COOPERATIVE MULTI-ROBOT LOCALIZATION: USING COMMUNICATION TO REDUCE LOCALIZATION ERROR

Valguima Odakura, Anna Helena Reali Costa
Laboratório de Técnicas Inteligentes - LTI
Departamento de Engenharia de Computacão e Sistemas Digitais - PCS
Escola Politécnica da Universidade de São Paulo
Av. Luciano Gualberto, trav 3, n. 158
Cidade Universitária, São Paulo-SP, Brasil

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Abstract: This paper presents a statistical algorithm for cooperative multi-robot localization based on a propagating detection model. The problem of multi-robot localization consists of localizing each robot in a group within the same environment, when robots share information to improve localization accuracy. Our approach is based on a well-known probabilistic localization approach, the Markov localization, that was originally designed to a single robot. A detection model can be incorporated in order to accommodate multi-robot cooperation in Markov localization. In this model, two robots exchange their pose beliefs whenever one robot detects another. We propose a novel detection model in that all robots in the group can benefit from a meeting of two robots through detection propagation. The technique has been implemented and tested in simulated environments. Experiments illustrate improvements in localization accuracy when compared with a previous multi-robot localization approach.

1 INTRODUCTION

Localization is one of the main abilities of an autonomous mobile robot. In order to perform their tasks, mobile robots need to know their pose, position (x, y) and heading θ, as accurate as possible. It is a challenging task, once localization is based on sensors readings, which are uncertain and noisy. Because of that, most of the robot localization approaches are based on probabilistic methods. In order to accomplish tasks more quickly and robustly than a single robot, multiple robots can be used (Cao et al., 1997; Parker, 2000), which poses the multi-robot localization problem.

The multi-robot localization problem consists of localizing each robot in a group within the same environment. It can be performed in two manners. Firstly, it can be done individually, where each robot solves its self-localization problem alone, based on its own resources. Secondly, it can be performed cooperatively, which takes advantage of multiple robots to improve positioning accuracy beyond what is possible with a single robot. In the latter case, cooperation is achieved by communication. The key idea of cooperation in multi-robot localization is that each robot can use measurements taken by all robots, in order to better estimates its own pose. In this way, the main difference between single robot and cooperative multiple robots localization is that multi-robot can have more information than single robot.

The information communicated by robots can be guided by robots detections, as in Roumeliotis and Bekey (2002) and Fox et al. (2000). Every time one robot detects another, they communicate and share their pose beliefs. In Fox et al. (2000), detection model performs pose update only for the two meeting robots. We argue that this information can be shared among all robots in the group, improving localization accuracy of all robots. Thus, the goal of this paper is to present an algorithm to multi-robot localization, based on a novel detection model that propagates a detection to all robots within a group.

The cooperative multi-robot localization problem addressed in this paper considers the following assumptions:

- Initial robots' poses are unknown.
- Robots know an environment model.
- Robots are equipped with proprioceptive sensors that measure their self-motion.
- Robots are equipped with exteroceptive sensors that monitor the environment, and detect and identify other robots.
- Robots are equipped with communication devices that allow them to exchange information.

This paper is organized as follows. In Section 2, the single robot localization problem is presented and the

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Markov localization technique is introduced. In Section 3 the localization problem is extended to a group of robots. The proposed detection model to improve multi-robot localization accuracy is described in Section 4. Experiments realized are shown in Section 5. Finally, in Section 6, our conclusions are derived and future works are presented.

2 LOCALIZATION APPROACHES

Mobile robot localization is the problem of estimating a robot pose within an environment based on observations. Observations consist of information about the robot's movement and about the environment. Information provided by sensors are inherently uncertain, so probabilistic techniques are needed to deal with this.

The probabilistic approach uses a probabilistic representation of the robot's pose, that is, robot's pose is modeled by a random variable and the state space of this variable consists of all the poses within the environment. In this context, mobile robot localization can be classified as local or global. In local localization, the probability distribution function used is a unimodal Gaussian. In consequence of this representation, the pose of the robot is assumed to be within a small area and the initial robot's pose has to be known. In global localization, robot's pose is represented by a multi-modal probability distribution, which allows determining robot's pose without knowledge of its initial pose.

