A SOLUTION TO THE VEHICLE ROUTING PROBLEM USING TABU SEARCH

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Abstract: This paper presents a model for solving the Vehicle Routing Problem using Tabu Search. The Vehicle Routing Problem aims to serve a set of clients by a fleet of vehicle through the creation of least-cost routes that satisfy some constraints. In this paper, only the vehicle capacity is considered. The objective of this model is to design least-cost routes to serve a set of clients with known demands in such a way that some defined constraints are satisfied. An application to construct this model is proposed using Tabu Search. Some experiments were generated and confirmed that an increase in diversification on search space politics can generate results that are more qualified.

1 INTRODUCTION

The Vehicle Routing Problem (VRP) can be described as a set of customers that have to be served by a fleet of vehicles, satisfying some constraints (Laporte, 1992; Xu and Kelly, 1996). The Routing problems are usually deal within the logistic context (Ho and Haugland, 2004; Xu and Kelly, 1996). Logistic can be defined as the provision of goods and services from a supply point to a demand point (Bodin, Golden and Assad, 1983). The transport is one of the most costly activities in logistic, typically varying in one or two thirds of the total costs (Ballou, 2001). As said by Thangiah and Petrovic (Thangiah and Petrovic, 1997) the cost of transportation is dependent upon the minimization of the distance traveled by the vehicles and the number of vehicles required for serving all the demands. Therefore, the necessity to improve the efficiency of this activity has great importance. The search for transportation cost reduction, through designing a routing model that offer more economic routes aiming the minimization of time, distance and associated cost (Barbarasoglu and Ozgur, 1999) besides the number of vehicles required is a frequent problem of the decisions making in the logistic context (Ballou, 2001). The importance of the routing problems is evident due to the magnitude of the associated distribution costs. In this context, a small percentage saved in these expenses could result in substantial saving total (Bodin, Golden and Assad, 1983). Section 2 presents some resolutions methods of this problem. Section 3 is an introduction to Tabu Search metaheuristic. Section 4 defines the proposed model and the formulation used in this paper is showed in section 5. The Tabu Search implementation and the politics of neighborhood generation are explained in section 6. Section 7 brings a description of the experiments that were made and, in section 8, the results are shown, followed by the final considerations of this work in section 9.

2 THE VEHICLE ROUTING PROBLEM

The VRP is NP-Hard (Lenstra e Rinooy Kan, 1981) and this property makes this type of problem very difficult to be solved by exact methods, because of the computational costs. Since VRP is Np-Hard to obtain good solutions in an acceptable time, heuristics are used and this is the reason why the

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majority of researchers and scientists direct their efforts in heuristics development (Thangiah and Petrovik, 1997; Nelson et al, 1985; Xu and Kelly, 1996). Osman and Laporte (Osman and Laporte, 1996) define heuristic as a technique, which seeks good solutions at a reasonable computational cost without being able to guarantee the optimality. Laporte et al (Laporte et al, 2000) define two main groups of heuristics: classical heuristics, developed mostly between 1960 and 1990, and metaheuristics. The classical heuristics are divided in three groups: constructor methods, two-phase methods and improvement methods. Since 1990, the metaheuristics have been applied to the VRP problem. To Osman and Laporte (Osman and Laporte, 1996) a metaheuristic is formally defined as an iterative generation process which guides a subordinate heuristic by combining intelligently different concepts for exploring and exploiting the search space in order to find efficiently near-optimal solutions. Several metaheuristics have been proposed to solve the VRP problem. Among these ones, Tabu Search are considered the best metaheuristic for VRP. To review some works with Tabu Search and others metaheuristics some readings are suggested (Cordeau et al, 2002; Gendreau et al, 1999; Tarantilis et al, 2005).

3 TABU SEARCH

It's a technique to solve optimization combinatorial problems (Glover, 1989; Glover and Laguna, 1997), which consists in an iterative routine to construct neighborhoods, emphasizing the prohibition of stopping in an optimum local. The process that Tabu Search seeks, for the best solution, is through an aggressive exploration choosing the best movement, for each iteration, independently if this movement improves or not the value of the actual solution. As said by Viana (Viana, 1998), in Tabu Search development, intensifications and diversification strategies are alternated though the Tabu attribute analysis. Diversification strategies direct the search to new regions, aiming to reach whole search space while intensification strategies reinforce the search in the neighborhood of a solution historically good.

