

A TREE BASED ALGEBRA FRAMEWORK FOR XML DATA SYSTEMS

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Abstract: This paper introduces a framework in algebra for processing XML data. We develop a simple algebra, called TA (Tree Algebra), for processing storing and manipulating XML data, modelled as trees. We present assumptions of the framework, describe the input and the output of the algebraic operators, and define the syntax of these operators and their semantics in terms of algorithms. Furthermore we define the relational operators and their semantics in terms of algorithms. Examples show that this framework is flexible to capture queries expressed in the domain specific XML query language. As can be seen the input and output of our algebra is a tree that is the input and output are XML document and the XML documents are defined as trees. We also present algorithms for many of the algebra operators; these algorithms show how the algebra operators such as join, union, complement, project, select, expose and vertex work on nodes of the XML tree or element and attributes of an XML document. Detailed examples are given.

1 INTRODUCTION

In this paper we develop an algebraic model for data management, which unifies in a single framework data presentation, data communications and data processing based on an XML Schema (Zisman 2000). Starting with the XML Schema of the data we develop domain specific XML algebra suitable for data processing of the specific data. The algebraic model we are developing, based on the existing standard (W3C XML Query Algebra), is a domain specific model of collaborative information processing over the Web. Even though it is domain specific, however, it will be generic since it will model an integrated architecture for distributed information processing. The input and output of our algebra are XML documents defined as a tree. We also present examples and algorithms for most algebra operator; these examples and algorithms show how the relational algebra and its operators work. Furthermore we apply *join*, *union*, *complement*, *project*, *select*, *expose* and *vertex* operations to nodes to form the XML tree as elements and attributes of XML (Bourret 2004).

The rest of the paper is structured as follows: Section 2 presents the related work for XML algebra. This leads up to our algebra description in Section 3. Section 4 presents our XML data model. Section 5 presents the algebra relational. The

examples for algorithms and input and output of the algebra operators are defined in Section 6. Section 7 presents a discussion of the algebra.

2 RELATED WORK

Table 1 provides a summary and review of algebraic methods discussed in this paper. It can be seen that all have some significant drawbacks, in particular in complexity and generality. In the next section we attempt to address these problems by developing a tree-based algebra, which is more versatile in its application. The algebraic models we have reviewed are IBM (Beech, M. & Rys 1999), Lore (McHugh *et al* 1997), YATL (Christophides, Cluet & Simeon 2000), Niagara algebra (Galanis *et al* 2001) and AT&T a (W3C February 2001).

Table 1: Comparison of Different Algebraic Models

Algebraic Model	IBM	YATL	Niagara	Lore	AT&T
Data Model	Standalone XML algebra	Logical Data Model, Graph Based	Standalone XML algebra	Integrated DBMS	Standalone XML algebra
Approach	Represents the collection of vertices	Integrates data from different sources	Operates on a set of bags of vertices	Cost-based query optimization	Based on the iteration operation, NRA
Distinctive Features	- Not system specific, XQuery support. - Provides an algebra operator that operates as graph-based data model.	- Ability to integrate data from different sources; can be used efficiently in querying distributed data. - YATL has Tree operations for transforming relation structure to Tree structure (Codd 1972)	- Simple and powerful algebraic expressions, optimised rules	- Optimised from cost-based perspective, dynamic schema structure. - Each query is transformed into a logical query plan using logical operators such as <i>Select, Project, Discover...etc</i>	- Built in types for detecting errors at query compile time. - SQL& OQL and NRA support XQuery. - Based on the iteration operation
Drawbacks	- Deficiency of Optimisation Rules, - Complex query structure, data type should be known at query compile time	- The integration is exclusively based on YATL data model and type system, and does not provide a well-defined list of the operators and explicit optimisations.	- The algebraic framework assumes that at the time of writing the query, the type is known for each vertex (attribute, element) [Fenkhauser, S. 2001] - Implementing a query using IBM model will lead to a complex structure.	- The physical operators are designed specifically for its own data model	- Query plan based on this algebra, once generated, is difficult to optimize
Special Operators	Reshaping Operators	Bind, Tree operators	-	Vindex, Lindex, once	-
W3C	Proposed	-	Proposed	-	Proposed

