LOCALITY AND GLOBALITY: ESTIMATIONS OF THE ENCRYPTION COLLECTIVITIES

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Keywords: Self-organization, collectivity, structure, architecture, interconnection, locality, globality, symmetry.

Abstract: In this paper we try to define a collectivity, to model and to measure it. Because N. Bourbaki names "collectivizing relation" the relation defining a set, we name collectivities only the sets selected or built by the help of the relations. The orthogonal interconnections model very well the collectivities. The behavior (structural self-organization) around the origin is different for homogenous and non-homogenous interconnections. How can we measure this behavior? A way is by locality and globality. The locality measures analytically by neighborhoods, neighborhood reserves, *Moore* reserves and synthetically by diameters, degrees, average distances. The globality is the behavior of an interconnection around a property. The globality vs. symmetry measures by the compactity, efficiency and interconnecting filling. The locality and the globality are among primary manifestations of the self-organization. In this way, collectivities modeled by self-organizing interconnections can contribute to changing our fundamental view of computers by trying to bring them nearer to the nature.

1 INTRODUCTION STRUCTURE AND ARCHITECTURE

A complexity system modelling means firstly the perception of a *self-organization* of the system and then the proper modelling. *To perceive a complex*, said Wittgenstein, *means to perceive the relations of its constituent parts in a determined way*. On the other hand, one of the characteristics of the nature is the *collectivity*. Through the computing terrain, Professor Moshe Sipper said in the foreword to a recent book, during the past few years a new wind has been sweep, *slowly changing our fundamental view of computers. We want them, of course, to be faster, better, more efficient - and proficient - at their tasks. But, more interestingly, we are trying to imbue them with abilities hitherto found only in nature, such as evolution, learning, development, growth, and* collectivity (Castro and Zuben, 2005). We can observe collectivities in the not living world (universe galaxies, solar systems, crystalline units) as in the living world (ant hills, bee swarms, nations).

What *properties* are behind the relations who tie the collectivities? Maybe is the gravity, the symmetry or the survival instinct? In a word, *structural self-organization*. The self-organization can be structural and functional. Our paper refers to the structural self-organization applied to the collectivities.

First let us define the collectivity. For this we must answer to another question: what is a *set*? A set "can be selected by a membership or by a *relation which substantiate the membership* or by bringing in the set field *elements which fulfill the relation*" (Drăgănescu, 1985). Because N. Bourbaki names "collectivizing relation" the relation defining a set, we name collectivities only the sets selected or built by the help of the *relations*. Therefore, we exclude the sets selected by the membership, the most general. A collectivity not means a set made, for example, of a star, a planet, a crystal, an ant, a bee and a man.

The relation which substantiates the membership of a collectivity is connected with its functionality: a collectivity is made of the least *functional entities*. For example, an interconnecting is made of nodes and

Lupu C., Niculiu T. and Franţi E. (2006).
LOCALITY AND GLOBALITY: ESTIMATIONS OF THE ENCRYPTION COLLECTIVITIES.
In *Proceedings of the Third International Conference on Informatics in Control, Automation and Robotics*, pages 486-493
DOI: 10.5220/0001205404860493

links which is equivalent with the graph definition (a set X of nodes and an application Γ of X in X which gives the set of connections). The *encryption collectivity* means a set S of signs and an application (key) K of S in S which gives encryptions.

In this paper we try to begin to study the collectivities *structural* and by the help of the *architecture* concept, a connection concept toward the relation/function. We start with the definition of the concept of structure (Nemoianu, 1967). The word structure comes from the Latin where there are the noun *structura*, with the meaning of building, and the verb *struere* (to build) with the past participle *structus*. In English and French the word has the same meaning: edifice, way to build. The abstraction of the word makes slowly: only in the XVII-XVIIIth centuries appears in the sense of *reciprocal relation of the parts or the constitutive elements of a whole, determining its nature, its organization*. The initial meaning of building maintains till now but abstracter sense will be dominated more and more. During the XIXth century, structure is generally opposite to function, like static to dynamic.

The end of the XIXth century brings a new meaning of the structure concept. It will begin to represent not a simple configuration, a "static" organization, but a *whole made by solidary elements, in which everyone depends on all other ones and can not be what it is than in and through them*. Evidently, it is a step forward. The *connection between parts* (the first meaning) is something less necessary, less outlined, more approximately, more vaguely and more generally than the *total interdependence system of each part with all other parts* (the second meaning). If the first meaning is a *sum*, the second is a *whole*. This turning point coincides with the penetration of the structure concept in the humanities. The term has been changed by a synonym, *Gestalt*, understood as form, pattern, structure, the making of parts which are determined by *whole*, system of its behavior *can not equal with the sum of the parts*. *Gestalt* is not related to organization or to plan, but with an organism, a whole, an *entelechy*. The *entelechy* is a term introduced by the Austrian psychologist Ehrenfels appointing the features (of geometric figures or melodies) by which they exceed the *sum* characteristics. A geometric figure remains itself even represented in other coordinate system, decreased, enlarged, color modified. This invariance of the transposing calls also *isomorphism*.

