

# A THEORETICAL PERFORMANCE ANALYSIS METHOD FOR BUSINESS PROCESS MODEL

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**Keywords:** Computational Model, Balance Equation System, Bottleneck, Business Process Model

**Abstract:** During designing a business process model, to predict its performance is very important. The performance of business operational process is heavily influenced by its bottlenecks. In order to improve the performance, finding the bottlenecks is critical. This paper proposes a theoretical analysis method for bottleneck detection. An abstract computational model is designed to capture the main elements of a business operational process model. Based on the computational model, a balance equation system is set up. The bottlenecks can be detected by solving the balance equation system. Compared with traditional bottleneck detection methods, this theoretical analysis method has two obvious advantages: the cost of detecting bottlenecks is very low because they can be predicted in design time with no need for system simulation; and it can not only correctly predict the bottlenecks but also give the solutions for improving the bottleneck by solving the balance equation system.

## 1 INTRODUCTION

In order to keep competitive in the marketplace, enterprises have to continuously increase the performance while reducing the cost. Business process modeling represents the business logic of an enterprise's business activities. It includes mainly three kinds of important components: activities, data processed by activities and resources allocated to or consumed by activities. Performance of business processes running in an enterprise influences heavily on performance of the whole enterprise. As a result, it is effective to improve enterprise performance by accelerating its business processes.

A large number of experiences accumulated in business practice indicate that keeping compatible data processing speed of relative activities and appropriate allocation of resources are two major factors for high performance. Otherwise, performance bottlenecks constraint the whole process performance. Given a manufacturing enterprise that has a high-speed producing department and a low-speed sales department, it will have to stop operating due to the overflow warehouse sooner or later. Once the root cause behind this phenomena is detected, the enterprise

can adjust the produce-store-sale process to gain the producing speed, sales speed and warehouse capacity suited by re-allocating resources. However, detecting business process bottlenecks is usually time consuming and prone to be inaccurate because of the complexity of business process and the inherent hidden characteristics of bottlenecks. Therefore, an effective and accurate approach is urgently needed to detect the bottlenecks in business process for performance improvement.

There are currently a lot of methods in use to detect the bottlenecks(Christoph, 2003)(Christoph, 2001)(Blake, 1995)(Lawrence, 1995). Two frequently used methods are measuring waiting time and calculating the utilization. Waiting time method is to look for the machine where the parts have to wait for the longest time. A common method is to look for the longest queue. However, this approach works only for linear systems containing only one type of part. The utilization based method is to measure the percentage of the time that a machine is active, and then define the machine with the largest active percentage as the bottleneck. Both of these two approaches have several drawbacks.

For measuring waiting time, the accuracy of this approach is compromised if the system contains

buffers of limited size. Furthermore, this approach analyzes only the processing machines of the manufacturing system. Other elements, such as human workers, supply and demand, do not have the concept of buffer, so that the waiting time measuring approach is not feasible for them. For measuring utilization, the measure result may be not accurate enough to be trusted because the difference between the utilizations of the machines may be very small. While this method is easy to be automated, the results are not always accurate (Luthi, 1997). Moreover, most of utilization bottleneck detection methods are based on analyzing the system data in running time or simulating the systems, which will result in great cost and time consuming.

In this paper, an abstract computational model is put forward aiming at analyzing business process performance effectively and efficiently. Firstly, a computational model is proposed to abstractly model business process as sets of transitions, data, data buffers, and connections between transitions, based on which most of the common performance related problems are described. Then, the problem of analyzing bottleneck transitions in a business process is theoretically induced as a Balance Equation solving problem and a solving algorithm for Balance Equation is proposed. Finally, real business performance problems are analyzed using above theoretical method to illustrate the application of this approach and verify its effect.

We start presenting the computational and analytical model in Section 2. Section 3 introduces the theoretical foundation, Balance Equation System, for business transition bottleneck detection and proposes the equation solving algorithm. Performance problems in real case are dealt with in Section 4 for illustrating and verifying the whole approach. Section 5 concludes this paper and gives direction of the future works.