3 DESCRIPTION OF OUR ALGEBRA

Our XML Algebra is a tree structure that consists of algebra operators (Greenwald *et al* 2003). Each node in the tree has only one parent node, but a parent node can have multiple children nodes. The XML Tree is interpreted top-down, so the root of the tree at the top of the XML Tree is where the final XML document output is produced. The leaves of the tree correspond to the different data sources accessed, which in our system prototype are assumed to be object-relational (Galanis *et. al* 2001, Scholl 1986). There are two types of operators in our algebra: firstly, the algebraic operators are *join, union, complement, project, select, expose and vertex*.

Every operator has a unique output of a tree, which makes it distinct from other algebra operators. Secondly, the relational operators are *universal, subsuming, equivalence and similarity*.

In the following sections we well explain in more detail all algebra operators and algebra relational. The algebra operators are based on W3C (standard algebra) and comprise XML-specific and special operations.

4 OUR XML DATA MODEL

Figure 1 shows an XML tree, in which there is a topmost, unique *element, collection*, known as the root of the XML document (ancestor) (Comon *et al* 1997). All elements are enclosed within X&topmost element, collection. Object1 and object3 sub-elements reside within the root node. This nesting of sub-elements can go to an arbitrary level. Figure 1 shows the tree data model.

Elements and attributes correspond to nodes in the XML tree. Directed, named edges connect nodes, with the tag of the corresponding element or attribute name acting as the name of an edge. For each node of the tree, except the root node, there is a backward edge leading to the parent node. Note that a parent node appears only between those connected with child nodes or leaf nodes. A path consists of the sequence of node names that one needs to follow in order to arrive at a node from the root node.

4.1 Concepts in the tree model

Root (ancestor or parent): the top node of the tree is identified as the root node.

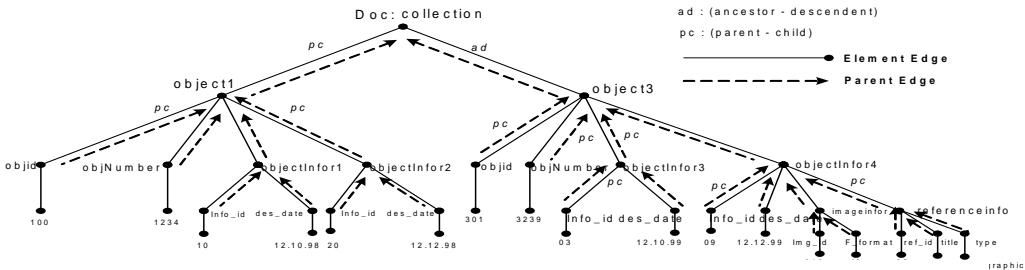


Figure 1: Tree data model

- Node (parent or child): an edge is a link from a parent node to a successor node, called child node.
- Leaf (child): child nodes, atomic values
- Path: a path from node V₁ to V_n sequence of nodes, where 1<=n<=20
- Descendants: represent all nodes that are children of current node or children of children of current node and so on.
- Ancestor of nodes V are parent, grandparent, etc, that is all nodes found on the path from node V to the root node. The ancestor represents all nodes that are parent of current node or parent of parent of current node and so on.

4.2 XML Document as Tree Structure

- XML document → Tree
- Element → Root node, parent, child node
- Leaf → child node, atomic values
- Attribute → function, values.

5 ALGEBRA RELATIONAL

In this section we introduce four types of relational algebra operators: *Universal*, *Similarity*, *Equivalence* and *Subsumption*. In the following sections we will explain these relations in more details.