The linguistic researchers contribute resolutely to the understanding and to the using of the structure concept unifying both meanings: the *coherent, coagulated globality* and the *relations system between local parts* or, in few words, the *globality* and the *locality*. This step in the evolution of the structure term opens a path to the identification between structure and essence of an object or a phenomenon. Wittgenstein writes in *Tractatus* that the *manner in which the objects depend some on the others in the state of affairs constitutes the structure of the state of affairs*.

Having in view the above, the *structure of a collectivity* can be self-organized *locally* and *globally*. For example, an interconnecting structure estimates locally by neighborhoods. Thus, the *locality is the behavior (structural self-organization) of a collectivity around an origin*. The origin can be temporal or spacial. The locality definition refers to the first meaning of the structure concept (the connection between parts). The *globality is the behavior (structural self-organization) of a collectivity around a property*. For example, the interconnections can be estimated and designed by the help of the *symmetry* properties. The globality definition concerns to the second meaning of the structure concept at which referred Wittgenstein (total interdependence system of each part with all other parts).

On the other hand, the *collectivity architecture*, a connection concept between the structure and the function, gives a *global meaning* to the collectivity with the aim to better understand the connection between the structure and the function of this collectivity. Thus, we can speak of the universe architecture, a crystallographic system architecture, a house architecture, a town architecture, a computer architecture, an interconnecting architecture, a communication architecture. The *architecture measures by the degree of membership to global properties*. The symmetry is a global property.

Helping the interconnection as a collectivity model we try to prove that the dichotomy locality-globality covers mathematically one of the structural meanings of the collectivity: the localization and the globalization, i.e. a *structural potential of a collectivity dynamics*, a *structural self-organization of a collectivity*. The dynamics of an encryption collectivity can help us to the decryption process.

2 INTERCONNECTION AS A COLLECTIVITY MODEL

The interconnections made of N nodes and L links model very well the collectivities. The nodes are the members of the collectivity which are tied by links. If there are the encryption collectivities the nodes are signs and the links are the set of encryption keys (a key is included in the set L). We shall limit, without losing too much of generality, to the orthogonal interconnections (Duato et al., 1997). The algebraic representation of an orthogonal interconnection can be made in a *mixed radix number system*, MRNS. Any number N can be represented in MRNS as a product of whole numbers, $N = m_r\ m_{r-1}\ ...\ m_1$.

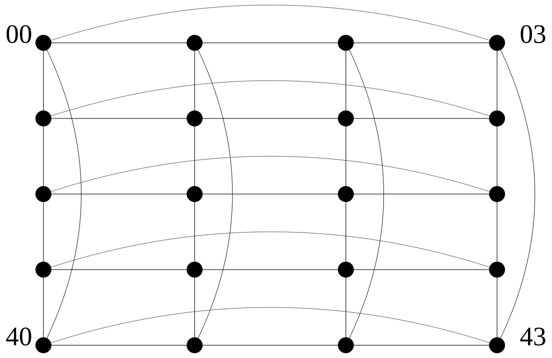

Figure 1: A GHT with $N = m_2 \cdot m_1 = 5 \cdot 4$.

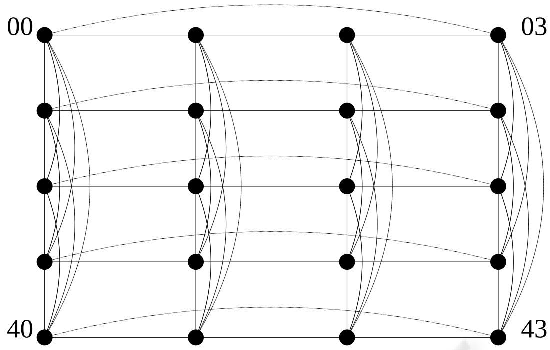

Figure 2: A GHS with $N = m_2 \cdot m_1 = 5 \cdot 4$. The interconnecting vector is $\left(X^{21}, X^{11}\right)$ and GHS is coded (CCS, T).

On the basis of this representation, to each node of an interconnection we can associate an address X, $0 \leq X \leq N-1$, made of r digits. Afterwards, we present some orthogonal interconnections as collectivities, i.e. sets selected or built by *relations*.