## 2 THE COMPUTATIONAL MODEL

To perform performance analysis, it's necessary to firstly abstract the system as a mathematical model. Such mathematical models keep critical elements but eliminate those unnecessary details. As a result, we can focus on the perspective closely related to the problem we concern. At the same time, we can utilize some mathematical mechanism to detect the bottleneck conveniently and accurately. A lot of theoretical models can act this role, for example, queue network(Donald, 1998), Petri Net(Ajmore,

1984)(Bacelli, 1993)(Salimifard, 2001), fluid model(Kelly, 2004) etc. However, all these models are not specially designed for analyzing the performance of business process model. They are complex and not easy to be used in the business process bottleneck detection.

To simulate the behavior of a business process and support further mathematical performance analysis, we use a computational model defined in Definition 2.1. In our definition, four kinds of elements are taken into consideration: transitions, data, connections and data buffers. Transitions are abstraction of activities in a business process, which perform data processing in a certain period of time. Data are abstraction of business items processed by business process activities. The basic unit of data is a data particle, which can be a group of transaction data with fixed composition structure in a business process. A connection is a directed linkage between two transitions representing that output data from the source transition provides input data for the target transition. A data buffer is associated with a connection to cache data for dealing with the run time un-synchronization between two connected transitions. A transition is periodically executed on a fixed time interval. In case that one of the input buffers is empty or an output buffer is full, the current transition operation is skipped. Later, after the time interval, the transition engine will come again to see if the transition can be performed.

**Definition 2.1** A Computational Model is a tuple  $(T, C, B, \text{Start}, \text{End}, I/O, V^R, V^E)$ , where,

1)  $T$  is a set of transitions. 2)  $C$  is a set of connections that connect two transitions. 3)  $B$  is a set of data buffers. Each data buffer may store multiple data particles. 4)  $\text{Start}$  is a set of initial data buffers, and the size of these buffers is supposed to be infinite. 5)  $\text{End}$  is a set of final data buffers, and the size of these buffers is supposed to be infinite. 6)  $I/O$  is a finite set. Each of the elements in the set is called a data input/output structure (D-I/O-S) which is defined for an individual transition. A D-I/O-S is denoted as the following form:  $T_i: (\bar{N}) \rightarrow (\bar{M})$ , where  $T_i$  is the transition,  $\bar{N}$  is the consumed data particles by transition  $X$ , and  $\bar{M}$  is the produced data particles. For example,  $T_1: (1,2,2) \rightarrow (3,4)$  means that each time transition  $T_1$  consumes three types of data particles and the number of each type separately is 1, 2, and 2, and produces two types of data particles and the number is 3 and 4. 7)  $V^R$  is a set of running speeds of all the transitions. 8)  $V^E$  is a set of effective speeds of all the transitions.

For a transition, there are two important concepts regarding its performance analysis: the running speed  $V^R$  of a transition and the effective speed  $V^E$  of a transition.  $V^R$  indicates the speed that the transition accesses the input buffers in order to perform an operation, i.e., the number of times that the transition accesses the input buffers trying to extract data particles from them within a time unit. It should be noticed that not every attempt to extract data particle can be accomplished. An attempt may fail due to empty input buffer or fill-up output buffer.  $V^E$  indicates the speed that the transition accesses the input buffers, gets data particles successfully, and performs data processing, and writes data into output buffers. From the definition, it obviously holds  $V^R \leq V^E$ .

Figure 2.1 shows the status of a running transition. In which the light-grey blocks indicates the pace of running, while the dark-grey blocks show the pace of processing. For the yellow block with red one in it, that is the case that the operation fails to meet the requirement for performing a transformation. The ratio  $V^R/V^E$  represents the power usage of the transition. The above two speeds are about the average running speed. In the real world situation, these speeds may vary with time.



Figure 2.1: Status of a running transition

Choice transition is a special kind of transition as denoted in Figure 3.2. It is used to represent multiple output choices for a transition. For the transition A, there are three possible output paths connecting with different next transitions. The transition C in the diagram is the choice node. It is associated with some judgment criteria in order to decide which connection the data should move with. It should be noticed that there is no buffer over the A-C connection. This is because a choice node is treated as a kind of super fast transition unit, such that every input data will be passed to the next level connections with no time delay.

### 3 THEORETICAL ANALYSIS METHOD

The essential capability of process model is to specify the behavior of a real business process. An individual transition represents a behavior of a business activity, and the execution logic of a set of transitions is used to capture the behavior environment of a business process. The performance of a business process can be scaled by different

approaches, such as throughput of an activity and task waiting time. In this paper, data throughput is regarded as a performance scale standard for a business process. That is to say, the more data is manipulated within a time unit, the higher performance of the business process will be.

