A SMOOTHING STRATEGY FOR PRM PATHS
Application to 6-axes MOTOMAN manipulator

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Abstract: This paper describes the use of the probabilistic motion planning technique SBL "Single-Query Bidirectional Probabilistic Algorithm with Lazy Collision Checking" or in motion planning for robot manipulators. We present a novel strategy to remedy the PRM "Probabilistic Roadmap" paths which are both excessively long and velocity discontinuous. The optimization of the path will be done first through Coarse optimal lazy A* optimization and then through a Fine cutting-triangles-edge one, the edges discontinuities are smoothed with cubic polynomials taking the robot's specific Dynamic and Cinematic constraints. The whole strategy is applied to the 6 axes robot Manipulator MOTOMAN SV3X.

1 INTRODUCTION

The motion planning problem has been in a great difficulty solving path planning for real industrial robot manipulator which are usually 6 or above degrees of freedoms (dofs). The traditional techniques (Latombe, 1991) can hardly solve problems until 4 or 5 dofs, while its known to be definitely impossible for 6 dofs in 3D space.

The introduction of random techniques in roadmap generation (Kavraki, 1996) has greatly simplified the planning problem, theirs greatest advantages being, easiness of understanding, implementing, they can deals with complicated scene and can solve problem with high number of dofs.

Their idea consists of randomly sampling the configuration space (Lozano-Péréz, 1979) of the robot in order to capture the connectivity of free space, static configurations are tested using fast hierarchical collision checking algorithms (Larsen, 1998), the configurations are then connected with a simple but very fast local planner to obtain the so called Roadmap. Roadmap techniques are generally classified in two branches, the Multiple Query and the Single Query, for the first one a roadmap is pre-calculated and used later for multiple queries, the second one does not precompute such roadmap but it explores the free space starting from given initial configuration in search for a final one, the constructed graph is valid only for one query. but PRM generated paths are know to be discontinuous and far from optimal.

Among all PRM variants, we have chosen the SBL (Sanchez-Ante, 2001) because it is the most successful one due to its speed and behaviour even in cluttered environment. Still, the path generated is far from optimal due to the fact that the algorithm generates nodes randomly and diffuses them homogeneously in the free space, which is not necessarily the shortest path. The SBL is called:

-Single query: the network is recalculated for each start/goal pair.

-Bidirectional: because the network construction is started simultaneously from the start and goal configuration.

-Lazy collision: because the collision checks are delayed until they are extremely needed (Bohlin, 2000).

Our work consists at optimizing the Raw PRM path in some sense by modifying the PRM path; this path is the optimal among those using only the nodes on the raw path and connected with straight line in C-space. Then using a fine optimization scheme we optimize further the path from the previous step, finally we smooth the sharp corners with cubic polynomial taking the robot specific dynamic and kinematic constraints. The result is an optimized smooth trajectory ready for execution.

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2 REALTED WORKS

The use of smoothing on paths was already suggested in 2D workspaces but less in 3D workspaces. In (Quinlan, 1993) the authors represent the path as an elastic band under tension forces to pull the path tight. Repulsion forces from the obstacles are added to keep the path from "hugging" the obstacles. B-splines are used in (Foley, 1990) (Kant, 1987) to construct smooth paths from a cell decomposition of the free space. Cubic splines are also used in (Tondu, 1999). The use of smoothing techniques on PRM paths has been already proposed, the authors often use shortcutting segment or parts of segment methods already used in (Laumond, 1990) (Berchtold, 1994). In most of the cases after such smoothing, paths are shorter but still not optimal in any sense and discontinuous in velocity.

3 COARSE OPTIMIZATION

Given a graph of nodes relating the initial and final nodes, many search technique are available to obtain the path composed of a sequence of nodes relating the start and goal nodes.

The A* algorithm obtains the optimal path out of the graph of nodes without the need to explore the whole graph, this is done is by guiding the search with a cost function describing heuristic rules.