5.1 Universal Element Relation

This relation is unary (\cup) and contains all information. In a case study for a museum objects

information system the universal element would include the collections, their objects, object information and exhibition and institutions as described in (ICOM 1995, CIDOC 2002) and presented by Ali Elbekai in his thesis.

5.2 Similarity Relation

Similarity relation (\sim) means that the major structures of the two trees are similar, for example the Doc4 tree is similar to the Doc3 tree, as in Figure 2. Also, the similarity relation is a binary relation. In general we can see the relation between any two XML tree is similarity if the following holds: the root node in Doc_n tree and the root node in Doc_m tree are similar, the parent node in Doc_n tree and the parent node in Doc_m tree are similar and the child nodes in the Doc_n tree and the child node in Doc_m tree are similar. In addition if we find that any parent node or child node in Doc_n tree or Doc_m tree is an unnamed node then this node matches the corresponding named node at the same level and position as are depicted in Figure 2.

5.3 Equivalence Relation

Equivalence relation (\approx) means that two trees are indistinguishable as in Figure 3 where the nodes in the Doc3 tree are equivalent to those in the Doc4 tree then we can identify that the Doc3 tree is equivalent to the Doc4 tree. The equivalence relation is a binary relation. In general we can explore that the relation is equivalence between any two XML trees if the following holds: the root node Doc_n tree and the root node Doc_m tree are equivalent, the

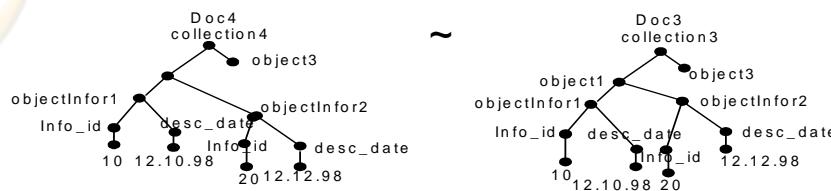


Figure 2: Similarity relational operator



Figure 3: An Equivalence relational operator

parent node Doc_n tree with parent node Doc_m tree are equivalent and the chid nodes Doc_n tree and child nodes Doc_m tree are equivalent. In addition if

the parent node or child node in both Doc trees is unnamed, then the unnamed node matches the corresponding node in the same level and position.

5.4 Subsumption Relation

Subsumption (\subset) means that one tree is a subset of the other, for example the Doc3 tree is a subset of the Doc1 tree in a binary relationship. If the nodes present in Doc3 tree are a subset of the nodes present in Doc1 tree then we can see the Doc3 tree is a subsumption of the Doc1 tree. In general we can conclude that the Doc_n tree is subsumption or equal of Doc_m tree if the following holds: the root node Doc_n tree exists in Doc_m tree, the parent node Doc_n tree exists in the Doc_m tree and the child node Doc_n tree exists in the Doc_m tree. In other words the Doc_n tree is part of Doc_m tree.

6 ALGEBRA OPERATORS

All operators in tree algebra take one tree or more as input and produce one tree of data as output (Roth, Korth & Silberschatz 1988).

6.1 Join Operator

A join (\oplus) is a binary operator, which takes two trees as input, and combines them into one tree as output. This combination is made whenever a certain expression holds true, that is the two tree collections are joined on a predicate. The predicate can be as generic as the conditions accepted by the selection operator. The query operation selects documents that meet the stated criteria from a collection of documents. It may also extract components from selected documents and construct new documents from these components. Also the join operator has the property of being commutative. More precisely $\text{Doc1 tree} \oplus \text{Doc2 tree} = \text{Doc2 tree} \oplus \text{Doc1 tree}$. Furthermore, it is associative which means $(\text{Doc1 tree} \oplus \text{Doc2 tree}) \oplus \text{Doc3 tree} = \text{Doc1 tree} \oplus (\text{Doc2 tree} \oplus \text{Doc3 tree})$. Figure 5 shows how to join any two trees in general. The syntax of the join operator is $\text{DOC}_n \text{ Tree} \bowtie [\text{condition}] \text{ DOC}_m \text{ Tree}$ where $(1 \leq n \leq 10)$ and $(1 \leq m \leq 10)$.