A *generalized hypercube*, GHC, is a collectivity with $N = m_r\ m_{r-1}\ ...\ m_1$ nodes interconnected in r dimensions. In every dimension i, $i = 1, 2, ..., r$, the m_i nodes are interconnected all by all, i.e. every node $X = (x_r\ x_{r-1}\ ...\ x_{i+1}\ x_i\ x_{i-1}\ ...\ x_1)$ is connected with the nodes addressed by $X' = (x_r\ x_{r-1}\ ...\ x_{i+1}\ x'_i\ x_{i-1}\ ...\ x_1)$, where $1 \leq i \leq r$, $0 \leq x'_i \leq m_{i-1}$ and $x'_i \neq x_i$. From GHC derives the *hypercube*, HC, with $N = m^r$, the *binary hypercube*, BHC, with $N = 2^r$ nodes, and the *completely connected structure*, CCS, with $N = m$ nodes. A *generalized hypertorus*, GHT, have $N = m_r\ m_{r-1}\ ...\ m_1$ nodes in r dimensions, in every dimension i, $i = 1, 2, ..., r$, the m_i nodes being interconnected in a torus, i.e. every node X is connected with the nearest neighbor nodes addressed by $X' = (x_r\ x_{r-1}\ ...\ x_{i+1}\ x'_i\ x_{i-1}\ ...\ x_1)$, where $1 \leq i \leq r$, $x'_i = |x_i \pm 1|_{modulo\ m_i}$. From GHT derives the *hypertorus*, HT, with $N = m^r$, the binary hypercubes also, and the *torus*, T, with $N = m$. A *generalized hypergrid* have $N = m_r\ m_{r-1}\ ...\ m_1$ nodes in r dimensions, in every dimension i, $i = 1, 2, ..., r$, the m_i nodes being interconnected in a *chain*, i.e. every node X is connected in a *grid* with the nodes addressed by $X' = (x_r\ x_{r-1}\ ...\ x_{i+1}\ x'_i\ x_{i-1}\ ...\ x_1)$, where $1 \leq i \leq r$; $x'_i = x_i \pm 1 | x_i \neq 0$ and $x_i \neq m_i - 1$; $x'_i = x_i + 1 | x_i = 0$; $x'_i = x_i - 1 | x_i = m_i - 1$. From GHG derives the *hypergrid*, HG, with $N = m^r$, the *chain*, C, with $N = m$ nodes and BHC again.

These are *homogenous* (at links) interconnections. As example of *non homogenous* interconnections we gave a variation of non-homogenous orthogonal interconnections, the *generalized hyper structures*, GHS (Lupu, 2002). A GHS is an interconnection in which every node X is connected in the dimension i, $1 \leq i \leq r$, to the nodes addressed by an interconnecting vector $\left(\cup_{j=1}^{k_i} X^{ij}\right) = (x_r\ x_{r-1}\ ...\ x_{i+1}\ x'_i\ x_{i-1}\ ...\ x_1)$. $\left(\cup_{j=1}^{k_i} X^{ij}\right)$ specifies that a node of GHS is connected by a *vector of unions of elementary interconnection structures*, instead of a *single* elementary interconnection structure in the homogeneous interconnections. This *interconnecting vector* has *r elements*, $1 \leq i \leq r$. So, this *interconnecting vector* is defined, on one hand, by the number of dimensions, r, and, on the other hand, by k_i elementary interconnection structures, $i = 1, 2, ..., r$, for which the unions $\left(\cup_{j=1}^{k_i} X^{ij}\right)$ are specified, $j = 1, 2, ..., k_i$. X^{ij} are homogeneous interconnections, like tori, T, grids, G, and completely connected structures, CCS, and must not be disjoint for a dimension.

In the figures 1 and 2 we give two examples of simple homogenous and non-homogenous interconnections. At homogenous regular interconnections, as the GHC or HT, the origin position does not matter. The interconnections are *spherical*, the diameter is the same. At irregular networks, as the generalized hypergrids and other non-homogenous interconnections, it matters where the position of the origin is. The ”structural” behavior around the origin is different for homogenous and non-homogenous interconnections. How can we measure this behavior? One way is by locality and globality.

3 LOCALITY: A FIRST SENSE OF COLLECTIVITY STRUCTURE

The collectivities having as a model the interconnections made of nodes and links can be estimated by *locality* and *globality*. The *locality is the spatial behavior of interconnection around an origin*. As in physics, where the gravity characterizes attraction of the objects, the locality *defines the interconnection*: nearer objects communicate better or nearer nodes interconnect easier. As we told above, the locality definition refers to the first meaning of the structure concept, the connection between parts (links of nodes). The locality measures analytically by neighborhoods, neighborhood reserves, *Moore* reserves and synthetically by diameters, degrees, average distances (Lupu, 2004a). We consider the locality to be classified firstly as *structural* (topological), and, secondly, as *functional*. Therefore, the locality of an interconnection will be defined by two localities: a *structural locality* and a *functional locality*.

