

ROBUST CONTROL OF INDUCTION MOTOR USING FAST OUTPUT SAMPLING TECHNIQUE

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Abstract: In this paper a design method based on robust fast output sampling technique is presented for the speed control of induction motor. The nonlinear model of induction motor model is linearized around various operating points to obtain the linear models. The input of the induction motor is the stator voltages and only the speed is considered as the output of the systems. A single controller is designed for these linear models. The nonlinear model of the induction motor is simulated with the proposed controller at these operating points. This method does not require the state of the system for feedback and is easily implementable.

1 INTRODUCTION

The induction motor is being used in many industrial applications due to its reliability, ruggedness, and low cost. Its mechanical reliability is due to the fact that there is no mechanical commutation (i.e. there are no brushes nor commutator to wear out as in a DC motor). Further more it can also be used in volatile environments since no sparks are produced as is the case with the commutator of a DC motor. For these and other reasons induction motor is widely used in many electric drive applications. However, the induction motor presents a challenging control problem. This is due to the fact that this dynamical system is nonlinear, two of the state variables (rotor fluxes/currents) are not usually measurable, and due to heating the rotor resistance varies considerably with a significant impact on the system dynamics. High performance drives for various applications using induction motors with vector control have become the preferred form of motive power in a number of applications. Vector control transforms the induction motor into a system that has the characteristics of a separately excited DC motor. Success of vector control techniques depend on the knowledge of instantaneous magnitude and position of rotating magnetic field (or flux) in the machine. Direct measurement of magnetic field using search coils, hall effect sensors require implantation of sensors in the air gap of the machine and hence result in increased complexity. Moreover, they are prone to errors caused by factors like

temperature variation and noise (Janson and Lorenz, 1992), (Verghese and Sanders).

In order to circumvent direct measurement of the magnetic flux, a number of flux estimation methods have been proposed in the last few years (Janson and Lorenz, 1992), (Sangwongwanich et al.). All these methods utilize terminal measurements of voltage and current along with (or without) rotor speed to arrive at an accurate estimate of the magnitude and position of the flux in the machine. These methods have been broadly classified as open loop observers and closed loop observers to have superior performance characteristics with respect to robustness and accuracy (Janson and Lorenz, 1992). Almost all of the literature uses more than one of the system states for the design of the controller.

In this paper a design of robust fast output sampling controller for induction motor control by measuring only the speed is proposed. This is done by sampling the speed signal at a faster rate than the input signal. It will be shown that using this technique a robust control for the multimodel representation of nonlinear model of the induction motor can be obtained. The outline of the paper is as follows. Section 2 presents dynamic model of an induction motor. Section 3 deals with a brief introduction of fast output sampling control technique where as the controller design is presented in Section 4. Section 5 presents simulation results and discussions followed by the concluding remarks.

2 DYNAMIC MODEL OF AN INDUCTION MOTOR

Under the commonly used assumptions, the behavior of the three phase, four pole, induction motor in the orthogonal field reference frame can be described by a set of non-linear equations as given below (Kraus, et al, 2002) ,(Mohan, 2000), (Mohan, 2001)

$$\begin{aligned} \frac{d}{dt}i_{ds} &= \sigma R_s i_{ds} + \omega_s i_{qs} + \beta L_m \omega_r i_{qs} \quad (1) \\ &\quad + \beta R_r i_{dr} + \beta L_r \omega_r i_{qr} + \sigma V_{ds} \\ \frac{d}{dt}i_{qs} &= -\omega_s i_{ds} - \beta L_m \omega_r i_{ds} - \sigma R_s i_{qs} \\ &\quad - \sigma L_m \omega_r i_{dr} + \beta R_r i_{qr} + \sigma V_{qs} \\ \frac{d}{dt}i_{dr} &= \beta R_s i_{ds} - \gamma L_m \omega_r i_{qs} - \gamma R_r i_{dr} \\ &\quad + \omega_s i_{qr} - \sigma L_s \omega_r i_{qr} - \beta V_{ds} \\ \frac{d}{dt}i_{qr} &= \beta L_s \omega_r i_{ds} + \beta R_s i_{qs} - \omega_s i_{dr} \\ &\quad + \sigma L_s \omega_r i_{dr} - \gamma R_r i_{qr} - \beta V_{qs} \\ \frac{d}{dt}\omega_r &= \frac{p}{2J_{eq}} (T_e - T_L) \\ T_e &= \frac{3P}{2} L_m (i_{qs} i_{dr} - i_{ds} i_{qr}). \end{aligned}$$