For a business process, the execution logic structure of its activities is usually known, which indicates that the data input/output capability of each activity in the process is known. This structure is defined as data input/output structure of computational model. Based on the above conditions, we can analyze the global performance of the process model, so that the analysis result can help design the running speed of each activity and the size of every data buffer. In order to analyze the performance of process model, we designed a balance equation system (BES). Based on BES, a lot of performance related business problems can be solved.

#### 3.1 Balance Equation System (BES)



Figure 3.1 Sequential Structure.

Figure 3.1 is a simple Computational Model with sequential execution logic. It includes two transitions, A and B, and a data buffer C. The data input/output structure of transition A is  $A: (N_{XA}) \rightarrow (M_{AB})$ , and the data input/output structure of transition B is  $B: (N_{AB}) \rightarrow (M_{BY})$ . Suppose the running speed of transition A and transition B separately are  $V_A$  and  $V_B$ . For data buffer C, when transition A puts  $V_{XA} * M_{AB}$  particles into the buffer, transition B will extract  $V_{AB} * N_{BY}$  particles from the buffer in the same time unit. In order to guarantee the process can be safely and normally executed in a long time, the  $V_A * M_{AB}$  and  $V_B * N_{BY}$  must be equal. Otherwise, the buffer C will produce two situations: one is the process is blocked because the buffer C is full; and the other is transition B is idle because the buffer is empty. The following is called Balance Equation for buffer C:

$$V_A M_{AB} = V_B N_{AB}$$

$$\text{Where } 0 \leq V_A \leq U_A \text{ and } 0 \leq V_B \leq U_B \quad (3.1)$$

Where,  $V_A$  and  $V_B$  are the effective speeds of transition A and B respectively;  $U_A$  and  $U_B$  are the upper bounds of  $V_A$  and  $V_B$  respectively. At the design time of a model,  $U_A$  and  $U_B$  are parameters for consideration. In the run time,  $U_A$  and  $U_B$  should

be  $V_A^R$  and  $V_B^R$  respectively.

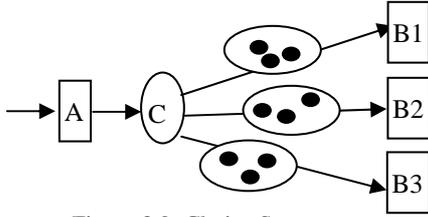


Figure 3.2: Choice Structure

Figure 3.2 is a Computational Model with a choice execution logic. The data input/output structure of transition A is  $A: (N_{XA}) \rightarrow (M_{AC})$ , and the data input/output structure of transition  $B_i$  ( $i=1, 2, 3$ ) is denoted as  $B_i: (N_{AB_i}) \rightarrow (M_{B_iY})$ . So the Balance Equation is as following:

$$\alpha_i V_A M_A = V_{B_i} N_{AB_i} \quad i=1,2,3 \quad (3.2)$$

$$\sum_i \alpha_i = 1 \quad (3.3)$$

The above Balance Equations describe the basic feature of the data particle flow over the computational model. All the balance equations of a computational model is called the Balance Equation System of the model.

Properties: The following are some basic properties of the balance equations.

- 1) There are exactly two non-zero elements for each equation.
- 2)  $V=0$  is a solution.
- 3) The solution set is convex.

In most cases, it holds in the model that the number of connections is larger than the number of transitions. This indicates that the BES is usually over constrained.

**Definition 3.1:** The computational model is called consistent if there is a non-zero solution for its BES.

### 3.2 Solvability of the BES

For any Balance Equation, we are going to study what is the condition that it has non-trivial solution. In order to give a general conclusion, an example is used to illustrate the main idea. Figure 3.3 is an example of Computational Model. It includes four transitions, four data buffers, and four connections. From transition A to transition B, there are two paths, A-C-B and A-D-B.

