A NOVEL ENTROPY METHOD FOR CLASSIFICATION OF BIOSIGNALS

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Abstract: The paper introduces entropy as a measure for 1D signals. We propose as entropy measure the relationship between the crest of the signal (i.e. its portion contained between the absolute minimum and maximum) and the energy of the signal. A linear transformation of 2D signals into 1D signals is also illustrated. The experimental results are compared to several fuzzy entropy measures and other well-known methods in literature. Experiments have been carried out on medical images from a large mammograms database; this choice is due to the high-degree of difficulty of this kind of images and the strong interest in the scientific community on medical images. The capability of the methods was tested in order to discriminate between benignant and malignant microcalcifications.

1 INTRODUCTION

The concept of entropy has been developed in thermodynamics in order to characterize the ability of a system in changing his status. Measures of system entropy are usually functions defined in the phase space and they reach the maximum or minimum value, depending on the contextual definition, whenever system variables are uniformly distributed.

This concept has been borrowed in communication systems for coding purposes and data compression . Entropy based functionals have been also used in image and signal analysis to perform deconvolution and segmentation , to measure the pictorial information and to define image differences .

Several entropy measures, defined on the feature space, have been introduced. The measure of the entropy in image classification is not a new one ; the idea of cross entropy has been used to define the distance which is popularly known as Kullback-Leibler information distance.

However, this distance between two distributions should not be considered as the true distance, because it is not symmetric and does not satisfy the triangle inequality. It may be mentioned that in early sixties connections among statistics, quantum mechanics and information theory have been thoroughly studied by several authors , using Shannon maximum entropy principle. Caianiello proposed that such a connection can be obtained in the natural meeting ground of geometry. In the following we present two classes of entropic measures: the fuzzy entropy and the Vitulano's entropy .

The choice to use mammograms is due to the development of new imaging methods for medical diagnosis that has significantly widened the scope of the images available to physicians.

The problem is to realize a mapping from the set of all possible images (image space) to the set – usually smaller – of all possible features value (feature space).

2 RELATED WORKS

Solutions proposed in literature follow different approaches and emphasize different aspects of the problem. In several CAD methods applied to mammography feature enhancement is carried out by evaluating the results due to wavelet transforms.

The general approach consists of multiple steps:

- computation of the forward wavelet transform of the image;
- nonlinear transformation or adaptively

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weighting of the wavelet coefficients;
- computation of the inverse transform.

A number of techniques have appeared in literature; differences among these approaches are related to the types of decomposition and reconstruction parameters taken into account.

Our contribution to the analysis and CAD methods is to introduce an entropy measure of the signal in order to cluster microcalcifications in mammograms.

3 METHODS

As previously stated, two different entropy measures are introduced and their properties outlined: fuzzy and Vitulano's method.

Fuzzy entropy method may be summarized as follows:

- computation of the fuzzy-entropy; fuzzy entropy is function of the distribution of the pixel grey levels. The entropy measures characterize the difference of the minimum intensity values distribution with respect to the mean intensity;
- application of a Bayesian classifier in the new feature space.

The Vitulano's entropy method requires almost three different steps:

- selection of nine disjoint Regions of Interest in a mammogram;
- transformation of the related 2D signal into a 1D signal;
- computation of the entropy features

3.1 Fuzzy entropies

In this study four different types of fuzzy entropies are introduced. The term "fuzzy" is due to their characteristic to satisfy some of the formal properties of the classical entropy (introduced by Shannon); even if they are computed on image features that are not probabilities.

The input image, $\boldsymbol{f} \equiv (f_1,\ldots,f_n)$, is represented as a linear signal after the transformation from raster to spiral indexing.

Moreover, we define the vector $\boldsymbol{h} \equiv (h_1,\ldots,h_n)$, where

$$h_i = \begin{cases} m - f_i & m > f_i \\ 0 & otherwise \end{cases}$$

and $m = mean(\boldsymbol{f})$ *for* $i = 1,\ldots n$

that represents all values that are below the mean value, m.

