A NEW FAMILY OF CONTROLLERS FOR POSITION CONTROL OF ROBOT MANIPULATORS *

Fernando Reyes, Jaime Cid, Marco Mendoza, Isela Bonilla
Benemérita Universidad Autónoma de Puebla
Grupo de Robótica, Facultad de Ciencias de la Electrónica
Apartado Postal 542, Puebla 72001, México

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Abstract: This paper addresses the problem of position control for robot manipulators. A new family of position controllers with gravity compensation for the global position of robots manipulators is presented. The previous results on the linear PD controller are extended to the new proposed family. The main contribution of this paper is to prove that the closed–loop system composed by full nonlinear robot dynamics and the family of controllers is globally asymptotically stable in agreement with Lyapunov's direct method and LaSalle's invariance principle. Besides the theoretical results, a real-time experimental comparison is also presented on visual servoing applications to illustrate the performance of the proposed family on a direct–drive robot of two degrees of freedom.

1 INTRODUCTION

The position control of robot manipulators, or also the so-called regulation problem is the simplest aim in robot control and at the same time one of the most relevant issue in practice of manipulators. This is a particular case of the motion control or trajectory control. The primary goal of motion control in joint space is to make the robot joints track a given time-varying desired joint position. On the other hand, the goal of position control is to move the robot end-effector to a fixed desired target, which is assumed to be constant, regardless of its initial joint position (Craig, 1989)(F. L. Lewis, 1993)(O. Khatib, 1989).

The PD control is the most widely used strategy for robot manipulators, because of its simplicity, it counts with theoretical support to justify the use of the PD in global positioning (M. Takegaki, 1981)(S. Arimoto, 1986)(C. Canudas, 1996). On the other hand, the PID control is another popular strategy, until now we do not have the required theoretical support backing to guarantee position control in a global sense (Kelly, 1995)(R. Kelly, 1996)(Kelly, 1999)(Y. Xu, 1995)(V. Santibañez, 1998).

However, the PD control with gravity compensation has serious practical drawback, for example: it requires the exact knowledge of the gravitational torque vector from robot dynamics. Although the structure of the gravitational torque vector can be easily obtained as the gradient of the robot potential energy due to gravity, some parameters can be uncertain such as masses and mass centers. Other draws is that the choice of the PD gains relies on the desired position (J. Alvarez, 2003)(A. Loria, 2002)(L. Sciavicco, 1996).

In recent years, various PD-Type control schemes have been developed for position control of robot manipulators. Among them the following can be cited: A PD controller with proportional and derivative gains as nonlinear functions of the robot states developed in (Y. Xu, 1995). In the reference (V. Santibañez, 1998) was proposed a Saturated PD controller to deliver torques within prescribed limits according to the actuator capability. A new class of nonlinear PID controllers with robotic applications was presented in (Seraji, 1998). (Kelly, 1999) presented a PD controller in generic task space. This controller was based on energy shaping methodology. Most recently, (J. Alvarez, 2003) proposed a saturated linear PID controller with semiglobal stability.

In view of the simplicity and applicability of the simple PD controller in industrial applications, the main motivation of this paper is in the theoretical and practical interest of obtaining controllers that lead to

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global asymptotic stability of the closed-loop system. The objective of this paper is to extend the previous results on the linear PD controller to a new family of position controllers. In addition to the theoretical issues of the proposed family, this paper also presents real-time experiments for position control on a direct-drive robot of two degrees of freedom.

This paper is organized as follows. Section 2 recalls the robot dynamics and useful property for stability proof. In the Section 3, the new family of position controllers and its analysis of global asymptotic stability is presented. Section 4 summarizes the main components of the experimental set-up. Section 5 contains the experimental results. Finally, some conclusions are offered in Section 6.