The evaluation function $\hat{f}$ is defined such that its value $\hat{f}(n)$ at any node n is an estimate of the minimum cost path constrained to go through n. This is calculated using an estimate of the cost of the minimal path from the start node to node n plus an estimate of the cost of the minimal path from node n to the goal node.

$$\hat{f}(n) = \hat{g}(n) + \hat{h}(n)$$

It is straightforward to calculate a value for $\hat{g}$ by summing the arc costs of the path already found from the start node to node n. Finding a value for $\hat{h}$ is not so simple and heuristic information obtained from the graph has to be relied upon. For example, the straight line distance from node n to ng can be used as the value for $\hat{h}$. If $\hat{h}$ is an underestimate of h then A* is guaranteed to find the minimum cost path and the algorithm is said to be admissible. A proof is given in (Doyle, 1995).

In the usual A* and graph searching algorithms require a graph containing all admissible edges or segment between nodes, whereas we have only edges between consecutive nodes of the PRM path so we will need to construct a graph out of the path nodes by testing the straight line segments between each pair, this may be costly especially in complicated scene where the number of triangles is extremely high. In order to avoid collision testing all edges, we propose to "lazify" the A* algorithm.

3.1 Postponing Collision Checks

The idea is delay segment collision testing until it is "necessary", the necessity in our case is when a node is sorted to be placed in the optimal path.

In the beginning all pair of nodes are assumed to have and admissible edge joining them except for the (n_s,n_g) as we know it is not admissible (this trivial solution should have seen tested before the launching the planner)

Initially a node can be expanded to all other nodes in the graph, except for the initial node that cannot be expanded to the goal node.

In traditional A* the use pointers to specify relation between nodes resulting from a certain node expansion, here we use a parent-child notion, each node expand as a father of the resulting child node.

Also, each of the raw PRM path nodes can be expanded to all other nodes with two exceptions:

- The starting node does not expand to the goal one.
- The goal node does not expand to any node.

Lazy A* algorithm

1. insert start node n_s in a list called OPEN
2. *While*
3. *n ← SORT*
4. *if TEST(n)* is true *then*
   4.1. *if n* is the goal node *then* terminate the algorithm & use pointers to extract optimal path *else EXPAND(n).*

   else REALOCATE(n)
5. *end while loop*

The OPEN that will hold all nodes to be expanded; nodes are added and removed in each iteration. The While loop is an indefinite loop, whereas standard A* the condition is the OPEN list not empty, this was so because we are sure that the algorithm will find a solution before OPEN is empty, the loop is terminated once the solution is found

3.2 List Sorting

The SORT procedure will return the node with minimum cost path constrained to go through this later in the OPEN list, this also means that we do not sort all the OPEN list in any given order., the minimum cost node n is also removed from OPEN.

3.3 Testing segments

Algorithm for TEST

1. *let* n_p the parent of node n
2. *if* edge E(n_p, n) belong to PRM path *return* true
3. *if CLEAR(E(n,n_p)) return* true
4. *return* false

The TEST(n) is an algorithm that will test if the segment relating the current node n to its father node (the one which has ENGENDERED n in OPEN) is clear or notIf the segment joining node n and its parent node belongs to the raw path generated by RPM planner then the segment is declared safe (there is no need to test the segment since it is have been tested during PRM planning,), otherwise the segment is tested for collision.

3.4 Reallocating a node

If the segment relating node n to its parent node happens to be colliding, then reallocated will try to add n as a child to another node already in CLOSED list. Among all nodes in CLOSED that have not already been tested with n, n will be added to the one with minimum cost path constrained to go n.

If there are no such node in CLOSED then n will remain "orphan" and out of the OPEN list until it is inserted back by some node.

Algorithm for REALLOCATE

1. for all nodes in CLOSED that have not been already tested with n
   1.1. choose node n_p with minimum cost path constrained to go through n
   1.2. set n_p as the father of node n
2. if a node n_p was found then insert node n in OPEN

3.5 Node Expansion

The EXPAND procedure is responsible for inserting a node's children to OPEN taking care the following conditions:

- If a child is already in CLOSED or in OPEN it will not be added to neither of the lists.
- If a child node is already in OPEN and its current cost path is higher than the cost of going through the current parent node n, this later is set as its new parent with the cost updated to the new one.