Starting from these definitions we define the following measure of fuzzy entropies:

$$G_0 = \frac{1}{\log 2} \times \left(-\eta \times \log(\eta) - (1-\eta) \times \log(1-\eta)\right) \quad (1)$$

$$G_1 = \frac{2\sqrt{e}}{e-1} \times \left(\eta \times e^{1-\eta} - \eta \times e^{\eta-1}\right) \quad (2)$$

$$G_2 = 4 \times \eta \times (1-\eta) \quad (3)$$

$$G_3 = -\frac{1}{\log n} \sum_{i=1}^{n} h_i \times \log h_i \quad (4)$$

with

$$\eta = \frac{1}{n} \sum_{i=1}^{n} |h_i|$$

or

$$\eta = \sqrt{\frac{1}{n} \sum_{i=1}^{n} h_i^2}\ .$$

It is noteworthy that η gives a measure of the distance between the constant function f=m and the function $\boldsymbol{h}$. Moreover, all these measures of entropy are convex and their values range in the interval [0,1]. The maximum of G_i (for i=0, 1 2) is equal to 1 and is reached for η=0,5 while the maximum of G_3 is reached for $h_i=n^{-1}$ and it is also equal to 1. Further details concerning fuzzy entropies here introduced can be found in Caianiello (see References).

3.2 The Vitulano's method

There are different methods meant to read the information contained in a digital image: in rows, in columns, or by recurring to specific paths. The choice of the scansion method is connected to the type of information that somebody wants to pick out from the image (e.g. a certain recurrence in a direction, the search of the points of maximum or minimum of the surface image in order to carry out the histogram, the time of the calculus etc).

Mapping a signal from 2D into 1D space is also one of the main step of our method; the use of the spiral method allowed us to perform the expected target (connected pixels, set of pixels that locate the regions of the image etc.).

We define: $A_{m,n}$ as the domain of the surface image where (m, n) are respectively the number of rows and columns of A.

Only out of simplicity of expression, we place m=n, i.e. A is a square matrix.

Definition 1

We define crown of the matrix C_1 the set of the pixels

$$C_1=\{a_{1,1}\ldots a_{1,n};\ a_{2,n}\ldots a_{m,n};\ a_{m,n-1}\ldots a_{m,1}\ ;\ a_{m-1,1}\ldots a_{2,1}\} \quad (5)$$

that is, the order set of pixels contained in the row m=1 of the matrix, in the n-th column, in the m-th row, in the first column except the pixel, $a_{1,1}$ since it is already contained in the first row.

Let P is a discrete mono dimensional signal, so that:

$P(x) = Px_1, Px_2, \ldots.., Px_i, \ldots Px_k$

Definition 2

Therefore, we define first differential

$\Delta^1 Px_i = Px_{(i+1)} - Px_i$ (6)

Definition 3

We define second differential:

$\Delta^2 Px_i = \Delta^1 Px_i - \Delta^1 Px_{(i-1)}$(7)

If we substitute the values $\Delta^1 Px_i - \Delta^1 Px_{(i-1)}$

$$\Delta^2 Px_i = \Delta^1 Px_i - \Delta^1 Px_{(i-1)} = Px_{(i+1)} - Px_i - (Px_i - Px_{(i-1)}) =$$

$$= Px_{(i+1)} - 2Px_i + Px_{(i-1)}$$

It is easy to verify that for every pixel belonging to a crown of the matrix, the second differential assumes value 0.

It is straightforward that considering three pixels, belonging to a crown, the relation (7) assumes value 0, and they are 4-connected with respect to the central pixel.

If we suppose $A_{m,n}$ a bidimensional signal and $C_1, \ldots\ldots, C_k$ the crowns contained in its domain, we define joined spiral to the signal Am,n the relation:

$$T = \underset{i=1,k}{U} C_i \quad (8)$$

where C_i is the i-th crown obtained from the matrix $A_{m,n.}$

It is important to observe that the relation (8) realizes a linear reversible transformation of a generic signal in a space 2-D in a signal in a space 1-D.

Therefore it follows:

$$A_{m,n} \underset{S^{-1}}{\overset{S}{\rightleftharpoons}} T_{mxn} \quad (9)$$

Due to (9) a one to one application is established between each of the elements tk belonging to T with each of the pixel $a_{i,j}$ of the matrix $A_{m,n}$.

The transformation S maintains the information regarding the form and the dimensions of the image domain, the topological information such as the number of the objects and their position, the area and the outline of the objects, etc.