2 ROBOT DYNAMICS

The dynamics of a serial n-link rigid robot can be written as (L. Sciavicco, 1996)(Vidyasagar, 1989):

$$M(\mathbf{q})\ddot{\mathbf{q}} + C(\mathbf{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{f}(\dot{\boldsymbol{q}},\boldsymbol{\tau}) = \boldsymbol{\tau} \qquad (1)$$

where $\boldsymbol{q}$ is the $n \times 1$ vector of joint displacements; $\dot{\boldsymbol{q}}$ is the $n \times 1$ vector of joint velocities; $\boldsymbol{\tau}$ is the $n \times 1$ vector of input torques; $M(\boldsymbol{q})$ is the $n \times n$ symmetric positive definite inertia matrix, $C(\boldsymbol{q},\dot{\boldsymbol{q}})$ is the $n \times n$ matrix of centripetal and Coriolis torques; $\boldsymbol{g}(\boldsymbol{q})$ is the $n \times 1$ vector of gravitational torques and $\boldsymbol{f}(\dot{\boldsymbol{q}},\boldsymbol{\tau})$ is the $n \times 1$ vector for the friction torques. The vector $\boldsymbol{f}(\dot{\boldsymbol{q}},\boldsymbol{\tau})$ is decentralized in the sense that $\boldsymbol{f}(\dot{\boldsymbol{q}},\boldsymbol{\tau})$ depends only on $\dot{q}_i$ and τ_i; that is,

$$\boldsymbol{f}(\dot{\boldsymbol{q}},\boldsymbol{\tau}) = \begin{bmatrix} f_1(\dot{q}_1,\tau_1) \\ f_2(\dot{q}_2,\tau_2) \\ \vdots \\ f_n(\dot{q}_n,\tau_n) \end{bmatrix}.$$

The friction torques $\boldsymbol{f}(\dot{\boldsymbol{q}},\boldsymbol{\tau})$ are assumed to be dissipate energy at all non-zero velocities, and therefore, their entries are bounded within the first and third quadrants. This feature allows to consider the Coulomb and viscous friction common models. At zero velocities, only static friction is present satisfying:

$$f_i(\mathbf{0},\tau_i) = \tau_i - g_i(\boldsymbol{q})$$

for $-\bar{f}_i \le \tau_i - g_i(\boldsymbol{q}) \le \bar{f}_i$, with $\bar{f}_i$ being the limit on the static friction torques for joint i (B. Armstrong-Hoélouvry, 1999)(Armstrong-Hoélouvry, 1991).

It is assumed that the robot links are joined together with revolute joints. Although the equation of motion (1) is complex, it has several fundamental properties which can be exploited to facilitate control system design. For the new controller, the following important property is used:

Property 1. The matrix $C(\boldsymbol{q},\dot{\boldsymbol{q}})$ and the time derivative $\dot{M}(\boldsymbol{q})$ of the inertia matrix satisfy (Vidyasagar, 1989)(Koditschek, 1984):

$$\dot{\boldsymbol{q}}^T \left[\frac{1}{2}\dot{M}(\boldsymbol{q}) - C(\boldsymbol{q},\dot{\boldsymbol{q}})\right]\dot{\boldsymbol{q}} = \mathbf{0} \;\; \forall\, \boldsymbol{q},\dot{\boldsymbol{q}} \in \mathbb{R}^n. \qquad (2)$$

3 A NEW FAMILY POSITION CONTROLLERS

This section presents the new family of controllers and its stability analysis. Consider the following control scheme with gravity compensation given by

$$\boldsymbol{\tau} = \nabla\mathcal{U}(K_p,\tilde{\boldsymbol{q}}) - \boldsymbol{f}_v(K_v,\dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) \qquad (3)$$

where $\tilde{\boldsymbol{q}} = \boldsymbol{q}_d - \boldsymbol{q} \in \mathbb{R}^n$ is the position error vector, $\boldsymbol{q}_d \in \mathbb{R}^n$ is the desired joint position vector, $K_p \in \mathbb{R}^{n\times n}$ is the proportional gain which is diagonal matrix, $K_v \in \mathbb{R}^{n\times n}$ is a positive definite matrix, also called derivative gain, $\mathcal{U}(K_p,\tilde{\boldsymbol{q}})$ represents the artificial potential energy, positive de- finite function, and $\boldsymbol{f}_v(K_v,\dot{\boldsymbol{q}})$ denotes the damping function, which is dissipative function, that is, $\dot{\boldsymbol{q}}^T\boldsymbol{f}_v(K_v,\dot{\boldsymbol{q}}) > 0$.