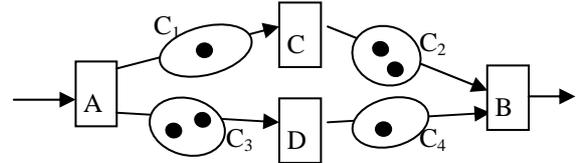


Figure 3.3: Computational Model Example

The data input/output structures of transition A, B, C, D separately are:

$$A: (N_{XA}) \rightarrow (M_{AC}, M_{AD});$$

$$B: (N_{CB}, N_{DB}) \rightarrow (M_{BY});$$

$$C: (N_{AC}) \rightarrow (M_{CB});$$

$$D: (N_{AD}) \rightarrow (M_{DB})$$

(MAC, MAD) denotes that transition A outputs data MAC and MAD in parallel. MAC is for path A-C, and MAD is for path A-D. (NCB, NDB) denotes that transition B can be executed only accepting two inputs from two paths in parallel. NCB is from C-B, and NDB is from path D-B. The balance equation system of this computational model is given as following:

$$\left\{ \begin{array}{l} V_A M_{AC} = V_C N_{AC} \end{array} \right. \quad (3.4)$$

$$\left\{ \begin{array}{l} V_C M_{CB} = V_B N_{CB} \end{array} \right. \quad (3.5)$$

$$\left\{ \begin{array}{l} V_A M_{AD} = V_D N_{AD} \end{array} \right. \quad (3.6)$$

$$\left\{ \begin{array}{l} V_D M_{DB} = V_B N_{DB} \end{array} \right. \quad (3.7)$$

From the above balance equations, the following two equations can be gotten:

$$\left\{ \begin{array}{l} V_A = \alpha_{AC} V_C = \alpha_{AC} \alpha_{CB} V_B \end{array} \right. \quad (3.8)$$

$$\left\{ \begin{array}{l} V_A = \alpha_{AD} V_D = \alpha_{AD} \alpha_{DB} V_B \end{array} \right. \quad (3.9)$$

, where  $\alpha = N/M$  is called transforming parameter of transition X and transition Y, and N is the data input of transition Y, and M is the data output of transition X. Comparing (3.8) and (3.9), we can get the consistency condition for equations (3.4)-(3.7) to have non-zero solution as:

$$\alpha_{AC} \alpha_{CB} = \alpha_{AD} \alpha_{DB} \quad (3.10)$$

**Definition 3.2:** Given a path in the computational model,  $T_1 \rightarrow T_2 \rightarrow \dots \rightarrow T_L$ , the transforming parameters of this path are defined as

$$\alpha_i = \prod_i \frac{N_{i+1}}{M_i} \quad (i=1, 2, \dots, L),$$

where  $N_{i+1}$  is data input of transition  $T_{i+1}$ , and  $M_i$  is the output data of transition  $T_i$ .

From the above example analysis, the following theorem is easy to be understood.

**Theorem 3.1:** The BES has non-trivial solution if and only if that for any two connected nodes on the model, all the paths connecting the two nodes have the same transition parameter.

**Example 3.1:** For the computation model in Figure 3.3, suppose its data input/output structure has the following two situations, and the running speed of every transition is  $(A,B,C,D)=(1,1,1,1)$ :

A: (1)->(2,1); B: (1,1)-> (1);  
C: (1)-> (1); D: (1)-> (1)  
A: (1)->(2,1); B: (1,1)-> (1);  
C: (1)-> (2); D: (1)-> (1)

For situation (1), the balance equation system is as following:

$$\begin{cases} 2V_A = V_C \\ V_A = V_D \\ V_C = V_B \\ V_D = V_B \end{cases}$$

Obviously, it has no solution for these equations. The transforming parameters of path A-C-B are

$\frac{1}{2} * \frac{1}{1}$ , and the transforming parameters of path A-

D-B are  $\frac{1}{1} * \frac{1}{1}$ . These parameters are not the same.

For situation (2), the balance equation system is as following:

$$\begin{cases} 2V_A = V_C \\ V_A = V_D \\ V_C = V_B \\ 2V_D = V_B \end{cases}$$

Then, we can solve the solution vector:  $(V_A, V_B, V_C, V_D) = \lambda(0.5, 1, 1, 0.5)$ , where  $1 \geq \lambda \geq 0$ . Similarly, we can get the transforming parameter of

path A-C-B as  $\frac{1}{2} * \frac{1}{1}$ , and  $\frac{1}{1} * \frac{1}{2}$  is the transforming

parameter of path A-D-B. Their parameters are the same.

Theorem 3.1 gives the theoretical statement on the feasibility of a computational model, which is the very fundamental rule for designing a computational model. In fact, a model with trivial solution is equivalent to that the model can not run safely in a

long time. The above analysis on the transition parameter also tells the following result.