The structural localities can be appreciated by *neighborhoods*. The neighborhoods can be classified as *surface* (radial) *neighborhoods* and *volume* (spherical) *neighborhoods*. The surface neighborhood of an interconnection is the number of nodes at a distance d, $SN_d(O) = N_d(O)$, where O is the origin chosen arbitrarily. The volume neighborhood is $VN_d(O) = \sum_{i=1}^{d} N_d(O)$. By neighborhoods, the structural locality can be evaluated analytically. Another measure, more synthetically, of the structural locality is the diameter: at the same number of nodes, the smaller diameter is the bigger locality is.

A problem, as we told above, is that the neighborhoods and the diameters depend on the *origin positions*. At homogenous regular interconnections, as the generalized hypercubes or hypertori, the origin position does not matter. At irregular interconnections, as the generalized hypergrids and other non-homogenous structures, it matters where the position of the origin is. The topographic model presented in (Lupu, 2004b) helped us to study the description and the behavior of the direct interconnections, homogenous and, especially, non-homogenous. The properties of interconnecting locality can be better ”read” by the *diameter contour patterns* in the *structural relief of the interconnection*.

We introduced a measure that helps us to reveal the interconnection relief, the *state of agglomeration*. The structural localities are more or less *agglomerated*, as in reality. The depth of the *valley* (minimum diameter) informs us about *maximum agglomerated locality*, and the height of the *peak* (maximum diameter) about the *minimum agglomerated locality*. Thus, *structural state of agglomeration of an interconnection node is given by the interconnection diameter computed with the origin in the corresponding node*. The *contour patterns* of structural states of agglomeration (of the diameters computed with the origin in every node) constitute a map with the *structural relief of the interconnection*.

The structural locality is an invariable information depending on the topology. A functional point of view on the interconnection locality can take into consideration the message routing distributions, $\Phi_O(d)$, where O is the origin and d is the distance.

As the structural locality, the functional locality measures also by neighborhoods: a *functional surface neighborhood*, $FSN_d(O) = \Phi_O(d) \times N_d(O)$, and a *functional volume neighborhood*, $FVN_d(O) = \sum_{i=1}^{d} \Phi_O(i) \times N_i(O)$. For the functional locality, there is also a synthetic measure like diameter, the *functional average distance*. The functional average distance helps the next definition: the *functional state of agglomeration of an interconnection node is given by the functional average distance of the interconnection computed with the origin in the corresponding node*. Shorter the functional average distance is, greater the state of functional agglomeration is! Using the *contour patterns* of the functional states of agglomeration we can draw a map depicting the *functional relief of the interconnection* (see next section).

The surface and volume neighborhoods, on the one hand, and the diameter or degree, on the other hand, are analytical and synthetic evaluation means of the intercommunication capability of interconnections, measuring the *structural locality*. By functional neighborhoods and, indirectly, by functional average distance, it expresses which part of the structural locality is used by communication process implemented on the network. In other words, the functional neighborhoods and the functional average distances express the *functional locality* of the interconnections.

Obviously, for a given interconnection, $SN_d \geq FSN_d$ and $VN_d \geq FVN_d$. The difference between the two types of neighborhoods represents what we named the *neighborhood reserve*. The neighborhood reserve is of *surface*, $SNR_d = SN_d - FSN_d$, or of *volume*, $VNR_d = VN_d - FVN_d$. Using the neighborhood reserve, we introduced a design/evaluation criterion of a topology by enunciating the following conjecture: *the intercommunication structural potential of an interconnection is optimally used in a communication process characterized by a routing distribution* Φ *if the neighborhood reserve is minimal*.

To evaluate the structural locality of an interconnection, besides the neighborhoods and neighborhood reserves, we proposed a simple measure: the *Moore reserve* based on the *Moore bound*. As it is known, the *Moore* bound is given as the *maximum number of nodes* which can be present in a graph of given degree l and diameter D: $N_{Moore} = 1 + l(((l-1)^D - 1)/(l-2))$. This bound is deduced from a complete l-tree with diameter D and is an *absolute limit* for a *diametrical volume neighborhood*, $VN_d(O) = \sum_{i=1}^{d} N_d(O)$, in *any graph (interconnection)* of l degree and D diameter. Except for the complete l-ary trees, this bound is rarely reached. *Petersen* graph, completely connected structures and rings with odd number of nodes are interconnections that reach the *Moore* bound. Therefore, it makes sense to compute for an interconnection how far is this bound: the farther away the *Moore* bound, the structural locality properties are worse. This is implemented by the *Moore reserves*.

The *surface Moore reserve* is defined by the difference between the number of nodes in a corresponding *Moore* tree at the distance d, with the degree in considered interconnection, and the surface neighborhood in considered interconnection: $SMR_d = l(l-1)^{d-1} - N_d$. The *Moore reserve* is defined by the difference between the *Moore* bound at the distance d and the volume neighborhood: $MR_d = N_{Moore}(d) - VN_d$.