The **control problem** can be stated by selecting the design functions $\mathcal{U}(K_p,\tilde{\boldsymbol{q}})$ and $\boldsymbol{f}_v(K_v,\dot{\boldsymbol{q}})$ such that the position error $\tilde{\boldsymbol{q}}$ and the joint velocity $\dot{\boldsymbol{q}}$ vanish asymptotically, i.e.,

$$\lim_{t\to\infty} [\tilde{\boldsymbol{q}}(t),\ \dot{\boldsymbol{q}}(t)]^T = \mathbf{0} \in \mathbb{R}^{2n}.$$

Proposition. Consider the robot dynamic model (1) together with the control law (3), then the closed-loop system is globally asymptotically stable and the positioning aim $\lim_{t\to\infty}\boldsymbol{q}(t) = \boldsymbol{q}_d \wedge \lim_{t\to\infty}\dot{\boldsymbol{q}}(t) = \mathbf{0}$ is achieved.

Proof: The closed-loop system equation obtained by combining the robot dynamic model (1) and control scheme (3) can be written as

$$\frac{d}{dt}\begin{bmatrix}\tilde{\boldsymbol{q}} \\ \dot{\boldsymbol{q}}\end{bmatrix} = \begin{bmatrix} -\dot{\boldsymbol{q}} \\ M^{-1}(\boldsymbol{q})\left[\nabla\mathcal{U}(K_p,\tilde{\boldsymbol{q}}) - \boldsymbol{f}_v(K_v,\dot{\boldsymbol{q}}) - C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}\right] \end{bmatrix} \qquad (4)$$

which is an autonomous differential equation, and the origin of the state space is its unique equilibrium point.

To carry out the stability analysis of equation (4), the following Lyapunov function candidate is proposed:

$$V(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}}) = \frac{1}{2}\dot{\boldsymbol{q}}^T M(\boldsymbol{q})\dot{\boldsymbol{q}} + \mathcal{U}(K_p,\tilde{\boldsymbol{q}}). \quad (5)$$

The first term of $V(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}})$ is a positive definite function with respect to $\dot{\boldsymbol{q}}$ because $M(\boldsymbol{q})$ is a positive definite matrix. The second one of Lyapunov function candidate (5), which can be interpreted as a potential energy induced by the control law, also is a positive definite function with respect to position error $\tilde{\boldsymbol{q}}$, because terms K_p and K_v are positive define matrixes. Therefore, $V(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}})$ is a globally positive definite and radially unbounded function.

The time derivative of Lyapunov function candidate (5) along the trajectories of the closed-loop equation (4) and after some algebra and considering property 1, can be written as:

$$\begin{aligned}\dot{V}(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}}) &= \dot{\boldsymbol{q}}^T M(\boldsymbol{q})\ddot{\boldsymbol{q}} + \frac{1}{2}\dot{\boldsymbol{q}}^T \dot{M}(\boldsymbol{q})\dot{\boldsymbol{q}} - \nabla\mathcal{U}(K_p,\tilde{\boldsymbol{q}})^T\dot{\boldsymbol{q}} \\ &= \dot{\boldsymbol{q}}^T\nabla\mathcal{U}(K_p,\tilde{\boldsymbol{q}}) - \dot{\boldsymbol{q}}^T\boldsymbol{f}_v(K_v,\dot{\boldsymbol{q}}) - C(\dot{\boldsymbol{q}},\boldsymbol{q})\dot{\boldsymbol{q}} \\ &\quad + \frac{1}{2}\dot{\boldsymbol{q}}^T\dot{M}(\boldsymbol{q})\dot{\boldsymbol{q}} - \nabla\mathcal{U}(K_p,\tilde{\boldsymbol{q}})^T\dot{\boldsymbol{q}} \\ &= -\dot{\boldsymbol{q}}^T\boldsymbol{f}_v(K_v,\dot{\boldsymbol{q}}) \le 0, \end{aligned} \quad (6)$$