where i_{ds}, i_{qs} are stator currents, i_{dr}, i_{qr} are rotor currents, V_{ds}, V_{qs} are stator voltages, ω_r is rotor angle velocity, ω_s is synchronous speed, T_e, T_L are electromagnetic and load torques, J_{eq} is inertia of the rotor, p is number of poles, α, β, σ and γ are all positive constants defined as:

$$\begin{aligned} \alpha &= L_s L_r - L_m^2, \\ \beta &= L_m / \alpha, \\ \sigma &= L_r / \alpha, \\ \gamma &= L_s / \alpha \end{aligned}$$

where L_s and L_r are stator and rotor inductances, R_s and R_r are stator and rotor resistances and L_m is the mutual inductance. The output is the rotor speed.

3 ON FAST OUTPUT SAMPLING FEEDBACK

3.1 Review on Fast Output Sampling Feedback

The problem of simultaneous stabilization has received considerable attention in the literature. Given a family of plants in state space representation (Φ_i, Γ_i) , $i = 1, \dots, M$, find a linear state feedback gain F such that $(\Phi_i + \Gamma_i F)$ is stable for $i =$

$1, \dots, M$, or determine that no such F exists. But the method is of use only in the case where whole state information is available.

One way of approaching this problem with incomplete state information is to use observer based control laws, *i.e.* dynamic compensators. The problem here is that the state feedback and state estimation cannot be separated in face of the uncertainty represented by a family of systems. Assuming that a simultaneously stabilizing F has been found, it is possible to search for a simultaneously stabilizing full order observer gain, but this search is dependent on the F previously obtained. If no stabilizing observer for this state feedback exists, nothing can be said because there may exist stabilizing observers for different feedback gains.

With the *Fast output sampling* approach proposed by (Werner and Furuta, 1995) it is generically possible to simultaneously realize a given state feedback gain for a family of linear, observable models. For fast output sampling gain L realize the effect of state feedback gain F , find the L such that $(\Phi_i + \Gamma_i LC)$ is stable for $i = 1, \dots, M$. If there exist a set of F 's, there should exist a common L for given family of plants. One of the problems with this approach is that large feedback gains tend to render the system very noise -sensitive. To overcome this problem the design problem can be posed as a multi-objective optimization problem in an LMI formulation proposed by (Werner, 1998).

Consider a plant described by a linear model

$$\dot{x} = Ax + Bu, \quad (2)$$

$$y = Cx, \quad (3)$$

with (A, B) controllable and (C, A) observable. Assume the plant is to be controlled by a digital controller, with sampling time τ and zero order hold, and that a sampled data state feedback design has been carried out to find a state feedback gain F such that the closed loop system

$$x(k\tau + \tau) = (\Phi_\tau + \Gamma_\tau F)x(k\tau), \quad (4)$$

has desired properties. Hence $\Phi_\tau = e^{A\tau}$ and $\Gamma_\tau = \int_0^\tau e^{As} ds B$. Instead of using a state observer, the following sampled data control can be used to realize the effect of the state feedback gain F by output feedback. Let $\Delta = \tau/N$, and consider

$$\begin{aligned} u(t) &= [L_0 \cdots, L_{N-1}] \begin{bmatrix} y(k\tau - \tau) \\ \vdots \\ y(k\tau - \Delta) \end{bmatrix} \\ &= Ly_k, \end{aligned} \quad (5)$$

for $k\tau \leq t < (k+1)\tau$, where the matrix blocks L_j represent output feedback gains, and the notation

\mathbf{L}, y_k has been introduced for convenience. Note that $1/\tau$ is the rate at which the loop is closed, whereas output samples are taken at the N -times faster rate $1/\Delta$.