Most approaches of local localization use Kalman filter to determine the pose of robots. In the Kalman filter approach, the robot's pose is described by using a Gaussian distribution. The Kalman filter technique has been shown to be accurate for keeping tracking of robot's pose (Leonard and Durrant-Whyte, 1991).

A global localization approach is ML – Markov localization. This localization technique maintains a probability distribution over the space of all poses of a robot in its environment, so it deals with multimodal distributions. Markov localization relies on the Markov assumption, which states that past sensor readings are conditionally independent of future readings, given the true pose of the robot, (Fox et al., 1999).

In ML, $p(\mathbf{x}_t = x)$ denotes the robot's belief that it is at pose x at time t, where $\mathbf{x}_t$ is a random variable representing the robot's pose at time t, and $x = (\mathsf{x}, \mathsf{y}, \theta)$ is the pose of the robot. This belief represents a probability distribution over all the space of $\mathbf{x}_t$.

ML uses two models to localize a robot: a motion model and an observation model. The motion model is specified as a probability distribution $p(\mathbf{x}_t = x|\mathbf{x}_{t-1} = x', \mathbf{a}_{t-1})$, where $\mathbf{x}_t$ is a random variable representing the robot's pose at time t, $\mathbf{a}_t$ is the action or movement executed by the robot at time t. The movement can be estimated, for example, by odometers on the wheels. The observation model is used to incorporate information from exteroceptive sensors, such as proximity sensors and camera, and it is expressed as $p(\mathbf{x}_t = x|\mathbf{o}_t)$, where $\mathbf{o}_t$ is an observation sensed at time t.

In ML the motion model is described as:

$$p(\mathbf{x}_t = x|\mathbf{x}_{t-1} = x', \mathbf{a}_{t-1}) = \sum_{x'} p(\mathbf{x}_t = x|\mathbf{x}_{t-1} = x', \mathbf{a}_{t-1}) p(\mathbf{x}_{t-1} = x'), \quad (1)$$

where $p(\mathbf{x}_t = x|\mathbf{x}_{t-1} = x', \mathbf{a}_{t-1})$ is characterized by a normal distribution:

$$p(\mathbf{x}_t = x|\mathbf{x}_{t-1} = x', \mathbf{a}_{t-1}) = \frac{1}{\sigma(\mathbf{a}_{t-1})\sqrt{2\pi}} \exp -\frac{(|x - x'| - \mathbf{a}_{t-1})^2}{2\sigma^2(\mathbf{a}_{t-1})}, \quad (2)$$

where $\sigma(\mathbf{a}_{t-1})$ is the standard deviation given $\mathbf{a}_{t-1}$.

The observation model is described as:

$$p(\mathbf{x}_t = x|\mathbf{o}_t) = \frac{p(\mathbf{o}_t|\mathbf{x}_t = x)p(\mathbf{x}_t = x)}{\sum_{x'} p(\mathbf{o}_t|\mathbf{x}_t = x')p(\mathbf{x}_t = x')}, \quad (3)$$

where $p(\mathbf{o}_t|\mathbf{x}_t = x)$ is characterized by a normal distribution. For a proximity sensor:

$$p(\mathbf{o}_t|\mathbf{x}_t = x) = \frac{1}{\sigma(\mathbf{o}_t)\sqrt{2\pi}} \exp -\frac{(d - \mathbf{o}_t)^2}{2\sigma^2(\mathbf{o}_t)}, \quad (4)$$

where d is the measured distance if the sensor detects an obstacle, $\mathbf{o}_t$ is the distance to the next obstacle in the map, and $\sigma(\mathbf{o}_t)$ is the standard deviation given $\mathbf{o}_t$.

The Markov localization algorithm is presented in Algorithm 1. In the beginning, $p(\mathbf{x}_0 = x)$ is the prior belief about the initial pose of the robot. If the initial pose is unknown, $p(\mathbf{x}_0 = x)$ is uniformly distributed around all possible poses.

In the next section, the multi-robot localization problem is presented based on the Markov localization approach.

3 MULTI-ROBOT LOCALIZATION

The cooperative multi-robot localization problem consists of localizing each robot in a group within the same environment, when robots share information in order to improve localization accuracy.

Representative recent works in cooperative multi-robot localization are from Roumeliotis and Bekey (2002) and Fox et al. (2000), that use Kalman Filter and Particle Filter as algorithms, respectively.