which is a globally negative semidefinite function and therefore, it is possible to conclude stability of the equilibrium point. In order to prove global asymptotic stability, the autonomous nature of the closed-loop equation (4) is exploited to apply the LaSalle's invariance principle (Khalil, 2002) in the region Ω:

$$\begin{aligned}\Omega &= \left\{\begin{bmatrix}\tilde{\boldsymbol{q}}\\ \dot{\boldsymbol{q}}\end{bmatrix} \in \mathbb{R}^{2n} : \dot{V}(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}}) = 0\right\} \\ &= \left\{\tilde{\boldsymbol{q}} \in \mathbb{R}^n, \dot{\boldsymbol{q}} = \boldsymbol{0} \in \mathbb{R}^n : \dot{V}(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}}) = 0\right\},\end{aligned}$$

since $\dot{V}(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}}) \le 0 \in \Omega$, $V(\tilde{\boldsymbol{q}}(t),\dot{\boldsymbol{q}}(t))$ is a decreasing function of t. $V(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}})$ is continuous on the compact set Ω, it is bounded from below on Ω. For example, it satisfies $0 \le V(\tilde{\boldsymbol{q}}(t),\dot{\boldsymbol{q}}(t)) \le V(\tilde{\boldsymbol{q}}(0),\dot{\boldsymbol{q}}(0))$. Therefore, the trivial solution is the unique solution of the closed-loop system (4) restricted to Ω, then it is concluded that the origin of the state space is globally asymptotically stable.

Remark I. A simple Case of Study: Suppose that the $\mathcal{U}(K_p,\tilde{\boldsymbol{q}}) = \tilde{\boldsymbol{q}}^T K_p\tilde{\boldsymbol{q}}$, and $\boldsymbol{f}_v(K_v,\dot{\boldsymbol{q}}) = K_v\dot{\boldsymbol{q}}$ then the popular controller PD is generated:

$$\boldsymbol{\tau} = K_p\tilde{\boldsymbol{q}} - K_v\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}). \quad (7)$$

Remark II: Next, are presented several controllers, generated all them of the following artificial potential functions:

$$\begin{aligned}\text{if } \mathcal{U}(K_p,\tilde{\boldsymbol{q}}) &= \sqrt{\ln(\cosh(\tilde{\boldsymbol{q}}))}^T K_p\sqrt{\ln(\cosh(\tilde{\boldsymbol{q}}))} \text{ and} \\ \boldsymbol{f}_v(K_v,\dot{\boldsymbol{q}}) &= K_v\tanh(\dot{\boldsymbol{q}}) \text{ then} \\ \boldsymbol{\tau} &= K_p\tanh(\tilde{\boldsymbol{q}}) - K_v\tanh(\dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q})\end{aligned}$$

$$\begin{aligned}\text{if } \mathcal{U}(K_p,\tilde{\boldsymbol{q}}) &= \sqrt{\cosh(\tilde{\boldsymbol{q}})}^T K_p\sqrt{\cosh(\tilde{\boldsymbol{q}})} \text{ and} \\ \boldsymbol{f}_v(K_v,\dot{\boldsymbol{q}}) &= K_v\sinh(\dot{\boldsymbol{q}}) \text{ then} \\ \boldsymbol{\tau} &= K_p\sinh(\tilde{\boldsymbol{q}}) - K_v\sinh(\dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) \\ \text{if } \mathcal{U}(K_p,\tilde{\boldsymbol{q}}) &= \sum_{i=1}^{n} k_{pi}[\tilde{q}_i\arctan(q_i) - \frac{1}{2}\ln(1+q_i^2)] \text{ and} \\ \boldsymbol{f}_v(K_v,\dot{\boldsymbol{q}}) &= K_v\arctan(\dot{\boldsymbol{q}}) \text{ then} \\ \boldsymbol{\tau} &= K_p\arctan(\tilde{\boldsymbol{q}}) - K_v\arctan(\dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}).\end{aligned}$$