For example, we assume t_k as the element to which corresponds the pixel $a_{i,j}$, so the pixels 4-connected to $a_{i,j}$, correspond to the elements in T for which the condition (10) is satisfied

$\Delta^2 t_n = 0$ or $\Delta^2 t_n = 8$ (10)

The pixels of a object in A are 4-connected, so :

The area of the object is given by the set V, whose elements satisfy the relation (10);

the contour of the object is given by the subset $V^1 \in V$ whose elements contain almost a ground pixel among its 8-connected pixels;

From the elements belonging to the set V^1 we are able to extract the following information:

- topological information – from the abscissa of a point belonging to *T*, we obtain the indexes of rows and columns of the pixels related to *A;*
- the shape of the object – from the elements related to $V^1 \in V$, we are able to describe the shape of the object contained in *A;*
- shape 3-D of the object – for each of the elements related to *V*, we compute: its location in the domain of A (index of row and column) and its grey level. So it is possible to have both the information over the 3-D shape and to reconstruct *V* pixel by pixel.

In a previous work , we have proposed the HER (Hierarchical Entropy based Representation) method, as the algorithm meant to realize the information retrieval from a multidimensional database.

Briefly the relevant point about HER that a 1-D signal, T, may be represented by a string F, such: that:

$T \approx F = m_1 e_1\ ;\ m_2 e_2\ ;\ldots\ldots..\ ;\ m_k e_k$ (11)

where $\{m\} = m_{1,\ldots\ldots,}\ m_k$ are the maxima extract in a hierarchical way from *T*;

and $\{e\} = e_{1,\ldots\ldots,}\ e_k$ are the energies associated to the maxima $m_i \in \{m\}$

Let's suppose a 1-D signal T, where m and M corresponding to the absolute minimum and maximum of T and ET its total energy .

It is important to underline that $\overline{m}$ and M aren't either the smaller or the bigger of the ordinate of the points of T, but the minimum and the maximum of the signal T (in a mathematical sense).

We define signal crest, C, the portion of the signal T between $\overline{m}$ and M.

In other words, the signal crest is obtained by placing the zero of the axes of ordinates equal to $\overline{m}$.

We assume Ec the energy of the crest signal C.

We apply to the signal C the method HER , obtaining $\{m_k\}$ and $\{e_k\}$ as the maxima and the energies of C respectively.

Let $e_i \in \{e_k\}$ the energy associated to the maximum $m_i \in \{m_k\}$

Definition 4

We define entropy of the signal T the relation:

$$S_T = \frac{\sum_{i=1}^{K} e_i}{E_T} \qquad (12)$$

It is straightforward that both the entropy of a constant signal (constant value of the function) and of a monotone signal (constant derivative) is equal to zero.

On the other hand the entropy equal to 1 corresponds to the maximum degree of disorder, i.e. there are not two points (x_i, x_{i+1}) in the signal domain that have the same ordinate.

4 EXPERIMENTAL RESULTS

An application on breast cancer mammograms was carried out in order to compare the behaviour of the different entropy measures above introduced.

4.1 Experiment with fuzzy entropies

Table 1 shows the mean values, μ, and the standard deviation, σ, of the distributions of G_0, G_1, G_2, G_3 and for the two classes of malignant (MM) and benignant (MB) microcalcification.

The result indicates that in the average the entropy of classes MM is greater than the entropy of class MB. The four measures have been used as a new features space, allowing a better discrimination among these classes of diseases.

Table 1

	μ_{MM}	σ_{MM}	μ_{MB}	σ_{MB}
G_0	3.25	0.79	1.48	0.78
G_1	12.47	0.91	9.20	3.00
G_2	2.00	0.55	0.94	0.45
G_3	15.00	1.10	11.70	1.70

4.2 Experiments with the entropy method

Because of the strong interest in the detection of microcalcifications in mammograms , we decided to test the Vitulano's method with this kind of signals.