**Lemma:** For any two nodes A and B in the model, if they are connected and the BES has non-trivial solution  $V$ , then the value  $V_A/V_B$  is a constant which is independent of different solutions of the system.

**Theorem 3.2:** If every two nodes on the model is inter-connected, then the solution for BES is unique in the sense that for every two solutions there is a constant  $C \geq 0$ , such that  $\hat{V}_1 = CV_2$ .

Based on theorem 3.2, the vector  $\hat{v} = v/\|v\|$  is denoted as the unit solution of the BES.

**Proposition:** If the model is connected, it holds  $V_i > 0$  for all  $i=1, 2, n$ , where  $V$  is a non-trivial solution for BES.

### 3.3 Algorithm for Solving BES

**Algorithm for solving BES**

**INPUT:**  
the transition number is  $n$  of this computational model;  
data input/output structure  $T_i: (N_i) \rightarrow (M_i)$ ;  
the running speed of every transition  $T_i$  is  $U_i$ ;  
 $N$  is the number of transitions in the computational model;

**OUTPUT:**  
effective running speed  $V_i$  of every transition  $T_i$

**Begin**  
**for**  $i=1,2,\dots,N$   
    initialize the value by setting  $V_i = U_i$   
**endfor**  
**while** ( there is a  $V_i$  changed ) **do**  
    **for**  $i = 1, 2, \dots, N$   
        **for**  $j = 1, 2, \dots, N$   
            **if** ((Connection( $T_i, T_j$ ) and ( $V_j M_{ij} < V_i N_{ij}$ ))  
                **then**  $\Delta V_i = (V_i N_{ij} - V_j M_{ij}) / N_{ij}$   
                     $V_i = V_i - \Delta V_i$  // such that  $V_j M_{ij} = V_i N_{ij}$   
                **endif**  
        **for**  $k = 1, 2, \dots, N$   
            **if** ((Connection( $T_i, T_j$ ) and ( $V_i M_{ik} > V_k N_{ik}$ ))  
                **then**  $\Delta V_i = (V_i M_{ik} - V_k N_{ik}) / M_{ik}$   
                     $V_i = V_i - \Delta V_i$  // such that  $V_i M_{ik} = V_k N_{ik}$   
                **endif**  
        **endfor**  
    **endfor**  
**endwhile**  
Return  $V$   
**End**

Figure 3.4: BES Solving Algorithm.

The algorithm in Figure 3.4 is an iterative one which will solve the BES. In the algorithm,  $V_i$  indicates the effective speed of the  $i$ -th node in the model. For a given node,  $V_j$  is the speed of its  $j$ -th input node, and  $V_k$  is the speed of its  $k$ -th output node.

In this algorithm, if a  $V_i$  converges to zero, which indicates all of  $V_i$  converges to zero, then the model does not have a non-trivial solution. During the computation process in the above algorithm,  $VM$  and  $VN$  is monotone descending, therefore,  $\Delta V_i \geq 0$  and  $\Delta V_i \geq 0$ . If a  $V_i$  keeps unchanged during the whole process of the algorithm, then this  $V_i$  is a bottleneck node for the model.

### 3.4 Bottleneck Transition Analysis

In the equation (3.1), each variable  $V_i$  has an upper bound  $U_i$ . In the language of computational model,  $V_i$  is the effective speed of the transition  $T_i$  and  $U_i$  is the running speed of it. Each solution  $V$  for the BES represents a group of steady solution for the performance of transitions on the model. If the effective speed of a transition is the first one to approach to its upper bound while others remain below their upper bounds, this transition is called the bottleneck of the system. It is possible that a system has more than one bottleneck.

Using theorem 3.2, the problem of finding the maximum system performance becomes the problem of solving  $\lambda$  from following equation,

$$V_{\max} = \max_{\lambda}(\lambda \hat{V}) \quad (3.11)$$

, where  $\hat{V}$  is used to denote the solution vector of a BES is a solution of a BES, and  $\lambda \geq 0$ , so that  $0 \leq \lambda V_i \leq U_i$ .

