PERFORMANCE ANALYSIS OF TIMED EVENT GRAPHS WITH MULTIPLIERS USING (Min, +) ALGEBRA

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Abstract: We are interested in the performance evaluation of timed event graphs with multipliers. The dynamical equation modelling such graphs are nonlinear in (min,+) algebra. This nonlinearity is due to multipliers and prevents from applying usual performance analysis results. As an alternative, we propose a linearization method in (min,+) algebra of timed event graphs with multipliers. From the obtained linear model, we deduce the cycle time of these graphs. Lower and upper linear approximated models are proposed when linearization condition is not satisfied.

1 INTRODUCTION

Timed event graphs (TEG's) are well adapted to model synchronization phenomena occuring in discrete event systems (Murata, 1989). Their behavior can be modelled by recurrent linear equations in (min, +) algebra (Baccelli et al., 1992). When the size of the model becomes very significant, techniques of analysis developed for these graphs reach their limits. A possible alternative consists in using timed event graphs with multipliers, denoted TEGM's. Indeed, the use of multipliers associated with arcs is natural to model a large number of systems, for example, when the achievement of a specific task requires several units of a same resource, or when an assembly operation requires several units of a same part.

To our knowledge, few works deal with the performance analysis of TEGM's. In fact, in the most of works the proposed solution is to transform the TEGM into an ordinary TEG, which allows the use of well-known methods of performances analysis.

In (Munier, 1993) the initial TEGM is the object of an operation of expansion. Unfortunately, this expansion can lead to a model of significant size, which does not depend only on the initial structure of the TEGM, but also on initial marking. With this method, the system transformation proposed under *single* server semantics hypothesis, or in (Nakamura and Silva, 1999) under *infinite* server semantics hypothesis, leads to a TEG with $|\theta|$ transitions ($|\theta|$ is the 1-norm of the elementary T-semiflow of the corresponding TEGM).

Another linearization method was proposed in (Trouillet et al., 2002) when each elementary circuit of graph contains at least one *normalized* transition (*i.e.*, a transition for which its corresponding elementary T-semiflow component is equal to one). This method increases the number of transitions. Inspired by this work, a linearization method without increasing the number of transition was proposed in (Hamaci et al., 2004).

A calculation method of cycle time of a TEGM is proposed in (Chao et al., 1993) but under restrictive conditions on initial marking.

The weights on the arcs of a TEGM are nonlinearly modelled in (*min*, +) algebra. Based on works given in (Cohen et al., 1998), we propose a new method of linearization without increasing the number of transition from the graph. The obtained (*min*, +) linear model allows to evaluate the performance of these graphs. According to initial marking, these performances are evaluated in an exact or approached way.

This article is organized as follows. Some concepts on TEGM's and their functioning are recalled in Section 2. The method of linearization is presented in Section 3. From the equivalent, or approached, TEG of a TEGM, we deduce the cycle time in the Section 4.

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2 RECURRENT EQUATIONS OF TEGM's

We assume that the reader is familiar with the structure, firing rules, and basic properties of Petri nets, see (Murata, 1989) for more details.

Consider a Petri net defined as a valued bipartite graph given by a five-tuple (P, T, M, m, τ) in which:

- P and T represent the finite set of *places*, and *transitions* respectively;
- A *multiplier M* is associated with each *arc*. Given $q \in T$ and $p \in P$, the multiplier M_{pq} (respectively, M_{qp}) specifies the weight (in $\mathbb{N}$) of the arc from transition *q* to place *p* (respectively, from place *p* to transition *q*). A zero value for M codes an absence of arc;
- With each place are associated an *initial marking* (m_p assigns an initial number of tokens (in $\mathbb{N}$) in place *P*) and a *holding time* (τ_p gives the minimal time (in $\mathbb{N}$) a token must spend in place *p* before it can contribute to the enabling of its downstream transitions).

We denote by ${}^\bullet q$ (resp., $q^\bullet$) the set of places upstream (resp., downstream) transition q. Similarly, ${}^\bullet p$ (resp., $p^\bullet$) denotes the set of transitions upstream (resp., downstream) place p.

An *event graph* is a Petri net whose each place has exactly one upstream and one downstream transition.

We denote W the incidence matrix of a Petri net. A vector $\theta \in \mathbb{N}^T$ such that $\theta \neq 0$ and $W\theta = 0$ is a T-semiflow. A T-semiflow θ has a minimal support *iff* there exists no other T-semiflow, θ', such that $\{q \in T \mid \theta'(q) > 0\} \subset \{q \in T \mid \theta(q) > 0\}$.

A vector $Y \in \mathbb{N}^P$ such that $Y \neq 0$ et $Y^t W = 0$ is a P-semiflow.

In the rest of the paper we assume that TEGM's are *consistent* (*i.e.*, there exists a T-semiflow θ covering all transitions : $\|\theta\| = T$) and are *conservative* (*i.e.*, there exists a P-semiflow Y covering all places: $\|Y\| = P$).

Remark 1 We disregard without loss of generality *firing times* associated with transitions of a TEG because they can always be transformed into holding times on places (Baccelli et al., 1992, §2.5).

With each transition q is associated a *counter variable*, denoted n_q : n_q is an increasing map from $\mathbb{R}$ to $\mathbb{Z} \cup \{+\infty\}$, $t \mapsto n_q(t)$ which denotes the cumulated number of firings of transition *q* up to time *t*.