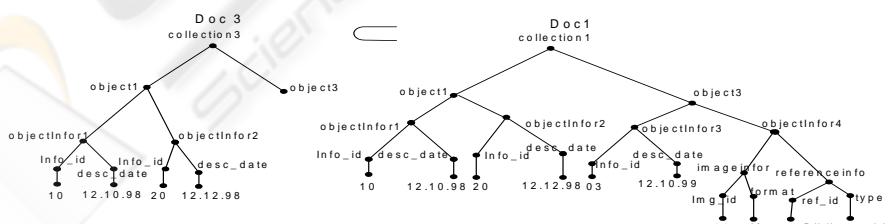


Figure 4: Subsumption relational operator

```

// Input two XML document or two DOC tree (DOCn, DOCm)
// Output DOCnm Tree = (DOCn Tree ⊕ DOCm Tree)
1. start from root node DOCn tree and root node DOCm tree
2. if root node has parent\child node
   2.1 perform depth-first algorithm
   2.2 if the parent node DOCn tree has child node and parent node DOCm tree has child node
      2.2.1 While child node DOCn tree agrees with child node DOCm tree repeat:
         2.2.1.1 join such child node (concatenation without repetition)
         2.2.1.2 set the output in DOCnm tree
         2.2.1.3 if child node in DOCn tree has leaf node agree with leaf node DOCm tree
            2.2.1.3.1 join such leaf node (concatenation without repetition)
            2.2.1.3.2 set joined leaf node to the output DOCnm tree
         2.2.1.4 join such leaf node in DOCn tree to leaf node DOCm tree
   2.3 set parent node to DOCnm tree
3. set no parent/child node and root node into DOCnm tree and terminate
4. end/terminate

```

Figure 5: An algorithm for Joining of two XML documents or two Doc trees

```

// Input two XML document or two DOC tree (DOCn Tree, DOCm Tree)
// Output DOCnm Tree = (DOCn Tree ∪ DOCm Tree)
1. start with root node DOCn Tree and root node DOCm Tree
2. set root node DOCn Tree or root node DOCm Tree in new DOCnm Tree
3. if root node has parent\child node in either DOC Tree
   3.1 perform depth-first algorithm
   3.2 if the parent node DOCn Tree = parent node DOCm Tree
      3.2.1 set parent node DOCn Tree or DOCm Tree in new DOCnm Tree
      3.2.2 while parent node DOCn Tree has child node or parent node
         DOCm Tree has child node
         3.2.2.1 set child node to new DOCnm Tree
         3.2.2.2 eliminate duplicate child node
   3.2.3 repeat
3.3 set null to new DOCnm and terminate
4. set null and terminate

```

Figure 6: An algorithm for Union two XML documents or two Doc Trees

6.2 Union Operator

The purpose of the Union operator (\cup) is to union two XML trees. It is a binary operator with two XML trees as input and one XML tree as output. This is depicted in Figure 6. Furthermore, the union operator is commutative. More precisely, $DOC1 \cup DOC2 \text{ tree} = DOC2 \text{ tree} \cup DOC1 \text{ tree}$. Also it is an associative operator, meaning that $(DOC1 \text{ tree} \cup DOC2 \text{ tree}) \cup DOC3 \text{ tree} = DOC1 \text{ tree} \cup (DOC2 \text{ tree} \cup DOC3 \text{ tree})$. Furthermore the output of the union operators is new XML tree data containing all the elements, root node, parent nodes and child nodes in the two input Doc trees data model without the duplication of any elements such as root nodes, parent nodes and child nodes. In general the syntax of the union is $DOCn \text{ Tree} \cup DOCm \text{ Tree}$, with limits ($1 \leq n \leq 10$) and ($1 \leq m \leq 10$). The union is disjoint: duplicates are purged.