4 HOMOGENEITY AND SYMMETRY

Based on the topographic model we estimate three bidimensional interconnections more and more *non-homogenous* and *asymmetrical*. Let us draw, in the first example, the functional relief for *uniform distribution* of bidimensional interconnection having 20 nodes on a dimension.

The unidimensional *elementary interconnection structure*, non-homogenous, *EIS*1, is the same in both dimensions being composed of a completely connected structure (nodes $0 \div 8$), a grid (nodes $8 \div 11$) and, again, of a completely connected structure (nodes $11 \div 19$). *EIS*1 has, in this way, 20 nodes ”symmetrically arranged”.

In the figure 3 we give the *contour patterns* for the uniform distribution. First, we notice the perfect symmetry in both dimensions thanks to the symmetry of the *EIS*, the same in both dimensions. According to this symmetry, we observe that the biggest part of the functional relief is formed of four tablelands having the same height, 5.5 nodes, orientated to the four *cardinal points*. In the middle of the interconnection, like a cross 4 nodes wide, four *canyons* deepen, with the average distance of 4.5 nodes. Right in the interconnection center there is a valley, the most agglomerated part of the structure, with a depth of 3.5 nodes. The biggest slope of the average distance $\bar{d}_U(O)$, to the interconnection middle, is 2 nodes, and the slopes crossing the canyons are 1 node.

The functional reliefs for the other distributions (structural and exponential) look likewise. The heights or the slopes are the difference.

Let us draw, in the second example, the functional relief of a bidimensional non-homogenous structure which has in the first dimension an elementary interconnection structure *EIS*2 being composed of a completely connected structure (nodes $0 \div 8$), of a grid (nodes $8 \div 11$) and, again, of a completely connected structure (nodes $11 \div 19$) and in the second dimension, the elementary structure *EIS*3 being composed of a torus (nodes $0 \div 8$), of a completely connected structure (nodes $8 \div 11$) and, again, of a torus (nodes $11 \div 19$). In the figure 4 we give the contour patterns of this interconnection for uniform distribution. The bidimensional interconnection is symmetrical too, though it has in the making of the elementary interconnection structures, *EIS*2 and *EIS*3, different homogenous sub-interconnections. The relief of this interconnection is *more varied*: four peaks, rather small tablelands, 7.5 nodes height, and a larger valley, of four nodes, separating the network in two along x_2 dimension and in the middle of x_1 dimension, 5.5 nodes depth. Still there are two saddles 6.5 nodes height between the peaks and, in the middle of the network, as in the previous example, the deepest valley (the most agglomerated part), 4.5 nodes depth.

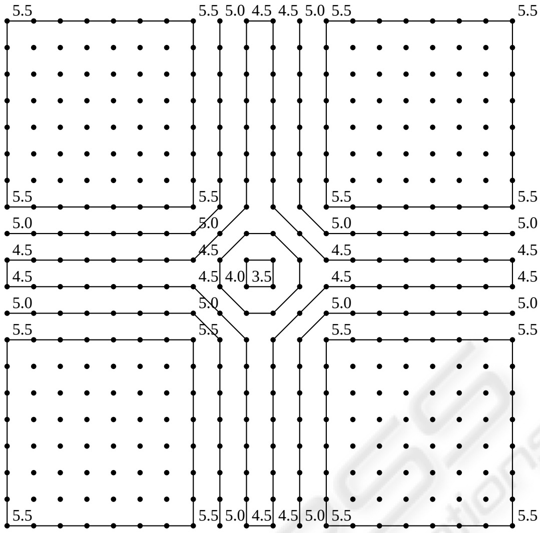

Figure 3: The functional relief for the bidimensional interconnection with the non-homogenous *EIS*1 for the uniform distribution. The *contour patterns* of the functional average distance $\bar{d}_U(O)$ are drawn.

The symmetry is not the same on the two interconnection axes, like in the first example. The symmetry, in present example, *differs* from an axis to the other and, therefore, is *weaker*.

In the last example is given a non-homogenous interconnection with a marked characteristic of *asymmetry*. Let us draw the functional relief of a non-homogenous bidimensional interconnection with 20 nodes per dimension. On the first dimension there is an elementary interconnecting structure *EIS*4 being composed of a completely connected structure (nodes $0 \div 5$), a grid (nodes $5 \div 12$) and a torus (nodes $12 \div 19$). On the second dimension the elementary interconnecting structure *EIS*5 is composed of a torus (nodes $0 \div 10$), a completely connected structure (nodes $10 \div 15$) and, again, a torus (nodes $15 \div 19$). In the figure 5 we give the contour patterns of this asymmetrical on both axes interconnection. The structure presents only partial symmetries on certain areas.