Remark III Jacobian Transpose Controller Approach: The position control in the space of Task-Oriented coordinate proposed by (M. Takegaki, 1981) can be address. A minor modification of the Lyapunov function leads to:

$$\begin{aligned}\text{if } \mathcal{U}(K_p,\tilde{\boldsymbol{x}}_r) &= \sqrt{\ln(\cosh(\tilde{\boldsymbol{x}}_r))}^T K_p\sqrt{\ln(\cosh(\tilde{\boldsymbol{x}}_r))} \\ \text{and } \boldsymbol{f}_v(K_v,\dot{\boldsymbol{q}}) &= K_v\tanh(\dot{\boldsymbol{q}}) \text{ then} \\ \boldsymbol{\tau} &= J_A(\boldsymbol{q})^T R(\theta)K_p\tanh(\tilde{\boldsymbol{x}}_r) \\ &\quad -K_v\tanh(\dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q})\end{aligned}$$

where $\boldsymbol{x}_r$ is the position of the robot in Cartesian coordinates, given by the direct kinematics $\boldsymbol{f}$ and $J_A(\boldsymbol{q}) \in \mathbb{R}^{n\times n}$ is a Jacobian matrix, defined from direct kinematics as:

$$J_A(\boldsymbol{q}) = \frac{\partial\boldsymbol{f}}{\partial\boldsymbol{q}}.$$

4 EXPERIMENTAL SET-UP

An experimental system for research of robot control algorithms has been designed and built at The Universidad Autónoma de Puebla, México; it is a direct-drive robot of two degrees of freedom (see Figure 1). The experimental robot consists of two links made of 6061 aluminium actuated by brushless direct-drive servo actuators from Parker Compumotor to drive the joints without gear reduction. Advantages of this type of direct-drive actuator includes freedom from backslash and significantly lower joint friction compared with actuators composed by gear drives. The motors used in the robot are listed in Table 1. The servos are operated in torque mode, so the motors act as a torque source and they accept an analog voltage as a reference of torque signal. Position information is obtained from incremental encoders located on the motors. The standard backwards difference algorithm applied to the joint positions measurements was used to generate the velocity signals. The manipulator workspace is a circle with a radius of 0.7 m.

Besides position sensors and motor drivers, the robot also includes a motion control board manufactured by Precision MicroDynamics Inc., which is used to obtain the joint positions. The control algorithm runs on a Pentium-II (333 Mhz) host computer.

Figure 1: Experimental robot.

Table 1: Servo actuators of the experimental robot.

Link	Model	Torque [Nm]	p/rev
1. Shoulder	DM1050A	50	1,024,000
2. Elbow	DM1004C	4	1,024,000

With reference to our direct-drive robot, only the gravitational torque is required to implement the new family of controllers (3), which is available in (Kelly, 1997):

$$\boldsymbol{g}(\boldsymbol{q}) = \begin{bmatrix} 38.46\sin(q_1) + 1.82\sin(q_1+q_2) \\ 1.82\sin(q_1+q_2) \end{bmatrix} \text{[Nm]}.$$

The vision system consists of a camera with a focal length $\lambda = 0.0048$ [m] and a FPG-44 frame processor board. A black disc was mounted on end-effector, the centroid of disc was selected as the object feature point.

The CCD camera was placed in front of the robot and its position with respect to the robot frame Σ_R was $\boldsymbol{o}_C = [0.15,\ -0.55,\ 0.55]^T$ [m].

