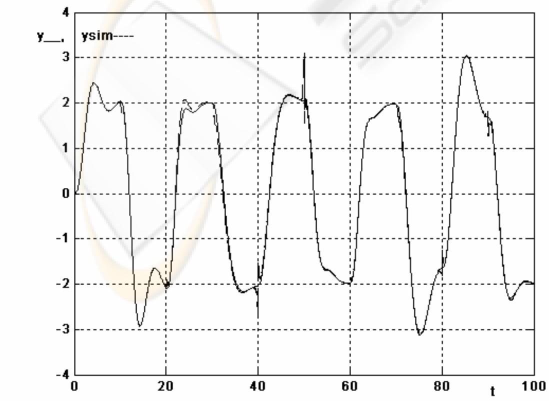

Figure 4: The outputs of system and model.

5 CONCLUSIONS

In the paper the optimal parameter estimator for linear continuous systems with time-varying parameters was presented. The identification method is based on the convolution of the input/output measurements with some chosen modulating function. Preprocessed data are used in *Moving Window Identifier* which operates on finite time interval [t-T ,t], for $T<t<T_0$. Hence the solution gives optimal parameters which are window average values. $\Theta^o(t, T)$ is function of time t and T.

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