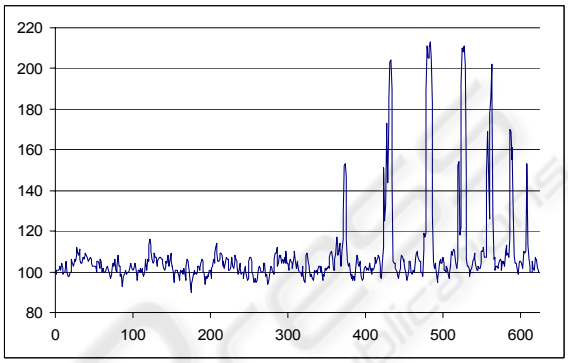
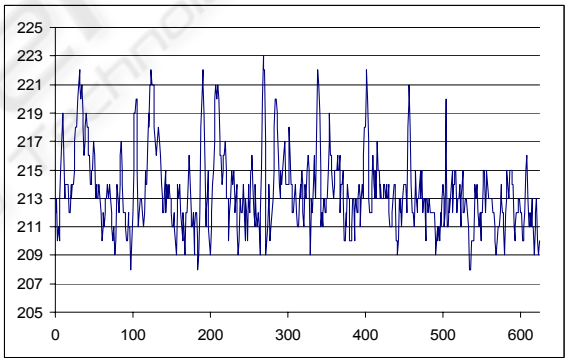


Figure 1a and b: Representation of the signal (obtained after the application of the spiral path) related to the parenchymal tissue for a benignant (top) and for a malignant (bottom) microcalcification.



The analysis of the signals of the microcalcifications highlights that for the malignant cases (Figure 1b) the impulses are characterized by a small amount of energy (impulse area), a significant shape and a remarkable value of the entropy in the bottom of the signal if compared to the signals of the benignant ones (Figure 1a).

The results are summarized on Table 2

Table 2: Results due to the Vitulano's method when applied to microcalcification

Method	Malignant		Benignant	
	# Errors	Percentage	# Errors	Percentage
Fourier	70	51%	62	51%
Vitulano's Entropy	2	98,4%	3	97,6%
HER	6	96%	5	96%
Fuzzy Entropy	22	85%	6	95%

5 DISCUSSION AND FINAL REMARKS

The experimental results show the role of the entropy in perception. In particular, the use of the Fourier transforms, wavelets and high pass filers do not show good performance unless of ad hoc tuning of given parameters. In fact, the shape, the power spectra or the approximation degree of the polynomial are not characteristics due to the nature of the signal.

For our purpose the degree of disorder (entropy) of the image is an important indicator; in fact the texture disorder (parenchymal tissue structures) in the suspicious region of the image represents a significant component for a physician in the diagnosis of malignancy or benignancy.

The methods proposed in this work get the guide reasons by observing that, when a malignant lesion comes up, not only it causes alterations in the parenchyma, but also increases its level of disorder.

The experimental results are shown in Figure 2; we selected 175 images corresponding to a benignant or a malignant microcalcification.

For each sample image we applied HER, by using the 70% of the crest energy; we wish to underline the fact that the results don't change significantly by assuming the 50% or 90% of the crest energy.

The graph of Figure 2 shows clearly two disjoined classes, corresponding to malignant and benignant microcalcifications.

The analysis of the results show that the number of the extracted maxima is bigger for a malignant lesion, but the energy value associated with each of the maxima is higher for a benignant microcalcification.

The comparison between the two signals shown in Figure 1 reveals that even if the number of the maxima is higher in the malignant lesion, the global value of the associated energy lessens with respect to the benignant case. By recurring to the principle of entropy-disorder we may conclude by saying that signals related to malignant microcalcifications are characterized by a bigger amount of the entropy with respect to benignant ones.

In other words, we feel that the alterations concerning the same tissue, can be a valid measure or an increasing of the malignancy of the lesion.

It is remarkable that the entropy measures of the signal do not require a large amount of operations, therefore it is less computational time consuming

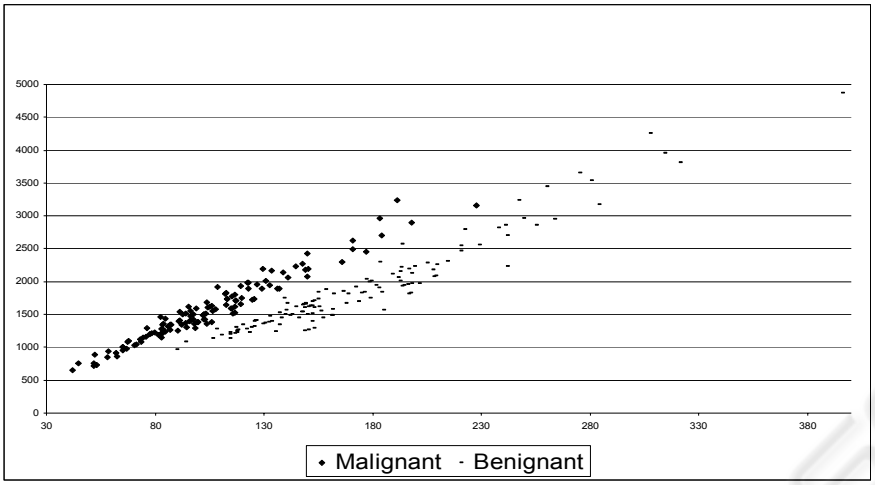


Figure 2: Graphical representation of the two classes of benignant and malignant microcalcifications.