We presented three bidimensional interconnections with the same number of nodes per dimension and

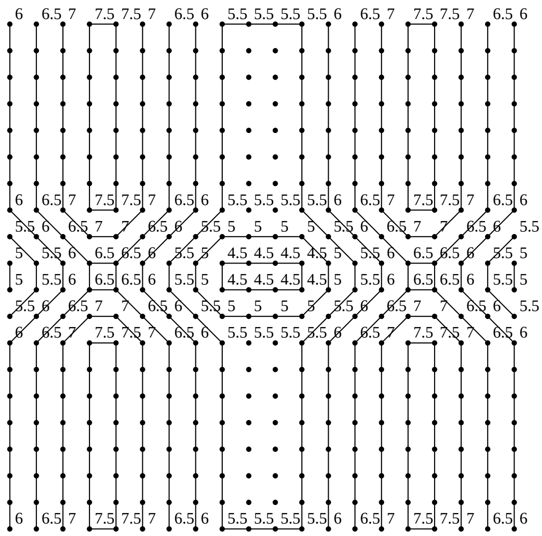

Figure 4: The functional relief for the bidimensional non-homogenous interconnection with the elementary structures *EIS*2 and *EIS*3. The *contour patterns* of the functional average distance $\bar{d}_U(O)$ for the uniform distribution are drawn.

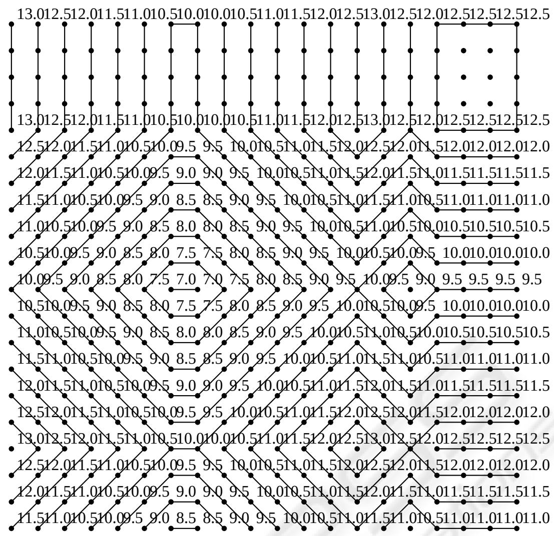

Figure 5: The functional relief of the bidimensional interconnection with elementary structures *EIS*4 and *EIS*5 for uniform distribution. The *contour patterns* of the functional average distance $\bar{d}_U(O)$ are drawn.

with elementary interconnection structures more and more *non-homogenous*. The functional reliefs proved these three interconnections have a more and more marked *asymmetry*, the structures having a more and more emphasized ”structural dynamism”, *structural self-organization*. This *structural dynamism* leads to a more and more powerful *structural self-organization* property. Therefore, the non-homogeneity leads, on the one hand, to the *asymmetry*, and, on the other hand, to the *more intense structural self-organization*.

5 GLOBALITY: A WAY FROM THE STRUCTURE TO THE ARCHITECTURE

One of the most important *properties* of any physical space structure is the *symmetry*. The *transformation* that keeps the structure of the space is named *automorphism*. Giving a space configuration, a structure, a form, an *interconnection*, we can emphasize a set of space automorphisms, which leave unchangeable this interconnection. Thus, the emphasizing automorphisms form a *group* which describes precisely the symmetry of the giving configuration.

The amorphous space has a *total symmetry* corresponding to the group of all automorphisms. The symmetry of an interconnection will be described, as we have told, by a subgroup of all automorphisms. The total symmetry of the space defined by n points (nodes, permutations) will be described by $S_{n!}$, while a *partial symmetry* is expressed by a subgroup (of permutations) included in $S_{n!}$. Therefore, symmetrical groups $S_{n!}$ model the symmetry of a space defined by n nodes and inversely. The total symmetry of a space is represented by a total interconnection, a completely connected structure with $n!$ nodes.

As an example, the plane figures have as constitutive symmetries only the identity, rotation, translation, reflection and reflection-translation. It is known that a rectangle has the following four symmetries: the identity, I; the two reflections S_1 and S_2 vs. non-parallel sides perpendicular bisectors, A_{S_1} and A_{S_2}; the rotation with $180°$, R. The four automorphisms can be represented by an interconnection, the vertexes of which are noted 1, 2, 3 and 4. With this, we equate the symmetries of the rectangle with following permutations (generators): $I = (1\ 2\ 3\ 4)$, $S_1 = (2\ 1\ 4\ 3)$, $S_2 = (4\ 3\ 2\ 1)$ and $R = (3\ 4\ 1\ 2)$. The four symmetries form a commutative group to the composition operation but, equating them with permutations, we notice that these symmetries form only a *subgroup* of the symmetric group of order 4, $S_{4!}$. In this way, we can examine the symmetry properties of plane figures, which divide the symmetric groups $S_{n!}$ in different subgroups. Let us note by G_S the groups (subgroups) of symmetries which divide the symmetric group $S_{n!}$.