4 USED MODEL

The model used in this paper is defined in a complete, undirected graph G=(V,A) where V= $(v_o,_{vi},...v_n)$ is a set of vertices and $A=((v_i,v_j):v_i,v_j \in V, i\neq j)$ is an edge set. Vertex v_0 represents a depot where a fleet of N_v vehicles of homogeneous capacity Q_v is located. All remaining vertices are customers to be served. A non-negative matrix C = (c_{ij}) is defined on A representing the distance between the vertices. This problem is symmetric, since the costs are same in both directions. A non-negative weight d_i, is associated with each vertex to represent the customer demand at $v_{i.}$ The routes must be designed in such way that each customer is visited once, only by one vehicle and every route must start and finish at the depot (Xu and Kelly, 1996; Barbarosoglu and Ozgur, 1999). In this model, the basic version of the problem is used, often called the capacitated vehicle routing problem, because the only constraint is the capacity of the vehicles (Toth and Vigo, 2002; Laporte, 1992), which means, the total demand of the route can't exceed the capacity Q_v of the vehicle.

5 MATHEMATICAL FORMULATION

The mathematical formulation used in this paper was based on the one described in Barbarosoglu and Ozgur (Barbarosoglu and Ozgur, 1999):

$$\text{Minimize} \sum_i \sum_j \sum_v c_{ij} X_{ij}^v \qquad (1)$$

Subject to:

$$\sum_i \sum_v X_{ij}^v = 1 \text{ for all j} \qquad (2)$$

$$\sum_j \sum_v X_{ij}^v = 1 \text{ for all I} \qquad (3)$$

$$\sum_i X_{ip}^v - \sum_j X_{pj}^v = 0 \quad \text{for all p,v} \qquad (4)$$

$$\sum_i d_i \left(\sum_j X_{ij}^v \right) <= Q_v \text{ for all v} \qquad (5)$$

$$\sum_{j=1}^{n} X_{0j}^v <= 1 \quad \text{for all v} \qquad (6)$$

$$\sum_{i=1}^{n} X_{i0}^v <= 1 \quad \text{for all v} \qquad (7)$$

$$X_{ij}^{v} \in Z \qquad \text{for all i,j, and v} \qquad (8)$$

where X_{ij}^{v} are binary variables indicating if arc(vi,vj) is traversed by vehicle v. The objective function of distance/cost/time is expressed by eq. (1). Constraints in eqs (2) and (3) together state that each demand vertex is served by exactly one vehicle. The eq. (4) guarantees that a vehicle leaves the demand vertex as soon as it has served the vertex. Vehicle capacity is expressed by (5) where Q_v is the capacity. Constraints (6) and (7) express that vehicle availability can't be exceeded. The sub tour elimination constraints are given in eq.(8) where Z can be defined by:

$$Z = \left\{ \left(X_{ij}^{v} \right) : \sum_{i \in B} \sum_{j \in B} X_{ij}^{v} <= |B| - 1 \quad \text{for} \quad B \subseteq V / \{0\}; |B| >= 2 \right\}$$

6 THE TABU SEARCH APPLICATION

The architecture proposed was based on Tabu Search and the following modules compose it:

Initial Solution: This module generates the initial solution, which will be used in Tabu Search application, and it's divided in:

a) Net Generation: This module is responsible for a net generation. In this application, two types of nets were created: for 50 and 75 customers and one depot. The necessary information for net generation (vertices, demands and coordinates) was obtained in problems 8 and 9 from Christofides and Eilon (Christofides and Eilon,1969). These problems describes two nets of customers and one depot. In problem 8 there are 50 customers and in problem 9 there are 75. In both nets, the cost of the arcs represents the Euclidian distance between each pair of customers. These problems are widely used for testing purposes by several authors (Gendreau, Hertz and Laporte, 1994; Christofides, Mingozzi and Toth 1979; Xu and Kelly,1996; Laporte et al, 2000). This fact is very important for the evaluation of the results obtained in this paper that are presented in the end of this paper.

b) Route generation: This module use the net obtained in the previous stage to generate a set of routes for the problem. These routes are implemented following the idea know as "The Nearest Neighbor" (Tyagi, 1968; Cook et al, 1998). The initial solution obtained in this module is represented as a net that connects all vertices representing the customers and the depot. The objective value obtained for the problem with 50 customers was 715.04 and for the other problem was 1077.75.

Tabu Search Application Module: In this module the Tabu Search algorithm is implemented aiming to minimize the routes costs. The stop criterion adopted is the maximum number of iteration without an improvement in the value of the objective function. The stage of neighborhood generation was implemented with the usage of three politics to create the movements: exchanges between routes, routes construction and delete/insertion of routes. The first politic implements the vertex exchange and it was called V1. The second politic implements the route construction and it was called V2 and third politic which consist in performing the insertion and the removal of vertex was called V3. From each generated neighborhood (V1, V2, V3), a movement that minimizes the value of the objective function is chosen. This movement must not be Tabu, but if it is in the Tabu List, it may be admitted when there is the possibility of applying the aspiration criterion function. In this paper the criterion used consists in: if the Tabu movement value is better than the actual solution best value then this movement is forgiven and accepted. If the solution of the actual neighborhood $f(s_x*)$ is the best one ever found, the variables that keep the best solution obtained f*(melhsol) and the Tabu List are updated. This procedure which consists in generating a neighborhood, choosing the movement that minimize the objective function, evaluating if it isn't Tabu or if it's Tabu but can be accept with the use of the criterion aspiration function, and optimize the best solution (if possible) is performed to the three politics. While the stop criterion isn't reached, the search is restarted attributing to the initial solution the f* value (melhsol).