5 EXPERIMENTAL RESULTS

To support our theoretical developments, this Section presents an experimental comparison of three position controllers on a planar robot.

We select in all controllers the desired position in the image plane as $[u_d\ \ v_d]^T = [500\ \ 355]^T$ [pixels] and the following initial position $[u(0)\ v(0)]^T = [360\ 400]^T$ [pixels] and $\dot{q}(0) = 0$ [degrees/sec]. The friction phenomena were not modeled for compensation purposes. The evaluated controllers have been written in C language. The sampling rate was executed at 2.5 msec. while the visual feedback loop was at 33 msec.

Figure 2 shows the experimental results of the controller

$$\boldsymbol{\tau} = J_A^T(\boldsymbol{q})R(\theta)K_p \tanh\left(\Lambda \begin{bmatrix} \tilde{u} \\ \tilde{v} \end{bmatrix}\right) - K_v \tanh(\Lambda \dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}). \quad (8)$$

The parameters of this controller were selected as $K_p = \text{diag}\{23.6,\ 3.95\}$ [Nm/pixels^2], $K_v = \text{diag}\{2.0,\ 0.2\}$ [Nm-sec/degrees] and $\Lambda = \text{diag}\{0.1,\ 0.1\}$. Figure 2 depicts the time evolution of feature error vector $[\tilde{u}\ \tilde{v}]^T$. The transient response is fast and it was around 0.6 sec. After transient, both components of the feature position error tend asymptotically to a small neighborhood of zero(3 and -1 pixels, respectively).

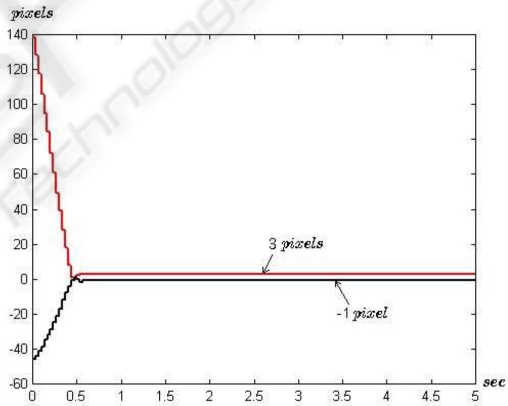

Figure 2: Feature position error trajectory in the image plane for controller (8).

The experimental results for the controller

$$\boldsymbol{\tau} = J_A^T(\boldsymbol{q})R(\theta)K_p \arctan\left(\Lambda \begin{bmatrix} \tilde{u} \\ \tilde{v} \end{bmatrix}\right) - K_v \arctan(\Lambda \dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) \quad (9)$$

are shown in Figure 3. The proportional and derivative gains were selected as $K_p = \text{diag}\{15.0,\ 2.8\}$ [Nm/pixels^2], $K_v = \text{diag}\{2.0,\ 0.1\}$ [Nm-sec/degrees] respectively and $\Lambda = \text{diag}\{0.1,\ 0.1\}$. The transient response is fast and it was around 0.5 sec. The components of the feature position error tend asymptotically to a small neighborhood of zero (2 and -3 pixels, respectively).

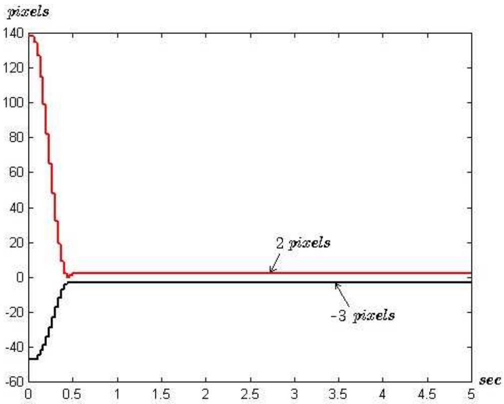

Figure 3: Feature position error trajectory in the image plane for controller (9).