We defined at the beginning of the paper that the *globality is the behavior (structural self-organization) of a collectivity around a property*. How does it define the globality of the plane figures vs. symmetry property? A quantitative appreciation, a measure of the *globality vs. symmetry*, which we note Γ_n, is given by the ratio of the order of group of symmetries and the order of symmetric group: $\Gamma_n = |G_S|/|S_{n!}|$. The inverse of Γ_n we denominated *group locality*, L_n, (Lupu and Niculiu, 2005).

The globalities must be compared at the same number of interconnecting nodes (same $S_{n!}$). For example, the globalities vs. symmetry of the tetragon and rectangle are the same for they refer to the same symmetric group, $S_{4!}$, while we can not say anything about globalities of the isosceles triangle and the square for they refer to the different symmetric groups, $S_{3!}$ and $S_{4!}$. The maximum globality will be obtained when $G_S = S_{n!} = 1$. Let us give three plane figures, an isosceles triangle, a trigon and an equilateral triangle, all having 3 interconnecting nodes, so referring to $S_{3!}$. The isosceles triangle has two symmetries, I and S, its globality being the least, $G_S/S_{3!} = 1/3$. The trigon has three symmetries, I, R_1 and R_2. Its globality is equal to 1/2. The equilateral triangle has 6 symmetries, I, R_1, R_2, S_1, S_2 and S_3. Its globality is the biggest, 1.

Instead of relying on the logic distances between the nodes (locality), we want to evaluate/design a interconnection (collectivity) based on *properties*. The globality put the properties, a constructive, synthetic principle, an *architectural principle*, before the distances, an analytic principle, especially tied to the locality. The logic distances "disappear" into a globality, which displays the properties. The locality principle helped us to design/evaluate new non-homogenous interconnection networks, as generalized hyper structures, and the globality principle helped us to imagine a *new interconnection paradigm* based on symmetrical morphemes and ensembles and that we will shortly introduce in next paragraphs.

The *morphological interconnection*, that we propose as a new model for a *collectivity*, have to *ensemble* in $S_{n!}$ *elementary entities*. We shall name these entities, *morphemes*, and the tying interconnection, *morphological interconnection*. If we use the architectural principle of globality vs. symmetry we shall name *symmetrical morphemes*, *symmetrical ensembles* and *symmetrical interconnection*.

The symmetrical morphemes, helping us to build symmetrical ensembles, are bidimensional or tridimensional forms emphasizing in a symmetric group $S_{n!}$ by the *Cayley* graphs (Akers and Krishnamurthy, 1989) of (sub)groups of symmetry, G_S. These groups of symmetry represent the symmetries of plane or tridimensional figures. For example, the symmetries of the *right line segment* are the identity $I = (1\ 2)$ and the reflection $S = (2\ 1)$. G_S has a *Cayley* graph with a transposition. The symmetries of the *isosceles triangle* are the same, the identity $I = (1\ 2\ 3)$ and the reflection $S = (1\ 3\ 2)$. The *Cayley* graph associated to the symmetries of the isosceles triangle is also with 2 nodes and a transposition, the only difference being the defining automorphisms symmetric groups, $S_{2!}$ for segment and $S_{3!}$ for isosceles triangle. The symmetries of the *trigon* are identity $I = (1\ 2\ 3)$ and two rotations $R_1 = (2\ 3\ 1)$ and $R_2 = (3\ 1\ 2)$. The *complete* (Lupu, 2004a) *Cayley* graph of the trigon symmetries subgroup is a directed graph. It is an overlap of two hamiltonian circuits (cycles as permutations) in the opposite direction, representing *minimal Cayley* graphs of the trigon symmetries. The symmetries of the *equilateral triangle* are the identity $I = (1\ 2\ 3)$, the rotation with $180°$ $R_1 = (2\ 3\ 1)$, the rotation with $240°$ $R_2 = (3\ 1\ 2)$ and the reflections $S_1 = (1\ 3\ 2)$, $S_2 = (3\ 2\ 1)$ and $S_3 = (2\ 1\ 3)$. The symmetric morpheme of the equilateral triangle has the globality $\Gamma = G_S/S_{3!} = 1$. The morpheme of the right line segment is a *linear morpheme*, of the triangle and the square are *plane* morphemes and the morphemes of the pyramid and the prism are *spatial* morphemes.

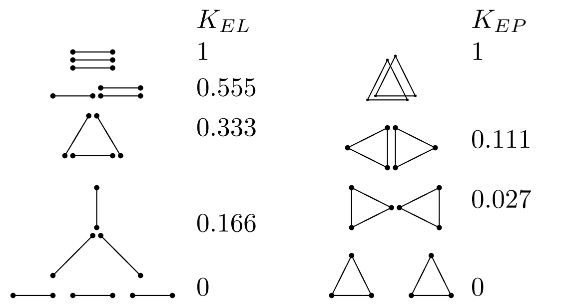

Figure 6: Compactity of the ensembles K_E realized by simple symmetric morphemes in architectural space $S_{3!}$.