6.3 Complement Operator

The complement operator (\perp), as a binary operator, operates on two XML trees as input, and produces one XML document or one XML tree as an output. In other words the XML document is a tree; the result of the complement of the two DOC trees is a new DOC tree containing all the nodes present in the first input DOC tree but not in the second DOC tree. That is the output of the complement operator is a new XML tree containing all element nodes (root node, parent nodes, children nodes, function, values) existing in the input DOCn tree data model and not existing in the DOCm tree. Figure 7 show how we can complement the two DOC trees or two XML document in general.

```

// Input two XML document or two DOC tree (DOCn Tree, DOCm Tree)
// Output DOCnm Tree = (DOCn Tree - DOCm Tree)
1. start from root node DOCn
2. If root node DOCn Tree and root node DOCm Tree has parent/child node
   2.1 Perform depth-first algorithm
   2.2 if DOCn Tree has parent node not existing in DOCm Tree
      2.2.1 set parent node DOCn Tree to the new DOCnm Tree
   2.2.2 while parent node DOCn Tree has child node not existing in DOCm Tree
      2.2.2.1 set child node DOCn Tree to DOCnm Tree
      2.2.2.2 if child node DOCn Tree has leaf node not existing in DOCm Tree
         2.2.2.2.1 set leaf node DOCn Tree to DOCnm Tree
      2.2.2.3 set null to DOCnm Tree
   2.2.3 repeat
2.3 set null to DOCnm Tree
3. set root node to DOCnm Tree and terminate
4. end/terminate

```

Figure 7: An algorithm for Complement two XML documents or two Doc Trees

```

// Input one DOC tree and the Output one DOC tree
1. start from the root node of DOC Tree
2. if root node has parent/child node
   2.1 Perform depth-first algorithm
   2.2 if projected node/selected node is equal to current node
      2.2.1 set the projected node as output to new DOC Tree
   2.3 node projected/selected not exist and terminate
3 end/terminate

```

Figure 8: An algorithm for Projection operator

The purpose of the complement operator is to compute the difference between the two input trees: DOCn Tree - DOCm Tree with limits ($1 \leq n \leq 10$) and ($1 \leq m \leq 10$). The complement operator is not commutative, which means $\text{DOC1 tree} - \text{DOC3 tree} \neq \text{DOC3 tree} - \text{DOC1 tree}$. Also it is not an associative operator.

6.4 Projection Operator

The input for the projection operator (π) is one XML tree, as it is unary. In general Figure 8 shows how the projection operator works on nodes of the DOC tree or elements of the XML document.

As can be seen from Figure 9 we take object3 as a parameter and the searching is performed on the root

node collection1 in Doc1 tree. The output of the projection operator is a new XML document or new DOC1p tree. For trees, projection may be regarded as eliminating nodes other than those specified. In the substructure resulting from node elimination, we would expect the (partial)

hierarchical relationship between surviving nodes that existed in the input collection to be preserved. Projection in tree algebra takes one collection $\langle C \rangle$ (tree) as input and $\langle P \rangle$ as parameters. Projection starts the search at the root node and follows the path parent node successor, the child node, until it finds the node projected. The syntax of the projection operator is defined as follows: $\pi \langle P \rangle \langle C \rangle$ where $\langle P \rangle$ is parameter and $\langle C \rangle$ is tree.

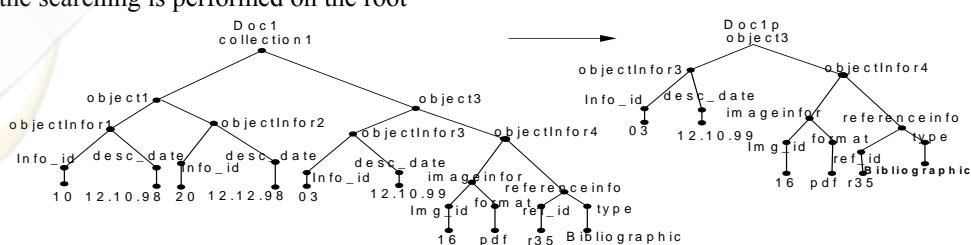


Figure 9: An example for Projection operator

```

// Input one DOC tree or one XML document
// Output one DOC tree or one XML document
1. start with entry point, it is the root node
2. perform depth-first algorithm
   2.1 if parameter is equal the specific node needed to expose
       2.1.1 return the specific node
       2.1.2 set specific node in the new tree
   2.2 exposed element not exist and terminate
3. end/terminate