The cost function is usually the distance from initial and final configurations criterion, the cost of the path through a stain number of nodes is the sum of the costs of individual segments or edges costs. We could choose the Euclidian norm of the edge, or more generally a weighted Euclidian norm

4 FINE OPTIMIZATION

Here we use a simple *cut-triangle-edge* scheme, many variant are found in (Berchtold, 1994); for each two consecutive segments E(n_1,n_2) , E(n_2,n_3) , we try to cut the triangle edge represented by $n_1n_2n_3$ (figure 1) with a segment E(n',n") where n', n" are at the middle of segment E(n_1,n_2) , E(n_2,n_3) respectively .This techniques will have two advantage:

1- Smooth the path by replacing the sharp angles with wider ones.
2- Optimize the path in any norm (triangle's inequality

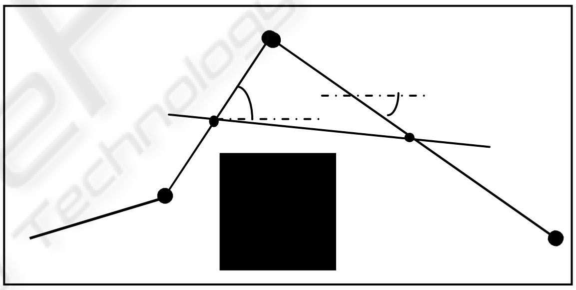


Figure 1: cutting-triangle-edge technique

Cutting-tiangle-edge Algorithm

1. Input path T
2. For each segment E(n_i, n_{i+1}) of path T ,i=1, m-1
   2.1. Calculate node n_i' at the middle of segment E(n_i, n_{i+1})
3. For each segment E(n_i',n'_{i+1}) , i=1, m-2
   3.1. If segment E(n_i',n'_{i+1}) is Clear
      3.1.1. replace segments E(n'_i, n_{i+1}) and E(n_{i+1}, n'_{i+1}) in T with E(n_i',n'_{i+1})
4. Return T

5 CUBIC SMOOTHING

Since the PRM path are inherently peace-wise curves. The transition from one line segment to the next induces an abrupt change in direction. The robot cannot execute the corresponding trajectory without coming at rest at each of these points. In order to avoid this, we should have a continuous joint velocity. So we have to smooth path , we will accomplish this taking care to modify the least possible the original one, to achieve this , we smooth the corners using a polynomial with a given degree and parameters.

First and second order polynomial are easy to calculate but they fit poorly, fourth order and higher polynomials provides seamless fit but they are very difficult if not impossible to calculate, that is why we believe the compromise would be to choose the *cubic polynomial* (Brock, 1999). The parameters are not complicated to calculate, the curve has C^2 continuity and it enables tighter approximation to the turn compared to quadratic polynomial. We will here explain the procedure for a single joint; the same applies to all remaining joints. The joint position turn is governed by the following cubic polynomial:

$$q(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \qquad (1)$$

The duration *d* of the cubic turn (Figure 2) is thus :

$$d = \frac{2|\Delta \dot{q}|}{\ddot{q}_{\max}} \qquad (2)$$

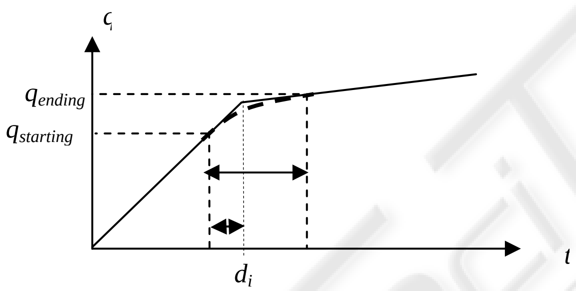


Figure 2: cubic smoothing.

$\Delta \dot{q}$ is the change in velocity of the two consecutive segments. The detailed description of the calculation of the start and the end configurations of the cubic turn and the cubic parameters are found in (Brock, 1999). The Cubic turns are tested for collision by sampling the polynomial to a number of configuration m linearly separated by a distance ε sufficiently small, if all m configurations are collision free than the cubic turn is collision free.

6 SIMULATION RESULTS

SBL planner, smoothing techniques are written in C++, the animation and visualization API was also implemented in C++ using COIN visualization library and its binding SoQt.