REFERENCES

F. Kossentini, M.J.T. Smith, C.F. Barnes, 1995. Image coding using entropy-constrained residual vector quantization. In IEEE Transactions on Image Processing ,4 , pp.1349-1357

V.Di Gesù, S.Roy, 2000. Fuzzy measures for image distance, in proc of Advances. In Fuzzy Systems and Intelligent Technologies , F.Masulli, R.Parenti, G.Pasi Ed.s, Shaker Publishing , pp.156 -164.

C. Di Ruberto, M. Nappi, S. Vitulano, 1998. Different Methods to Segment Biomedical Images. In Pattern Recognition Letters, 18, nos. 11/13, pp. 1125-1131.

Qian Zhao et alt., 2004. Multi-resolution source coding using Entropy constrained Dithered Scalar Quantization. In Proc. of DCC'04 IEEE.

M. Toews, t. Arbel, 2003. Entropy of likelihood feature selection for image correspondence. In Proc. ICCV'03 IEEE.

Melloul M., Joskowicz L., 2002. Segmentation of microcalcification in X-ray mammograms using entropy thresholding. In CARS 2002, H.U.Lemke et al. Editors.

Kullback S.C., 1959. Information Theory and Statisitcs, New York, N.Y.

E.T.Jaynes, 1962. In Brandies Theor.Phys.Lectures on Statistical Physics, Vol. 3, New York, N.Y.

E.R.Caianiello, 1983. Theories:Lett.Al Nuovo Cimento, Vo.38, 539.

A.Casanova, S.Vitulano, 2004. Entropy As a Feature In The Analysis And Classification Of Signals. In Series on Software Engineering and Knowledge Engineering, N.15 - World Scientific ISBN 981-256-137-4.

Heath M. et alt., 1998. Current status of digital database for screening mammography, (Kluwer Academic Pub.), pp. 457-460.

W.Richardson, 1992. Non linear filtering and multiscale texture discrimination for mammograms. In Mathematical Methods Med. Imag., Proc. SPIE 1768 pp. 293-305.

A. Laine et alt., 1992. Multiscale wavelet representation for mammography feature analysis. In Mathematical Methods Med. Imag., Proc. SPIE 1768 pp. 306-316

W.Qian et alt., 1993. Tree structured non linear filter and wavelet transform for microcalcification segmentation in mammography. In Biomedical Iimage Processing and Biomedical Visualization, Proc. SPIE 1905 pp. 509-520.

Y. Yoshida et alt., 1994. Automated detection of clustered microcalcifications. In digital mammograms using wavelet processing techniques in Medical Imaging, Proc. SPIE 2167pp.868-886.

R.N. Strickland, H.Hahn, 1996. Wavelet Transforms for detecting microcalcifications. In Mammograms IEEE Trans. on Medical imaging, Vol.15, No.2

A.Casanova, V.Savona, S.Vitulano, 2004. The Role Of Entropy. In Signal Analysis And Classification: An Application To Breast Diagnosis Medicon2004, Ischia.

A.Casanova, M.Fraschini, 2003. HER: Application on Information Retrieval. In Series on Software Engineering and Knowledge Engineering Vol. 15: 150-159 ISBN:981-238-587-8.

Vitulano S. et alt,. 2000. A hierarchical representation for content-based image retrieval. In Jour. Of Visual Lang. and Comp., 5, pp. 317-326.

I.W. Bassel, ,1992. Mammographic analysis of calcification Radiol. Clin.No.Amer. , Vol. 30 pp 93-105.

M. Heath, K.W. Bowyer, D. Kopans et alt., 1998. Current status of the digital Database for Screening Mammography. In Digital Mammography, Kluwer , pp 457-460 Academic Pub.

H. Barman et alt., 1993. Using simple local Fourier domain models for computer-aided analysis of mammograms. In Proc. 8th Scandinavian Conf. Image Anal. Norway pp. 479-486.