```

Figure 10: An algorithm for exposing specific node of Doc tree

6.5 Select Operator

The purpose of a select (σ) operation is to filter out tuples in the XML algebra satisfying an expression given as a predicate. The select operator is a unary operator to take one XML document or one tree as input and produce one XML document or one tree as output. The selection operator allows us to determine a subset over a collection of documents. It applies a given condition to each member of the collection of nodes and returns a result node or collection of nodes consisting of those members for which the condition evaluates true. Selection in tree algebra takes tree as input, and a $\langle P \rangle$ as parameter, and returns an output tree $\langle C \rangle$. Formally, the syntax of select is defined as follows: $\sigma \langle P \rangle \langle C \rangle$. The result of selection is a tree. We can take the parameter and start our selection by depth-first from the root node of the XML tree and successor, the parent nodes and successor child nodes until the selection condition is satisfied.

6.6 Expose/Vertex Operators

The Expose operator (ε) has one XML document or one Doc tree as input, as it is a unary operator and produces one Doc tree as the output. The purpose of the *Expose* operator is to retrieve specific elements of the XML document or specific nodes of the Doc tree. The Expose operator accepts as its parameters a list of path expression to be exposed from the document on which it operates, with the path expression in entry-point notation. Figure 10 shows how we can expose specific nodes of the Doc tree in general.

The Expose operator accepts as its parameter the element (parent, child) exposed from the tree data model on which it operates, with the path in entry point notation, and follows the path root node, parent node and successor child nodes until the element required to be exposed is found. The output of an Expose operator imposes a new ordering, the same as the order of its arguments. Once the nodes denoted by the path are reached, a new element content is constructed. In general the

new syntax of the expose operation is:

Expose[edge1<element>]([element: expression])

The Vertex operator (v) has one XML document or Doc tree as input, as it is a unary operator, and creates the actual XML vertex that will encompass everything created by the Expose operator. It arranges the element content according to the order indicated by its input. It creates the XML to which nodes can be connected, as well as the named edges that lead to the newly defined tree.

7 DISCUSSION

The results from our literature review gave us a choice of an existing solution for applications in our context of either (1) Lore algebra, adopting an existing standard for building an original solution, (2) the standard XML algebra of W3C, or (3) developing a more targeted solution, that is a domain specific algebra, to handle our requirements. All the considerations, given earlier in the review, have already been taken into account by the team working on the standard XML algebra (W3C February 2001) and XQuery (W3C 2004). In fact, the standard algebra is largely based on the AT&T model with some additional features and revisions in the spirit of Niagara. So, the standard is a satisfactory starting point for us in our efforts to develop domain specific algebra and was therefore adopted as the basis for our way forward. However, it does not make sense to implement the full algebra defined by the standard for one single specific task or even for a class of similar tasks. Because of this we have developed a formal data model as a restricted version of the universal algebra suitable for a representative class of problems. Of course, such an approach may lead to a non-universal model, but at the same time, it is feasible and does produce a more effective solution for particular classes of problems. On this basis, we started with the XML Schema (W3C September 2001) of the data to develop domain specific XML algebra more suitable for data processing of the specific data and then we used it for implementing the main offline components of the system. There are two types of operators in our algebra: firstly, the algebraic operators are *join*, *union*, *complement*,

project, select, expose and vertex. Every operator outputs a tree, which makes it distinct from other algebra operators. Secondly, the relational operators are *universal, subsuming, equivalence, and similarity*. This means our work is based on the tree based algebra framework for XML data systems. Also, our algebra has a sound data structure and a simple representation of the data. A contribution of this work is that it introduces an algebra that operates on a new data model, because our algebra employs XML trees as data sources and targets. Our algebra framework can be used in integrated architectures for distributed information processing and its components will be XML schema driven. Furthermore, as a test framework for our integrated approach we will prototype a system for the exchange of information between several independent museums for organising virtual exhibitions over the Web. Also, we plan to extend the algebra to support some of the more advanced features of the XML query language.