To show how a fast output sampling controller (5) can be designed to realize the given sampled-data state feedback gain, we construct a fictitious, lifted system for which (5) can be interpreted as static output feedback. Let (Φ, Γ, C) denote the system (2) at the rate $1/\Delta$. Consider the discrete-time system having at time $t = k\tau$ the input $u_k = u(k\tau)$, state $x_k = x(k\tau)$ and output y_k as

$$x_{k+1} = \Phi_\tau x_k + \Gamma_\tau u_k, \quad (6)$$

$$y_{k+1} = \mathbf{C}_0 x_k + \mathbf{D}_0 u_k, \quad (7)$$

where

$$\mathbf{C}_0 = \begin{bmatrix} C \\ C\Phi \\ \vdots \\ C\Phi^{N-1} \end{bmatrix}, \mathbf{D}_0 = \begin{bmatrix} 0 \\ C\Gamma \\ \vdots \\ C \sum_{j=0}^{N-2} \Phi^j \Gamma \end{bmatrix}.$$

Assume that the state feedback gain F has been designed that $(\Phi_\tau + \Gamma_\tau F)$ has no eigenvalues at the origin. Then, assuming that in intervals $k\tau \leq t < (k+1)\tau$

$$u(t) = Fx(k\tau), \quad (8)$$

one can define the fictitious measurement matrix

$$\mathbf{C}(F, N) = (\mathbf{C}_0 + \mathbf{D}_0 F)(\Phi_\tau + \Gamma_\tau F)^{-1}, \quad (9)$$

which satisfies the fictitious measurement equation $y_k = \mathbf{C}x_k$. For \mathbf{L} to realize the effect of F , it must satisfy

$$\mathbf{L}\mathbf{C} = F. \quad (10)$$

Let ν denote the observability index of (Φ, Γ) . It can be shown that for $N \geq \nu$, generically \mathbf{C} has full column rank, so that any state feedback gain can be realized by a fast output sampling gain \mathbf{L} .

If the initial state is unknown, there will be an error $\Delta u_k = u_k - Fx_k$ in constructing the control signal under state feedback. One can verify that the closed loop dynamics are governed by

$$\begin{bmatrix} X_{k+1} \\ \Delta u_{k+1} \end{bmatrix} = \begin{bmatrix} \Phi_\tau + \Gamma_\tau F & \Gamma_\tau \\ 0 & \Psi(L) \end{bmatrix} \begin{bmatrix} x_k \\ \Delta u_k \end{bmatrix}. \quad (11)$$

where $\Psi(L) = \mathbf{L}\mathbf{D}_0 - F\Gamma_\tau$.

Thus, one can say that *the eigenvalues of the closed loop system under a fast output sampling control law (5) are those of $\Phi_\tau + \Gamma_\tau F$ together with those of $\mathbf{L}\mathbf{D}_0 - F\Gamma_\tau$.*

One feature of fast output sampling control that makes it attractive for robust controller design, is the fact that a result similar to the above can be shown to hold when the same state feedback is applied simultaneously to a family of models representing different operating conditions of the plant.

3.2 Integral Action

The fast output controller (5), can be used to realize the effect of state feedback. If step disturbances are to be rejected with zero steady state error, then the controller must integrate the tracking error. A pure state feedback control law does not include integral action, but can be made to do so by introducing a new state ζ that integrates the error (Werner, 1996). This can be realized in discrete time using:

$$\zeta_{k+1} = \zeta_k + r_k - y_k$$

where r_k stands for the sampled reference input. A discrete time state space representation for the augmented system is

$$\bar{x}(k+1) = \bar{\Phi}\bar{x}(k) + \bar{\Gamma}u(k) + \bar{\Gamma}_r r(k), \quad (12)$$

where $\bar{x}(k) = [x(k)^T \zeta_k]^T$ and

$$\begin{aligned} \bar{\Phi} &= \begin{bmatrix} \Phi_\tau & 0 \\ -C & I \end{bmatrix}, \\ \bar{\Gamma} &= \begin{bmatrix} \Gamma_\tau \\ 0 \end{bmatrix}, \\ \bar{\Gamma}_r &= \begin{bmatrix} 0 \\ I \end{bmatrix}. \end{aligned}$$

State feedback gain F and the integrator gain F_I can be collected from a new state feedback gain $\bar{F} = [F \ F_I]$ to yield the control law

$$\begin{aligned} u(k) &= \bar{F}x(k) = Fx(k) + F_I \zeta_k \\ &= L y_k + F_I \zeta_k \end{aligned} \quad (13)$$

with a resulting closed loop matrix:

$$\bar{\Phi}_\tau + \bar{\Gamma}_\tau \bar{F} = \begin{bmatrix} \Phi_\tau + \Gamma_\tau F & \Gamma_\tau F_I \\ -C & I \end{bmatrix},$$