Suppose  $\lambda_0$  is a constant which can be applied to (3.11). From above analysis, we can get that the bottleneck transitions for the computational model are those transitions  $T_i$  such that

$$\lambda_0 \hat{V}_i = U_i \quad (3.12)$$

When the model is running at its maximum performance, only the transitions satisfying (3.12) can reach its maximum performance such that  $V_i^R = V_i^E$ . For other transitions, it holds  $V_i^R > V_i^E$ . The power usage rate of transition  $T_i$

$$\rho_i = \frac{\lambda_0 \hat{V}_i}{U_i} \quad (3.13)$$

From (3.13), it can be seen that only the bottleneck nodes can reach their maximum power usage. For example 3.1, if its data input/output structure is situation (2), its bottlenecks have two transitions, B and C, because their utilizations are the highest.

At the run time of the model, if all of the bottleneck nodes are the input and output nodes, it indicates there is potential to improve the performance of system by improving the IO performance only. Otherwise, improving the system performance will have to depend on better system node performance or better system structure.

## 4 APPLICATIONS OF BOTTLENECK DETECTION

In real situation, there are a lot of business problems caused by bottlenecks. However, these problems in a real business process usually fall into some patterns. In this section, we focus on two major patterns: Input Data Flush pattern and Block Caused by a Paused Transition pattern, and analyze them based on the solution given above.

### 4.1 Analyzing Input Data Flush

In real business world, the volume of data input to a business process varies from time to time and usually large numbers of data suddenly flush into for processing. We classify this kind of pervasive running status as a pattern called Input Data Flush pattern. For a business process in this pattern, it really suffers from sudden flush of input data which results in buffer jam, transition breakdown, and so on. So it's necessary to analyze and predict the impact of flush input data and suggest some approaches to counteract the negative effect. The theoretical method proposed in Section 3 can be leveraged to fulfill this demand.

Given a business process represented in computational model as shown in Figure 4.1, and suppose we have detect the bottleneck transition  $T_i$  by solving the Balance Equation System of such process, we can predict the operation behavior of each transitions as below:

If transition  $T_j$  is a subsequent transition of  $T_i$  and there is a path from  $T_i$  to  $T_j$ , then  $T_j$  will be light-load transition sooner or later since low performance  $T_i$  will limit the input data volume of  $T_j$ .

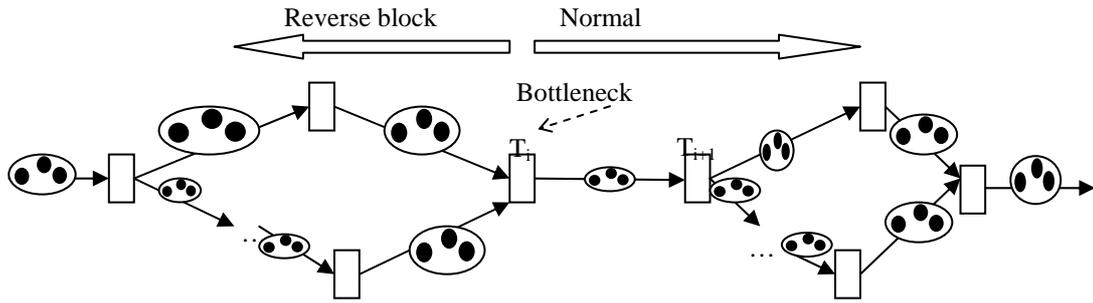


Figure 4.1 Input Data Flush Pattern

If transition  $T_j$  is a previous transition of  $T_i$  and there is a path from  $T_j$  to  $T_i$ , then firstly  $T_j$  will be heavy-load transition due to large input volume but gradually slow down because of buffer jams on the path from  $T_j$  to  $T_i$ .

There is another critical issue to be analyzed for this pattern. If one trying to accelerate the bottleneck transition  $T_i$  for better process performance, it seems wonderful but in fact dangerous if no deep analysis and extra step are taken. Given there is a high performance transition  $T_{i+1}$  that is the direct subsequent transition of  $T_i$ , the performance of  $T_{i+1}$  is prevented from full release by bottleneck transition  $T_i$  before  $T_i$ 's acceleration, then the low performance transitions after  $T_{i+1}$  can run safely without worrying about buffer jam. However, once we accelerate  $T_i$ , then  $T_{i+1}$  can fully release its performance and buffer jam might produce between  $T_{i+1}$  and its low performance subsequent transitions, which will result in new round system block. By further analysis based on solving the Balance Equation System, we can decide the upper speed limit for  $T_{i+1}$  after  $T_i$ 's acceleration so as to avoid new round block and specially configure  $T_{i+1}$  to meet its upper speed limit.