REFERENCES

- Beech, D, Malhotra A, & Rys, M, (eds.) A formal data model and algebra for XML, *Comm W3C* (1999).
- Bourret, Ronald. *XML and Database* (2004) at: <http://www.rpbourret.com/xml/XMLAndDatabases.htm>
- Christophides, V, Cluet, S & Simeon, J, On wrapping, query languages and efficient XML integration, *ACM SIGMOD Conf Management Data*, Dallas 141-152, May (2000).
- CIDOC Group (July 2002) <http://www.willpowerinfo.myby.co.uk/cidoc/cidoc0.htm>
- Codd, E F, Relational Completeness of Data Base, *Data Base Systems*, Prentice Hall **6** 65-98. (1972).
- Comon, H, Dauchet, M, Gilleron, R, Jacquemard, F, Lugiez, D, Tison, S, & Tommasi, S, *Tree Automata Techniques and Applications*, at: <http://www.grappa.lille3.fr/tata>. (1997)
- Fenkhauser, M, Simeon, J, & Woder, P, An algebra for XML Query, *In Proc. FST TCS*, New Delhi, December (2000).
- Fernandez, M, Simeon, J, & Wadler, P, A semi-monad for semi-structured data, *Int Conf Database Theory* 263-300 (2001).
- Galanis, L, Viglas, E, DeWitt, D J, Naughton, J F, & Maier, D, *Following the paths of XML Data: An Algebraic Framework for XML Query Evaluation*, Tech Rep Univ Wisconsin (2001).
- Greenwald, M B, Moore, J T, Pierce, B C, Schmitt, A: *Language for Bi-Directional Tree Transformations*.
- Tech Rep MS-CIS-03-08, Dept Comp Inf Sci, Univ Pennsylvania. (Aug 2003).
- ICOM, *International Guidelines for Museum Object Information (IGMO)*: CIDOC Information Categories (October (1995) <http://www.cidoc.icom.org/guide>
- McHugh, J, Abiteboul, S, Goldman, R, Quass, D, & Widom, J, Lore: A Database Management System for Semi-structured Data. *SIGMOD* **3**(26) 54-66 (1997).
- McHugh, J, & Widom, J, *Query optimization for Semi-structured data*, Tech Rep, Stanford Univ Database Group, August (1998). <http://www-db.stanford.edu/pub/papers/qo.ps>.
- Roth, M A, Korth, H F, & Silberschatz, A, Extended algebra and calculus for nested relational databases, *ACM TODS* **13** 389-417 (1988).
- Scholl, M H, Theoretical foundations of algebraic optimization utilization unnormalized relation, in: *ICDT'86, LNCS* **234** 409-420. (1986).
- W3C, *the XML Query Algebra*, Working Draft, <http://www.w3.org/TR/2001/WD-query-algebra-20010215>. February (2001).
- W3C, *XML Schema: Formal Description*, Working Draft, September (2001).
- W3C, *XQuery 1.0: An XML Query Language*, Working Draft (23 July 2004) <http://www.w3.org/TR/2004/WD-xquery-20040723/>
- Zhang, X. and Rundensteiner, E A, *XML Algebra for the Rainbow System*, Tech Rep WPI-CS-TR-02-24. Worcester Polytechnic Inst, July (2002).
- Zisman, A, An Overview of XML, *Comp Control Eng J* **11**(4) (2000).