3.3 Multimodel Synthesis

For multimodel representation of a plant, it is necessary to design controller which will robustly stabilize the multimodel system. Multimodel representation of plants can arise in several ways. When a nonlinear system has to be stabilized at different operating points, linear models are sought to be obtained at those operating points. Even for parametric uncertain linear systems, different linear models can be obtained for extreme points of the parameters or for

family of different models. The models are used for stabilization of different machine models

Now consider a family of plant $S = \{A_i, B_i, C_i\}$, defined by

$$\dot{x} = A_i x + B_i u, \quad (14)$$

$$y = C_i x, \quad i = 1, \dots, M. \quad (15)$$

By sampling at the rate of $1/\Delta$, we get a family of discrete-time systems $\{\Phi_i, \Gamma_i, C_i\}$.

Consider the family of discrete-time systems given by Eqns.(14) and (15) having at time $t = k\tau$ the input $u_k = u(k\tau)$, state $x_k = x(k\tau)$ and output y_k as

$$x_{k+1} = \Phi_{\tau i} x_k + \Gamma_{\tau i} u_k, \quad (16)$$

$$y_{k+1} = C_{0i} x_k + D_{0i} u_k, \quad (17)$$

Assume that $(\Phi_{\tau i}, \Gamma_i)$ are controllable. Then we can find a robust state feedback gains F such that $(\Phi_{\tau i} + \Gamma_i F)$ has no eigenvalues at the origin.

Then, assuming that in intervals $k\tau \leq t < k\tau + \tau$

$$u(t) = Fx(k\tau), \quad (18)$$

one can define the fictitious measurement matrix

$$C_i(F, N) = (C_{0i} + D_{0i} F)(\Phi_{\tau i} + \Gamma_{\tau i} F)^{-1}, \quad (19)$$

which satisfies the fictitious measurement equation $y_k = C_i x_k$. For robust fast output sampling gain L to realize the effect of F , it may satisfy

$$LC_i = F, \quad i = 1, \dots, M. \quad (20)$$

The Eqn.(20) can be written as

$$L \tilde{C} = \tilde{F}, \quad (21)$$

where

$$\tilde{C} = [C_1 \quad C_2 \quad \dots \quad C_M],$$

$$\tilde{F} = [F \quad F \quad \dots \quad F].$$

The robust state feedback gain F can be obtained from the augmented matrix (12) by considering a family of plants $\bar{S} = \{\bar{\Phi}_i, \bar{\Gamma}_i, \bar{\Gamma}_{ri}\}$, defined by.

$$\bar{x}(k+1) = \bar{\Phi}_i \bar{x}(k) + \bar{\Gamma}_i u(k) + \bar{\Gamma}_{ri} r(k), \quad (22)$$

3.4 An LMI Formulation of the Design Problem

When this idea is realized in practice *i.e.* fast output sampling gain L have been obtained by realizing the state feedback gain F , two problems are required to be addressed. The first one is apparent from(11). With this type of controller, the unknown states are

estimated implicitly, using the measured output samples and assuming that initial control is generated by state feedback. If initial state causes an estimation error, then decay of this error will be determined by the eigenvalues of the matrix $(LD_{0i} - F\Gamma_{\tau i})$ which depends on L and whose dimension equals the number of control input. For stability these eigenvalues have to be inside the unit disc, and for fast decay they should be as close to the origin as possible. This problem must be taken into account while designing L .

The second problem is that the gain matrix L may have elements with large magnitude. Because these values are only weights in linear combination of output samples, large magnitudes do not necessarily imply large control signal, and in theory and noise free simulation they pose no problem. But in practice they amplify measurement noise, and it is desirable to keep these values low. This objective can be expressed by an upper bound ρ on the norm of the gain matrix L .