### 4.2 Analyzing Block Caused by a Paused Transition

When a transition of a process model is paused because of exception, the process will be blocked. However, the block will not immediately be propagated to the whole process because the size of data buffer is not zero. The block propagation will spend some time. In a long time, the final result is that all the system is paused. For this situation, we called it a Block Caused by a Paused Transition pattern which is specified with Figure 4.2. For this pattern, it is difficult to detect the problem from the superficial phenomenon. And it is not easy to simulate all the situations because there are a lot of

transitions and large number of paths. We must provide theoretical analysis methods to handle this pattern.

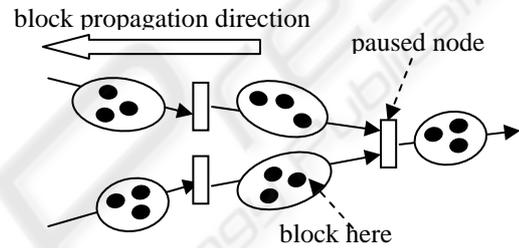


Figure 4.2: Block Caused by a Paused Node

On the one hand, it will be useful to predicate the situation, so that we can know how to control it. On the other hand, some methods should be provided to repair the paused system once this situation happens. No matter we are preventing or repairing the problem, computing the running speed of the transitions is the foundation. Based on the running speed, we will provide a method about how to compute block propagation time.

Suppose a transition  $T_{i+1}$  is paused, the block will be propagated backward along a path. If a connection from transition  $T_i$  to  $T_{i+1}$  exists, and the size of data buffer between them is  $C_i$ , and the effective speed of transition  $T_i$  is  $\lambda \hat{V}_i$ , and the data input/output structure of transition  $T_i$  is  $(N_i) \rightarrow (M_i)$ , then the

**block propagation time** from  $T_i$  to  $T_{i+1}$  is  $\frac{C_i}{\lambda \hat{V}_i M_i}$ .

Suppose  $T_1 \rightarrow T_2 \rightarrow \dots \rightarrow T_{n+1}$  is a path, and the  $T_{n+1}$  is the paused node, then the block propagation time  $BPT(T_{n+1})$  from  $T_{n+1}$  to  $T_1$  can be computed using the following formula:

$$BPT(T_n) = \sum_{i=1}^n \frac{C_i}{\lambda \hat{V}_i M_i} = \frac{1}{\lambda} \sum_{i=1}^n \frac{C_i}{\hat{V}_i M_i}$$

According to the above formula, we can have the following conclusion:

1) If the value of  $\lambda$  is smaller, which indicates the system is running at low performance level, a process will spend longer time to be fully blocked once a node is paused;

2) If the data buffer is the larger, the block propagation time will be longer.

## 5 CONCLUSION

Detecting and improving business process bottlenecks are critical for enterprise performance improvement. This paper aims at a quantitative analysis method for effectively and efficiently detecting business process bottlenecks. A computational model is proposed to abstract key elements in business process model, including transition, data, buffer and connections. Based on this computational model, a Balance Equation System is put forward to detect bottlenecks and the solutions about how to improve the bottlenecks are given. Furthermore, this approach was used to some concrete applications. We classified the common applications into some patterns, including large number input data impact, block caused by a paused node. Through using the theoretical analysis method, these application patterns can be solved well. Compared with traditional performance analysis method, the theoretical analysis method in this paper is easier and much more practical.

In a real business process, the performance of a transition (the running speed of the transition) may not be a constant, but depends on the resource allocated to it. The more resource is allocated, the better transition node performs we will have. From the resource allocation point of view, a critical resource can be allocated to multiple transitions. Many applications run on the same server is the typical example for this, in which applications are transitions and the CPU power of the server is the resource. How to best allocate the limited resource to make the system achieve the highest performance is an important topic for further study. In another situation, the system performance may not be able to meet the business requirements even though each individual transition has reached its maximum performance. This raises the problem of system structure optimization. The challenge here is how to change the system structure (the connection structure of the model) to meet a specific high requirements on performance. Changing the bottleneck node into a parallel sub-structure can be a example for this kind of structure optimization. The analytical power of computational model will

provide a solid foundation for dealing with this kind of problems.

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