Algorithm 1 Single Robot Markov Localization Fox et al. (1999)

for each pose x **do**
  Initialize $p(\mathbf{x}_0 = x)$
end for
loop
  if robot receives an odometer reading **then**
    for each pose x **do**
      $p(\mathbf{x}_t = x|\mathbf{x}_{t-1} = x', \mathbf{a}_{t-1}) = \sum_{x'} p(\mathbf{x}_t = x|\mathbf{x}_{t-1} = x', \mathbf{a}_{t-1})p(\mathbf{x}_{t-1} = x')$
    end for
  end if
  if robot receives a sensor reading **then**
    for each pose x **do**
      $p(\mathbf{x}_t = x|\mathbf{o}_t) = \frac{p(\mathbf{o}_t|\mathbf{x}_t = x)p(\mathbf{x}_t = x)}{\sum_{x'} p(\mathbf{o}_t|\mathbf{x}_t = x')p(\mathbf{x}_t = x')}$
    end for
  end if
end loop

The Markov localization (Fox et al., 1999) was initially designed for a single robot. One extension of this work, aimed to solve the multi-robot problem is presented in Fox et al. (2000). In order to accommodate multi-robot cooperation in Markov localization it is necessary to add a detection model to the previous observation and motion models.

The detection model (Fox et al., 2000) is based on the assumption that each robot is able to detect and identify other robots and furthermore, the robots can communicate their probabilities distributions to other robots. Let's suppose that robot m detects robot n and measures the relative distance between them, so:

$$p(\mathbf{x}_n = x|\mathbf{x}_m = x', \mathbf{r}_m) = p(\mathbf{x}_n = x) \sum_{x'} p(\mathbf{x}_n = x|\mathbf{x}_m = x', \mathbf{r}_m)p(\mathbf{x}_m = x'), \quad (5)$$

where $\mathbf{x}_n$ represents the pose of robot n, $\mathbf{x}_m$ represents the pose of robot m and $\mathbf{r}_m$ denotes the measured distance between robots. The calculation $\sum_{x'} p(\mathbf{x}_n = x|\mathbf{x}_m = x', \mathbf{r}_m)p(\mathbf{x}_m = x')$ describes the belief of robot m about the pose of robot n. Similarly, the same detection can be used to update the pose of robot m.

The probability distribution $p(\mathbf{x}_n = x|\mathbf{x}_m = x', \mathbf{r}_m)$ is characterized by a normal distribution. For a proximity sensor:

$$p(\mathbf{x}_n = x|\mathbf{x}_m = x', \mathbf{r}_m) = \frac{1}{\sigma(\mathbf{r}_m)\sqrt{2\pi}} \exp - \frac{(|x - x'| - \mathbf{r}_m)^2}{2\sigma^2(\mathbf{r}_m)}, \quad (6)$$

where $\mathbf{r}_m$ is the measured distance when robot m detects robot n and $\sigma(\mathbf{r}_m)$ is the standard deviation given $\mathbf{r}_m$.

Once a detection is made according to the detection model, the two robots involved in the process share their probabilities distributions and relative distance. This communication significantly improves the localization accuracy, if compared with a less communicative localization approach.

The multi-robot localization algorithm proposed by Fox et al. (2000) presents the advantage of performing global localization (see algorithm 2). Robots in the group cooperate to find their poses, communicating their pose beliefs when they meet. One disadvantage of this approach is that this information is shared only by the two meeting robots and it is not used by the other robots in the group.

A different detection model is presented by Roumeliotis and Bekey (2002). Every time two robots meet, their meeting is used not only to update their poses, but also to propagate to all robots in the group. So, robots pose are estimated with all information available at all time. However, the work uses Kalman filter as localization technique, and is unable to perform global localization.

The work presented in this paper aims at combining the advantages of the previous works, in the way that: (1) it can localize robots with unknown initial poses, as Fox et al. (2000) and (2) communication is used to propagate the information derived from a meeting of two robots to the other robots in the group, so becoming similar to the work of Roumeliotis and Bekey (2002). In order to achieve this goal, a new detection model is presented in the next section.