When trying to deal with these problem, it is better not to insist on an exact solution to the design(20): one can allow a small deviation and use an approximation $LC_i \approx F$, which hardly affects the desired closed-loop dynamics, but may have considerable effect on the two problems described above. Instead of looking for an exact solution to the equalities, the following inequalities are solved

$$\|L\| < \rho_1,$$

$$\|LD_{0i} - F\Gamma_{\tau i}\| < \rho_{2i}, \quad i = 1, \dots, M,$$

$$\|LC_i - F\| < \rho_{3i}. \quad (23)$$

Here three objectives have been expressed by upper bounds on matrix norms, and each should be as small as possible. The ρ_1 small means low noise sensitivity, ρ_2 small means fast decay of estimation error, most important - ρ_3 -small means that fast output sampling controller with gain L is a good approximation of the originally designed state feedback controller. If $\rho_3 = 0$ then L is exact solution.

Using the Schur complement, it is straight forward to bring these conditions in the form of LMI (Linear Matrix Inequalities) proposed by (Werner, 1996)

$$\begin{bmatrix} -\rho_1^2 I & L \\ L^T & -I \end{bmatrix} < 0,$$

$$\begin{bmatrix} -\rho_{2i}^2 I & (LD_{0i} - F\Gamma_{\tau i}) \\ (LD_{0i} - F\Gamma_{\tau i})^T & -I \end{bmatrix} < 0,$$

$$\begin{bmatrix} -\rho_{3i}^2 I & (LC_i - F) \\ (LC_i - F)^T & -I \end{bmatrix} < 0. \quad (24)$$

In this form, the function `mincx()` of the LMI control toolbox for MATLAB can be used immediately

to minimize a linear combination of ρ_1, ρ_2, ρ_3 . The following approach turned out to be useful. If the actual measurement noise is known, the magnitude of \mathbf{L} is fixed accordingly. Likewise eigenvalues of $(\mathbf{L}\mathbf{D}_{0i} - F\Gamma_{\tau i})$ less than 0.05 cause no problem. So we can fix ρ_1, ρ_2 and only ρ_3 is minimized subject to these constraints.

In this form the LMI Tool Box of MATLAB can be used for synthesis as suggested by (Geheinet, et al., 1995).

The fast output sampling feedback controller obtained by the above method requires only constant gains and hence is easier to implement. The example of multi machine system dynamics is used to demonstrate the method.

4 CONTROLLER DESIGN

The design procedure assumes a linear model. The linearization was carried out about an operating point of the non-linear induction motor model (1), with the data provided in the appendix for three induction machine models.

Since the nonlinear system is to track a constant reference speed, the linear system composed of the error states, obtained at this nominal speed would be equivalently required to track a reference of zero. The continuous linear models are discretized and the discrete models are represented by the following equation

$$\Delta x(k+1) = \Phi_{\tau i} \Delta x(k) + \Gamma_{\tau i} \Delta u(k), \quad (25)$$

$$\Delta y(k) = C_i \Delta x(k), \quad i = 1, 2, 3 \quad (26)$$

where

$$\Delta x(k) = [\Delta i_{ds} \quad \Delta i_{qs} \quad \Delta \omega_r \quad \Delta i_{ds} \quad \Delta i_{qs}]^T$$

$$\Delta u(k) = [\Delta V_{ds} \quad \Delta V_{qs}]^T$$

with the speed of the induction machines being the output. The appropriate control action can now be computed using Eqn. (13).

However, for tracking of a reference speed, the integral action $F_I \zeta_k$ would require the error in the speed of the system. This is realized by comparing the actual speed with the reference speed.

and

$$\Delta u(k) = F_I \zeta(k) + L \Delta y_k \quad (27)$$

From the incremental control $\Delta u(k)$, actual control $u(k)$ is obtained.

$$u(k) = u_0 + \Delta u_k$$

5 SIMULATION RESULTS AND DISCUSSIONS

The system (1) is linearized about three operating points using MATLAB® to get a linear system of form (14), Discretization of (A_i, B_i, C_i) at an interval of $\tau = 0.04$ sec. and adding an integrator leads to the augmented system (22) for which a common state feedback gain must be found. This problem can be solved by convex programming methods, for which software tools are available in the *LMI toolbox for Matlab* (Geheinet, et al., 1995). This yields a state feedback gain:

$$\begin{aligned} \bar{F} &= [F \ F_I] = \\ &= \begin{bmatrix} -0.1227 & 25.8298 & 37.6501 & \cdots \\ -2.9221 & 72.7314 & 42.1827 & \cdots \\ \cdots & 27.5299 & 38.8149 & 7.4603 \\ \cdots & 74.0091 & 42.0990 & 1.2915 \end{bmatrix} \end{aligned}$$