7 EXPERIMENTS DESCRIPTION

The objective function of this paper, showed in section 5, intends to minimize the distance associated to the arcs. The flexibility of the Tabu Search algorithm makes possible that several mechanisms of neighborhood generation can be utilized aiming the diversification of the search for good solutions. The Tabu Search allows the usage of intensification and diversification strategies in the search of good solutions (Glover and Laguna, 1997).

These strategies can be used through the learning of the model dynamic and parameters *NbMax* (number of iterations without improvement in objective function) and Tabu List size adjustments. In this work, searching a bigger variety of solutions, the experiments were done using the three politics of neighborhood generated alone or in association. Five types of experiments had been done with different combinations of neighborhood generation politics. The politics used in each experiment are shown in the Table 1.This table shows that the experiments 1, 2 and 3 aim to evaluate the solutions generated through the isolated execution of each politic. The experiments 4 and 5 evaluate the generated solutions through the politic association. After V1, V2 and V3 execution, the politics V2 and V3 had presented the best results, so they were grouped in the experiment 4. In experiment 5 the three politics were used together to generate the neighborhood. The objective of doing the experiments with the politics together is to analyze if when they are used with association they can influence themselves and produce a different neighborhood and possibly better solutions than when they are used isolated. For each experiment the Tabu Search algorithm was used with different size of Tabu List (250, 500, 750, 1000, 1500, 2000, 3000) and several *NbMax* values (100, 300, 500, 700, 900). Forty experiments, of each type were generated, totalizing 200 experiments for each net (50 customers and 75 customers). The experiments were made in a machine with Window XP, AMD Atlhon processor with 264 Mb of RAM memory.

Table 1: Politics used in experiments.

Exp.	Politcs	Description of Politics
1	V1	Exchange of vertex
2	V2	Route Construction
3	V3	Removal and Insertion of vertex
4	V2,V3	Route construction, Removal and Insertion of vertex
5	V1,V2,V3	Exchange; Route construction; Removal and Insertion of vertex

8 RESULTS

From the experiments done it was evidenced that with the flexibility of Tabu Search it's possible to obtain a great diversity of solutions that points as the best solutions the ones obtained when the politics were used together. When the politics were used together, they diversified the search in the space, because they generated more possible solutions to be investigated in the neighborhood. The behaviors of generated solutions in both cases have a similarity in the solutions quality. The graphics show the cost of the best solution obtained to each variation of *NbMax* and size of Tabu List used.

Using V1 politic, the *NbMax* intensification allowed finding solutions that minimize the initial solution, independently of Tabu List size. To the 50 customer net, using *NbMax* = 900 and Tabu List = 250 the result (654.88) was close to the result obtained (652,16) when Tabu List = 2000. To the 75 net customer, the best result was obtained with *NbMax* = 900. The politic V2 used by the experiment 2 had a constant behavior, not flexible and not influenced by the variation of the parameters, characterized by a poor neighborhood and consequently keeping the same value for all the experiments independently of the variation in Tabu List and *NbMax*. This behavior was observed in both problems. To the 75 net customers this politic presented the best results between the isolated politics (949.85). The experiments that used V3, primarily kept their constant results, presenting a significant reduction in a second moment and then keeping it constant until the end. To 50 customers net the reduction occurred with *NbMax* = 500 and Tabu List = 500, to 75 customers net it happened with *Nbmax* = 750 and Tabu List = 250. To the 50 customers net, this politic generated the best result between the politics that were used isolated (617.57). However, the results generated by these three isolated politics weren't the best. The average solutions obtained in each politic improved in 12% the value of the initial solution. In experiment 4, where the politics V2 and V3 were used together a significant reduction was obtained for both nets. To the 50 customers net, the best cost was 582,75 and to the 75 customers net was 904,11. It showed that the combination of the politics can generate greater quantities of movements diversifying the possible solutions. Although the behavior of these politics were not influenced by *NbMax* and Tabu List size variations, the results obtained were much better that those obtained through the execution of isolated politics.