The tests were run on a 1.7GHZ Pentium 4 PC with 128 Mbytes of main memory running Linux OS. We have constructed a 3D CAD model for the 6 axes MOTMAN SV3X manipulator.For the PRM planner, the maximum number of milestones allowed to be generated is 10000 and the distance threshold ρ for the segment discretization was set to 0.015. We have simulated a robot Gross movement from initial configuration to a final configuration in the scene shown in figure 3a, 3b respectively. Running the SBL planner for the given query gives the results shown in table 1.

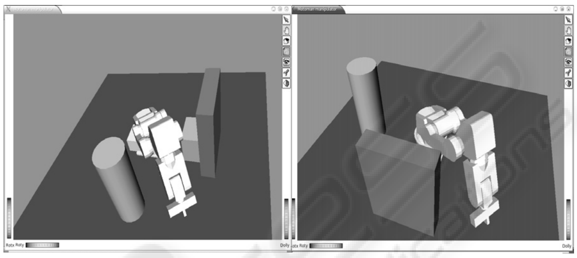

(a) (b)
Figure 3: configurations, (a) initial, (b) final .

Table 1: result of SBL planner query for the given example.

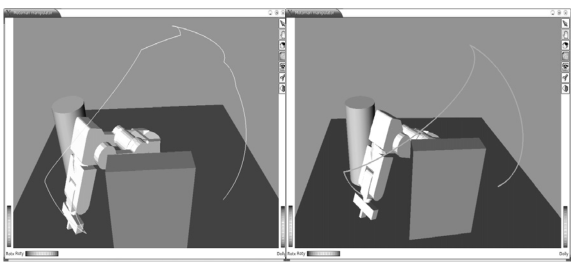
Total -time (s)	#nods In trees	#nodes In path	#total CC	#CC in path	# sampled nodes	total CC time(s)
0.39	55	9	448	123	136	0.38

Here the generated raw path is a 7 node path, figure 4a shows path traced by a point on the end effector along the SBL path. It is apparent that the path is lengthy and requires optimization.

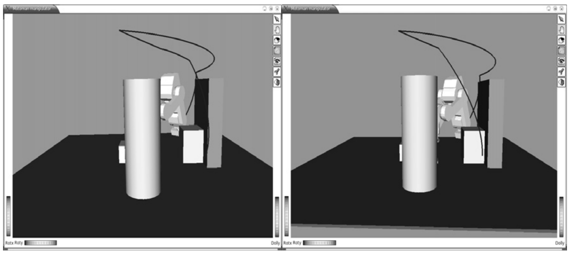
6.1 Lazy A* application

Using a simple Euclidian path cost, the path is shown on figure 4b with a cost of 6.66959.

If we place a weight of 10 on the shoulder axis weighted Euclidian norm, while we put a weight on only one, this lead to more Minimization of Euclidian distance along that axis,. The path is shown in figure 5a and its cost being 9.58155.


(a) (b)
Figure 4: (a) raw trajectory, (b) Euclidian norm A* path.

L_∞ norm minimization: In this example we penalize the maximum joint movement of the edge between two nodes; the generated path is shown in figure 5b with a cost of 5.7734.



a) b)

Figure 5: a) lazy A* with shoulder axis penalty path; b) L_∞ minimization lazy A* path.

6.2 Lazy A* versus A*

In order to evaluate the efficiency of the lazy A* with the standard A* we have run the SBL planner for 10 times for the same query, hence 10 different path have been generated. We have calculated the number of collision checks required for each technique in order to find the optimized path in some sense. The results are:

- average Collision checks for Lazy A*: 107.7
- average Collision checks for A*: 823.4

The ratio of the two averages is ≈1/8

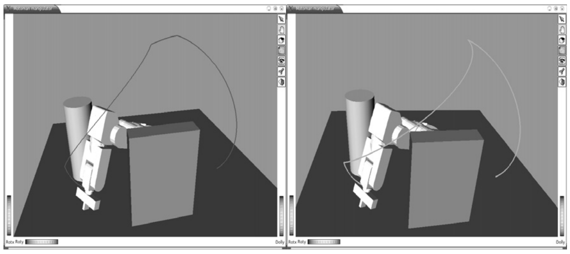
- Lazy A* time is = 0.25 s
- A* time is = 0.95

Using scenes that are extremely complicated will favour more and more the use of the lazy A* algorithm.