4 DETECTION MODEL

In this section it is explained the communication structure proposed to the detection model of multi-robot localization. We share the motivation from Fox et al. (2000) to investigate how localization accuracy can be improved exploring shared information among robots. However, we argue that all the robots in a group (bigger than two robots) can benefit from the shared information derived from a single detection (when robot m meets robot n). Two questions raise: *What information is useful for non-meeting robots? How can this information be shared?*

Suppose a robot k in the group. When robot m meets robot n, robot k can conclude that its pose is not the robot m and n poses, once only one robot can occupy the same space in the environment at the same time. Robot k can also conclude that its pose is not in the way between the two meeting robots, otherwise, robot m would have detected robot k instead of robot n. It is supposed that the detection sensor can sense robots in front of the detecting robot. For example, the detection sensor could be a camera (pointing forward) to identify the robot and a proximity sensor to measure the distance.

The answer to the second question is to propagate the probabilities distributions of the two meeting robots to the other robots in the group. It can be performed by the robot m, that executes the detection. When robot n updates its pose based on the information communicated by robot m, it communicates back its updated probability distribution, p_n. Robot m then calculates a probability distribution that will be communicated to the non-meeting robots:

$$p_d(\mathbf{x}_k = x'') = (1 - (p_m(\mathbf{x}_m = x') + p_{mn}(\mathbf{x}_m = x', \mathbf{x}_n = x) + p_n(\mathbf{x}_n = x))), \quad (7)$$

where $p_d(\mathbf{x}_k = x'')$ is the information communicated to the non-meeting robots, $p_m(\mathbf{x}_m = x')$ is the probability distribution of robot m, $p_n(\mathbf{x}_n = x)$ is the probability distribution of robot n and $p_{mn}(\mathbf{x}_m = x', \mathbf{x}_n = x)$ is the probability distribution of a robot being in the way between the two meeting robots (see algorithm 3).

Figure 1 illustrates an example of this situation. In the example there are 3 robots, $R1$, $R2$, $R3$ in their actual poses. The robots have different knowledge about their poses: robot 1 is certain about its pose, robot 2 is completely uncertain and robot 3 is in doubt about 2 poses. The belief of robot 3 is represented by the shaded cells. If robot 1 meets robot 2, robot 2 becomes certain about its pose and robot 3 keeps its previous pose knowledge. However, if the detection information is shared with robot 3, it becomes certain about its pose, because the possibility of being between robot 1 and robot 2 is eliminated.

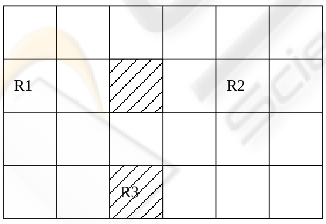

Figure 1: Example: poses of 3 robots. R1 is certain about its pose, R2 is completely uncertain and R3 is in doubt about its pose (the two dashed cells).

The example shows the improvement in group localization obtained if a detection information is propagated to all robots in the group. This new communication allows better localization results than in Fox et al. (2000). Experimental results presented in the next section allow comparative analysis.

Algorithm 2 Multi-robot Markov Localization Fox et al. (2000)

for each robot $\mathbf{x}_m$ **do**
  for each pose x **do**
    Initialize $p(\mathbf{x}_0 = x)$
  end for
  loop
    if robot receives an odometer reading **then**
      for each pose x **do**
        $p(\mathbf{x}_t = x|\mathbf{x}_{t-1} = x', \mathbf{a}_{t-1}) = \sum_{x'} p(\mathbf{x}_t = x|\mathbf{x}_{t-1} = x', \mathbf{a}_{t-1}) p(\mathbf{x}_{t-1} = x')$
      end for
    end if
    if robot receives a sensor reading **then**
      for each pose x **do**
        $p(\mathbf{x}_t = x|\mathbf{o}_t) = \frac{p(\mathbf{o}_t|\mathbf{x}_t = x)p(\mathbf{x}_t = x)}{\sum_{x'} p(\mathbf{o}_t|\mathbf{x}_t = x')p(\mathbf{x}_t = x')}$
      end for
    end if
    if robot m detects robot n **then**
      for each pose x **do**
        $p(\mathbf{x}_n = x|\mathbf{x}_m = x', \mathbf{r}_m) = p(\mathbf{x}_n = x) \sum_{x'} p(\mathbf{x}_n = x|\mathbf{x}_m = x', \mathbf{r}_m) p(\mathbf{x}_m = x').$
      end for
      for each pose x **do**
        $p(\mathbf{x}_m = x|\mathbf{x}_n = x', \mathbf{r}_m) = p(\mathbf{x}_m = x) \sum_{x'} p(\mathbf{x}_m = x|\mathbf{x}_n = x', \mathbf{r}_m) p(\mathbf{x}_n = x').$
      end for
    end if
  end loop
end for

5 EXPERIMENTS

In order to evaluate the localization results obtained with the cooperative multi-robot localization approach proposed in this paper we perform some experiments. We compare our approach (algorithm 3) to a previous approach (algorithm 2). In all experiments our approach outperforms the algorithm 2.