A first symmetric ensemble characteristic appreciates its *compactity*. The maximal compactity of an ensemble will be obtained when all morphemes will have all nodes, links, surfaces and volumes interconnected. There are four basic rules of morphemes interconnecting: *common nodes* (CN), *common links* (CL), *common surfaces* (CS) and *common volumes* (CV). In this way, the compactity is a measure of morphemes interconnecting in an ensemble. The compactity is minimal for CN interconnecting and maximal for CV interconnecting. Let us note the ensembles compactity with K_E and it will express different for the three types of morphemes: $K_{EL} = \Gamma^2 \frac{m \cdot n}{N_M}$, $K_{EP} = \Gamma^3 \frac{s \cdot m \cdot n}{L_M \cdot N_M}$ and $K_{ES} = \Gamma^4 \frac{v \cdot s \cdot m \cdot n}{NS_M \cdot L_M \cdot N_M}$, where Γ is the globality; n is the number of nodes interconnected, $n = 0 ... \frac{N_M}{\Gamma}$; m is the number of link interconnected, $m = 1 ... \frac{L_M}{\Gamma}$ ($m = 1$ for no link interconnected); s is the number of surfaces interconnected, $s = 1 ... \frac{NS_M}{\Gamma}$ ($s = 1$ for no surface interconnected); v is the number of volumes interconnected, $v = 1 ... \frac{1}{\Gamma}$ ($v = 1$ for no volume interconnected); N_M is the nodes number of the morpheme; L_M is the edges number of the morpheme; NS_M is the surfaces number of the morpheme. In the figure 6 we give some examples of symmetric ensembles structured in the architectural space $S_{3!}$ with linear and plane morphemes. It also mentions the compactity

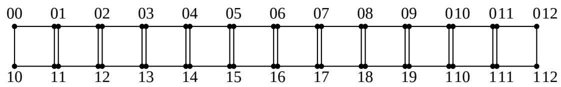

Figure 7: A GHG build by the rule CL in architectural space $S_{4!}$ from the 12 symmetrical morphemes of the tetragon.

K_{EL} for linear ensembles and K_{EP} for plane ensembles. About the other ensembles characteristics, the *interconnecting efficiency in pure ensembles* and the *capacity of filling*, we shall write in another paper.

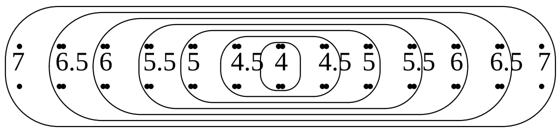

Figure 8: $\overline{d_U}$ functional relief of the ensemble of the fig. 7.

After a short evaluation of the symmetrical ensembles by *outside* measurements involving the globality and the geometry of the symmetric morphemes, let us appreciate by *inside* measurements which will offer a view on the on the communicability of them. The symmetrical ensembles are build in $S_{n!}$ of symmetrical morphemes which have a property or more, tied by some general rules. For example, in the figure 7 we give a generalized hypergrid assembled in $S_{4!}$ of 12 symmetrical morphemes of the tetragon. A generalized hypergrid, GHG, is assembled in two dimensions by rule CL and for the algebraic representation we used MRNS. In the figure 8, using the topographic model mentioned above we obtained a functional relief with an uniform routing distribution.

6 CONCLUSION

In this paper we tried to approach in other way the problem of encryption. Instead of occupying, for example, with the algorithms (functional self-organization) (Lupu et al., 2005), we questioned what hides behind the algorithms. A possible answer is the (encryption) collectivities modeled as interconnections (structuralized self-organization). Our principal aim was to define the collectivities, then to model and to measure them. The collectivity is a *privilege of structuralized nature* (living and not living). A collectivity is at least an interconnection. Locality and globality are among the most general structural measures, the primitives of an interconnection which models a collectivity. The locality supposes an origin and the globality, a property. The locality is the structural self-organization around an origin and the globality, around a property. The architecture, a connection concept between the structure and the function of the collectivity, measures by the degree of membership to global properties, like symmetry. Helping with these concepts, self-organization, structure, architecture, function, interconnection, locality and globality, we tried to model and to measure a collectivity. *Discovering the rules that govern the future interconnection environment is a major challenge* (Zhuge, 2005) and, maybe, one of the future interconnection environments is the collectivity model.

ACKNOWLEDGEMENTS

The work of this paper was done with financial support from SCRIPT Project 8/2005 from Romanian Security Program. The authors would like to thank one of the anonymous reviewers for its valuable comments.

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