In the experiment 5, which was done with the three politics together, the best solutions were found. It was 569.39 to the 50 customers net and 888.91 to the 75 customers net. For each *NbMax* value increase, better solutions were found. In both problems the best solution was obtained with *NbMax* = 900. Comparing the experiments 4 and 5 it is possible to say that the politic V1 was the responsible for the variations of the solutions. Although the isolated behavior of this politic was the worst one, it is possible to conclude that the presence of this politic, associated with the other two politics, allowed to find the best solution for both nets. The reason of experiments 4 and 5 good results can be determined by the fact that the different neighborhood generation politics, when used in association, increase the model flexibility, providing greater diversification and quantity of generated neighborhood.

These results were compared with some classical heuristics: {CW}Clark & Wright algorithm (Clarke and Wright,1964),{DV}Desrochers and Verhoog algorithm (Desrochers and Verhoog,1989), {MJ}Mole and Jamenson heuristic (Mole and Jamenson, 1976), {GM}Gillet and Miller algorithm (Gillet and Miller,1974) and with some of the best results of Tabu Search implementations:{PF} Pureza and França(Pureza and França,1991), {GHL} Gendreau, Hertz and Laporte (Gendreau, Hertz and Laporte, 1994) and {XK}Xu and Kelly (Xu and Kelly,1996). The results were obtained in Gendreau, Hertz and Laporte (Gendreau, Hertz and Laporte, 1994) and Xu and Kelly (Xu and Kelly, 1996). The best results obtained for problems 8 and 9 were generated by Tabu Search implementations. The comparison of the results is presented in Figure 1 and Figure 2. The results obtained in these experiments were considered satisfactory since there is a difference of less than 10% above the others results. The experiments show that the use of the diversification, in the neighborhood generation, allows an improve in the solution quality. The Tabu Search offers resources that enhance these qualified solutions. Observing only the papers that use Tabu Search, it´s easy to see that the implementations which use more memory resources and others sophisticated neighborhood produced the best results. So, the model development in this work, that uses a simple neighborhood structure, didn´t reach the best results of the others implementations.

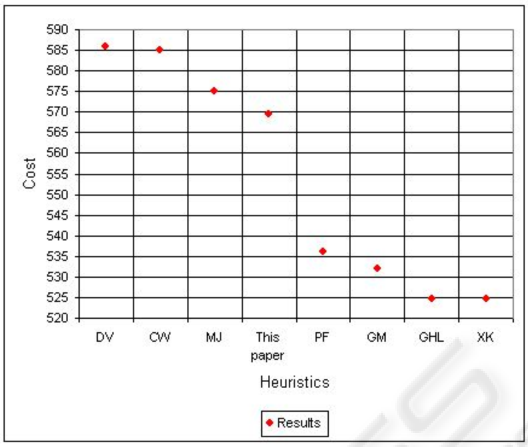

Figure 1: Papers results used in comparison for problem 8.

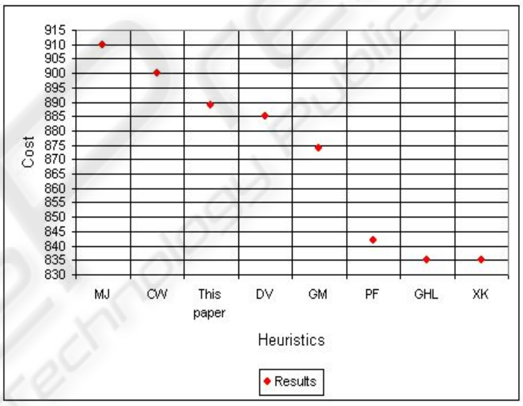

Figure 2: Papers results used in comparison for problem 9.

9 FINAL CONSIDERATIONS

In this paper, it was proposed a model for solving the Vehicle Routing Problem, which intended to generate least-cost routes to attend a set of customers respecting the constraints defined on the model. Here, the only constraint considered was the vehicle capacity. This model was implemented through the generation of an architecture that uses Tabu Search for solution generation. Two modules compose this architecture: initial solution generation module and Tabu Search module. The distance net generation was based on the vertices, demands and coordinates from Christofides and Eilon (Christofides and Eilon, 1969) problems 8 and 9. To the initial solution generation, it was applied a modified version of the algorithm based on Nearest Neighbor heuristic (Cook et al, 1998; Tyagi, 1968), contemplating the vehicle capacity constraint. The Tabu Search perform module utilized the initial solution to generate a set of routes that optimize the objective function model, through successive neighborhood generation. The flexibility of Tabu Search, allowed three neighborhood generations mechanisms, intending to diversify the process of generating routes for the model. The experiments indicate as better results the ones where all politics were used in association. The robustness and flexibility of Tabu Search makes possible to diversify and improve the process of search in space. The usage of varied neighborhood generation politics and the refinement of these politics, according to the objective model, make possible to expect that quality solutions can be found.

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