The experiments are conducted with simulated robots. Each robot is equipped with a proximity sensor to measure the distance to the walls in the environment, and a detection sensor, that can identify other robots and measure their relative position (x, y). All robot sensors are assumed to be corrupted by gaussian noise. The robots know an environment model and they do not know their initial poses in the environment. All the robots move simultaneously through the environment and they keep moving until all robots find their poses, that is, until their localization errors become near zero.

Algorithm 3 Novel Multi-robot Markov Localization

for each robot $\mathbf{x}_m$ **do**
  for each pose x **do**
    Initialize $p(\mathbf{x}_0 = x)$
  end for
  loop
    if robot receives an odometer reading **then**
      for each pose x **do**

$$p(\mathbf{x}_t = x|\mathbf{x}_{t-1} = x', \mathbf{a}_{t-1}) = \sum_{x'} p(\mathbf{x}_t = x|\mathbf{x}_{t-1} = x', \mathbf{a}_{t-1}) p(\mathbf{x}_{t-1} = x')$$

      end for
    end if
    if robot receives a sensor reading **then**
      for each pose x **do**

$$p(\mathbf{x}_t = x|\mathbf{o}_t) = \frac{p(\mathbf{o}_t|\mathbf{x}_t = x)p(\mathbf{x}_t = x)}{\sum_{x'} p(\mathbf{o}_t|\mathbf{x}_t = x')p(\mathbf{x}_t = x')}$$

      end for
    end if
    if robot m detects robot n **then**
      for each pose x **do**

$$p(\mathbf{x}_n = x|\mathbf{x}_m = x', \mathbf{r}_m) = p(\mathbf{x}_n = x) \sum_{x'} p(\mathbf{x}_n = x|\mathbf{x}_m = x', \mathbf{r}_m) p(\mathbf{x}_m = x').$$

      end for
      for each pose x **do**

$$p(\mathbf{x}_m = x|\mathbf{x}_n = x', \mathbf{r}_m) = p(\mathbf{x}_m = x) \sum_{x'} p(\mathbf{x}_m = x|\mathbf{x}_n = x', \mathbf{r}_m) p(\mathbf{x}_n = x').$$

      end for
      for each robot k but robot m and n **do**
        for each pose x **do**

$$p(\mathbf{x}_k = x) = \sum_{x''} p(\mathbf{x}_k = x) p_d(\mathbf{x}_k = x'').$$

        end for
      end for
    end if
  end loop
end for

Tests are performed in two different environments. The first environment is shown in Figure 2. It is a symmetric hallway, similar to that used in Fox et al. (2000). Since this environment is symmetric, robots need to deal with ambiguities to be able to localize themselves. For example, a single robot with unknown initial pose, has to pass the open space on corridor A, or it has to pass through all other corridors, B, C and D, in order to uniquely determine its pose.

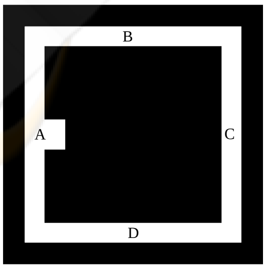

Figure 2: Test 1: Symmetric hallway environment with 4 corridors denoted by A, B, C and D. Black represents obstacles and walls and white represents free space.

The environment has dimensions 12×12 meters. The robot environment model is a grid-based model, where each cell has dimensions of 1×1 meters, and angular resolution of 90 degrees. It results in a state space of dimension $12 \times 12 \times 4 = 576$ states. Experiments are conducted with 8 robots, moving simultaneously through the corridors. All the robots run clockwise, following the corridors.

Figure 3 presents the localization errors per distance travelled for both methods. Results are averaged by eight runs of the experiment. The solid line refers to the algorithm 2 and the dashed line refers to our algorithm. Initially all robots are uncertain about their poses and their localization errors are high. As they move, they become more certain about their actual pose, so their localization errors diminish. For the algorithm 3, at distance 12 meters all robots are certain about its poses, whereas for the algorithm 2 it takes 18 meters.