6.3 Lazy A* versus random shortcutting

Random shortcutting (Laumond, 1990) segments techniques are frequently used to reduce the number of segments on the path, but the resulting path is not necessarily optimal in any sense.

Figures 5a and 5b show the result of the application of the Lazy A* and the Shortcutting method respectively, in the chosen Euclidian norm, only the A* managed to find the Optimal path, the cost function being 6.3212, the cost for the Shortcutting method is 7.7305.



a) b)

Figure 6: a) shortcutting path. b) Lazy A* path.

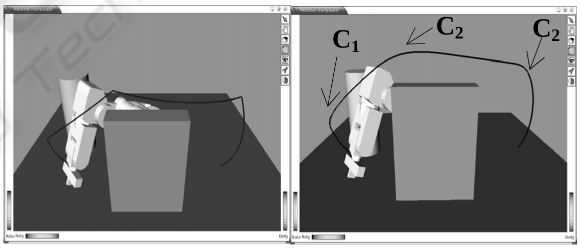
6.4 Cutting-triangle-edge application

We apply this technique on the optimal A* path (figure 4b), it is clear that the application of this technique will not dwarf any optimization gained from the A* because it only shortcut parts of segments which is more optimized in any norm used (this is the triangle' inequality which is a condition of normality).

The results are:

- the algorithm managed to cut all segment
- the cost of path generated is : 5.77343
- the time taken is : 0.05 seconds

Figure 7a shows the generated path.



a) b)

Figure 7: a) cutting-triangle-edge path: b) cubic smoothing path.

6.5 Cubic smoothing and trajectory generation

Up to now, we where dealing with C-space *spatial paths*, in order to execute the path parameterized with time or simply the trajectory, we need to associate joint velocities with each segment of the path. It is clear that if we require the robot to follow a C-space straight line segment of the path, then we should synchronize all joints with constant velocities so that the joint movement is linear.

What we have done is chose a velocity *v* that is smaller or equal than any of the physical maximum joint velocities. v is fixed for all segments making the path. Using v and the maximum joint displacement in the given segment, we calculate the segment duration d_i, d_i is used to calculate the joint velocities for the remaining 5 joints. In this way, we guarantee our piece-wise path following and prevent assigning later joint velocities that exceed actual ones.

Choosing v to be $0.10s^{-1}$, we calculated the 6 joints velocities and durations for all 4 segments making the path issued from the fine optimization step (figure 7a).

The total duration of the trajectory is 9.6246 seconds. Using this trajectory along with typical maximum normalized acceleration of $0.20s^{-2}$ we calculated the 3 cubic durations and polynomial's parameters. Table 2 shows the calculated cubic durations for the three corners smoothed.

Figure 7b shows the path with smoothed corners.

Table 2: cubic smoothing durations

		q_1	q_2	q_3	q_4	q_5	q_6
c_1	d(s)	0.1851	0.3350	0.1122	0.0125	0.1466	0
c_2	d(s)	0.2867	1.3977	0.1125	0.0462	0.1749	1.0765
c_3	d(s)	0.4878	0.1580	0.4982	0.1064	0.5288	0.9235

The cubic duration that is null means no cubic turn is needed due to the velocities of the two consecutive segments being equal. It is clear that the total trajectory duration can be reduced by using higher joint velocities, but the cubic deviation will be consequently higher.

7 CONCLUSION AND FUTURE WORKS

The results show that the PRM paths can be optimized through any criterion thought the Lazy A* algorithm instead of blind shortcutting techniques, the proposed lazy A* calculate the optimal path with minimum collision checks compared to standard A*, the remaining edges are smoothed thought cubic polynomial resulting in minimum deviation from original path, the final trajectory is an optimized smooth trajectory ready for execution.

We are implementing the approach on the physical MOTOMAN SV3X 6-axes manipulator, and also working on a scheme to optimize furthermore the PRM path by using an enhanced A* algorithm along with other techniques.

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