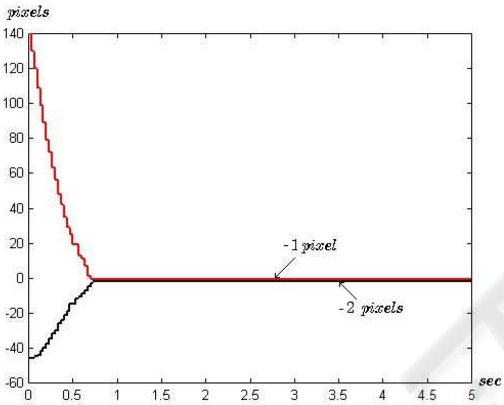

Figure 4: Feature position error trajectory in the image plane for controller (10).

Finally the experimental results for the controller

$$\boldsymbol{\tau} \;=\; \boldsymbol{f}(K_p,\tilde{u},\tilde{v}) - K_v\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) \qquad (10)$$

where

$$\boldsymbol{f}(K_p,\tilde{u},\tilde{v}) \;=\; \begin{bmatrix} f_1(K_p,\tilde{u},\tilde{v}) \\ \vdots \\ f_n(K_p,\tilde{u},\tilde{v}) \end{bmatrix}$$

$$f_i(K_p,\tilde{u},\tilde{v}) \;=\; \begin{cases} f_p(\tanh)_i & \text{if } \; |f_p(\sinh)_i| \geq \sigma_i^+ \\ f_p(\sinh)_i & \text{if } \; |f_p(\sinh)_i| < \sigma_i^+ \end{cases}$$

for all $i = 1, 2, \ldots, n$ and

$$\begin{bmatrix} f_p(\tanh)_1 \\ \vdots \\ f_p(\tanh)_n \end{bmatrix} \;=\; J^T(\boldsymbol{q})K_pR(\theta)\tanh\left(\Lambda\begin{bmatrix}\tilde{u}\\ \tilde{v}\end{bmatrix}\right)$$

$$\begin{bmatrix} f_p(\sinh)_1 \\ \vdots \\ f_p(\sinh)_n \end{bmatrix} \;=\; J^T(\boldsymbol{q})K_pR(\theta)\sinh\left(\Lambda\begin{bmatrix}\tilde{u}\\ \tilde{v}\end{bmatrix}\right)$$

are shown in Figure 4. The gains were selected as $K_p = \text{diag}\{25.85,\ 3.93\}$ [Nm/pixels^2], $K_v = \text{diag}\{0.17,\ 0.018\}$ [Nm-sec/degrees] respectively and $\Lambda = \text{diag}\{0.1,\ 0.1\}$. The transient response was around 0.75 sec. The components of the feature position error tend asymptotically to -1 and -2 pixels, respectively.

To compare the experimental results obtained for the three controllers we use the $\mathcal{L}_2$ norm (Khalil, 2002) of the feature position error. For the controller (8) $\mathcal{L}_2[\tilde{u}\ \tilde{v}]^T = 3.50$ [pixels], then for the controller (9) $\mathcal{L}_2[\tilde{u}\ \tilde{v}]^T = 3.99$ [pixels], finally for the controller (10) $\mathcal{L}_2[\tilde{u}\ \tilde{v}]^T = 2.47$ [pixels]. Therefore the smallest $\mathcal{L}_2$ norm corresponds to the controller (10), then this controller presents the best steady state performance.

6 CONCLUSIONS

This paper has introduced a new family of position control algorithms for robot manipulators. It is supported by a rigorous stability analysis, the theoretical results establish conditions for ensuring global regulation. The simple PD controller can be a particular member of this new scheme when its proportional gain is a diagonal matrix. Applications for Visual Servoing have been shown.

For stability purposes, the tuning procedure for the new scheme is sufficient to select a proportional gain as diagonal matrix and derivative gain as symmetric positive definite matrix in order to ensure global asymptotic stability.

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