Another measurement from the experiments is the average distance travelled by all the robots over the eight runs until they find their poses. It is 10.08 ± 1.32 meters for the algorithm 2 and 7.95 ± 1.14 for algorithm 3. It is clear that our algorithm presents smaller localization errors and distance travelled than the other approach.

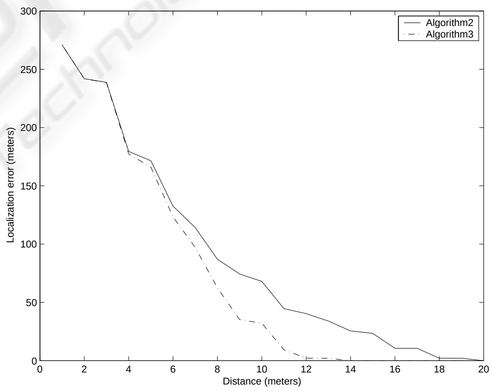

Figure 3: Localization error: the distance travelled by the robots until their localization errors are reduced to (near) zero (x-axis) per the average localization error (y-axis).

The second environment used for tests is shown in Figure 4. The environment is different from the first one, once it is an open area, and the trajectories of the robots are not restricted to follow a corridor. The environment has dimensions 11×11 meters. The robot environment model is a grid-based model, where each cell has dimensions of 1×1 meters, and angular resolution of 90 degrees. It results in a state space of dimension $11 \times 11 \times 4 = 484$ states. Experiments are conducted with four robots, performing random walk through the environment.

Figure 5 presents the localization errors per distance travelled for both methods. Results are avera-

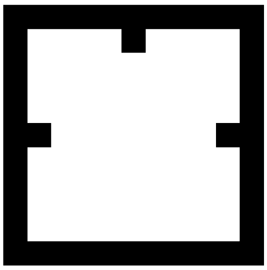

Figure 4: Test 2: Open environment.

ged by eight runs of the experiment. The solid line refers to the algorithm 2 and the dashed line refers to our algorithm. Initially all robots are uncertain about their poses and their localization errors are high. As they move, they become more certain about their actual pose, so their localization errors diminish. For the algorithm 3, at distance 20 meters all robots are certain about its poses, whereas for the algorithm 2 it takes 30 meters.

Another measurement from the experiments is the average distance travelled by all the robots over the eight runs until they find their poses. It is 12.56 ± 0.95 meters for the algorithm 2 and 11.44 ± 0.16 for algorithm 3. It is clear that our algorithm presents smaller localization errors and distance travelled than the other approach.

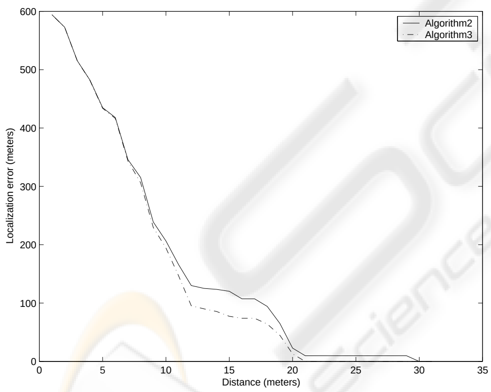

Figure 5: Localization error: the distance travelled by the robots until their localization errors are reduced to (near) zero (x-axis) per the average localization error (y-axis).

6 CONCLUSION AND FUTURE WORK

We have presented a statistical approach for cooperative multi-robot localization. The experimental results demonstrate that our approach, when compared to a previous multi-robot localization method, reduces the uncertainty in localization significantly and reduces the distance travelled by the robots in order to find their poses.

A limitation of our work is the increase in the amount of data needed to communicate in order to update robots' poses. Thus, in future work, we are interested in exploring the tradeoff between communication and localization accuracy.

Another point to be explored is an active detection approach. It means that when a robot knows its pose it can communicate it to all the robots within the group, and they can look for the right robot in order to exchange pose information with it and improve their pose beliefs.

A computationally efficient version of ML is the Particle Filter – PF (Thrun et al., 2001). As shown in Fox et al. (2000), Markov localization can be extended to PF. So, we pretend to extend the localization approach proposed here to perform PF localization.

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