Using (21), the fast output sampling gain is calculated as

$$L = \begin{bmatrix} -6.128 & 22.643 & -29.69 & 5.858 & \cdots \\ -2.075 & 10.899 & -23.982 & 12.78 & \cdots \\ 22.365 & -9.549 & -21.152 & 6.531 & \cdots \\ \cdots & 19.153 & -15.18 & -19.264 & 13.476 & \cdots \\ \cdots & 18.73 & -6.51 & -18.668 & 11.02 & \cdots \\ \cdots & 21.415 & -10.677 & -23.443 & 10.585 & \cdots \\ \cdots & 21.403 & -31.972 & 12.87 & \cdots & \cdots \\ \cdots & 22.504 & -33.03 & 15.132 & \cdots & \cdots \end{bmatrix}$$

Now, using Eqn. (27), $\Delta u(k)$ is obtained and $u(k)$ for the nonlinear model which is $u_0 + \Delta u(k)$ is also obtained. This is used in the simulation of the nonlinear model with 10% change in load.

Fig. (1) shows the speed profile of the closed loop systems of all three models for the load disturbance of -10%. Fig. (2) shows the speed profile of the closed loop systems of all models for the load disturbance of +10%. It can be seen that in all cases the designed robust controller is able to bring back the rotor speed to the rated value.

6 CONCLUSION

In this paper, a design scheme of the control of induction motor using fast output sampling control is developed. The rotor speed is taken as output. The output feedback control is applied at an appropriate sampling rate to the nonlinear model of the plant.

As shown in plots, the proposed controller is able to stabilize the speed around the operating points for the change in load torque (T_L) in spite of change in plant model.

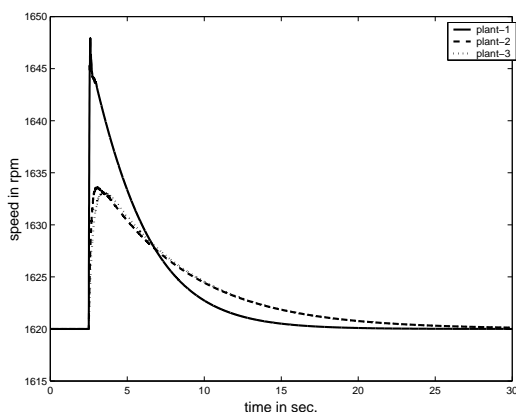


Figure 1: Response of various models to -10% load disturbance

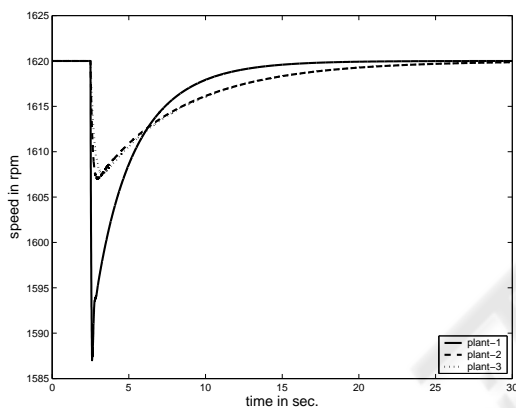


Figure 2: Response of various models to +10% load disturbance

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APPENDIX

A INDUCTION MOTOR PARAMETERS

The following parameters are used for simulation of the induction motor.

- Power : 3 HP/2.4 KW
- Voltage: 460 Volts (L-L, RMS)
- Frequency: 60 Hz
- Phases: 3
- Full- Load Current: 4 A
- Full- Load Speed: 1710 rpm
- Full- Load Efficiency: 88.5%
- Power Factor: 80.0%
- No. of Poles: 4
- slip=1.72%

Model Parameters	Model-1	Mode-2	Mode-2
R_r	1.4Ω	46.2Ω	74Ω
R_s	1.2Ω	1.43Ω	1.77Ω
X_{ls}	4.5Ω	6.72Ω	5.25Ω
X_{lr}	4.5Ω	6.72Ω	4.57Ω
X_m	142Ω	158Ω	139Ω