In the following, we assume that counter variables satisfy the *earliest firing* rule, *i.e.*, a transition q fires as soon as all its upstream places $\{p \in {}^\bullet q\}$ contain enough tokens (M_{qp}) having spent at least τ_p units of time in place p. When the transition q fires, it consumes M_{qp} tokens in each upstream place p and produces $M_{p'q}$ tokens in each downstream place $p' \in q^\bullet$.

Assertion 1 The counter variable n_q of a TEGM (under the earliest firing rule) satisfies the following *transition to transition* equation:

$$n_q(t) = \min_{p \in {}^\bullet q,\, q' \in {}^\bullet p} \lfloor M_{qp}^{-1}(m_p + M_{pq'} n_{q'}(t - \tau_p)) \rfloor. \quad (1)$$

Let us note the presence of inferior integer part to preserve integrity of Eq. (1). In general, a transition q may have several upstream transitions ($\{q' \in {}^{\bullet\bullet} q\}$) which implies that its associated counter variable is given by the *min* of *transition to transition* equations obtained for each upstream transition.

Example 1

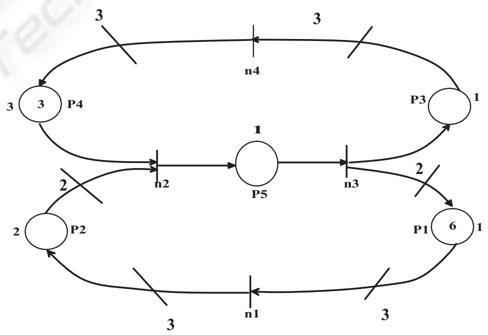

Figure 1: A TEGM.

The counter variables associated with transitions of TEGM depicted in the figure 1 satisfy the next system equations:

$$\begin{cases} n_1(t) &= \lfloor \frac{6+2n_3(t-1)}{3} \rfloor, \\ n_2(t) &= \min(\lfloor \frac{3n_1(t-2)}{2} \rfloor, 3 + 3n_4(t-3)), \\ n_3(t) &= n_2(t-1), \\ n_4(t) &= \lfloor \frac{n_3(t-1)}{3} \rfloor. \end{cases}$$

In the case of ordinary TEG's, the *transition to transition* equation given in Eq. (1) becomes:

$$x_q(t) = \min_{p \in {}^\bullet q,\, q' \in {}^\bullet p} (m_p + x_{q'}(t - \tau_p)). \quad (2)$$

This equation is linear in the algebraic structure called (*min*, +) algebra. This structure, denoted $\mathbb{Z}_{\min}$, is defined as the set $\mathbb{Z} \cup \{+\infty\}$, equipped with the *min* as additive law (denoted $\oplus$) and with the usual addition as multiplicative law (denoted $\otimes$). The neutral element of the law $\oplus$ (resp., $\otimes$) is denoted $\varepsilon = +\infty$ (resp., $e = 0$). More generally, the (*min*, +) algebra is a *dioid* (Baccelli et al., 1992).

A *dioid* $(\mathcal{D}, \oplus, \otimes)$ is a semiring in which $\oplus$ is *idempotent* $(\forall a,\ a \oplus a = a)$. Neutral elements of $\oplus$ and $\otimes$ are denoted ε and *e* respectively.

From the Eq.(2) obtained for each transition, one can express a TEG as the following recursive matrix equation:

$$x(t) = A \otimes x(t-1), \quad (3)$$

where A is a square matrix with coefficient in $\mathbb{Z}_{\min}$, and $x(t)$ is the vector of the counter variables associated with transitions of the graph. See (Baccelli et al., 1992) for more details on the representation of TEG's in the dioid $\mathbb{Z}_{\min}$.

3 LINEARIZATION OF TEGM'S

A TEGM is *linearizable* if there exists a change of variable $n_q(t) = \theta_q x_q(t)$ such that $x_q(t)$ satisfies a (*min*,+) linear recurrent equation knowing that:

- $n_q(t)$ is the counter associated with transition q of TEGM,
- θ_q is the component of T-semiflow associated with transition q ($\theta_q \in \mathbb{N}^*$).

Proposition 1 A TEGM is *linearizable* if

$$\forall q \in T, \forall p \in^\bullet q, \qquad \lfloor \frac{m_p}{M_{qp}} \rfloor \in \theta_q \mathbb{N}. \quad (4)$$

Proof: According to assertion (1), we have for each transition q of a TEGM:

$$n_q(t) = \min_{p \in {}^\bullet q,\, q' \in {}^\bullet p} \lfloor M_{qp}^{-1}(m_p + M_{pq'} n_{q'}(t - \tau_p)) \rfloor.$$

Using the change of variable $n_q(t) = \theta_q x_q(t)$, and by distributivity of the multiplication with respect to the *min* operator, we have:

$$x_q(t) = \min_{p \in {}^\bullet q,\, q' \in {}^\bullet p} \tfrac{1}{\theta_q} \lfloor (\tfrac{m_p}{M_{qp}} + \tfrac{M_{pq'}}{M_{qp}} n_{q'}(t - \tau_p)) \rfloor.$$

From relation

$\frac{\theta_q}{M_{pq'}} = \frac{\theta_{q'}}{M_{qp}}$, obtained for consistent and conservative TEGM (see (Munier, 1993)), we have

$$x_q(t) = \min_{p \in {}^\bullet q,\, q' \in {}^\bullet p} \tfrac{1}{\theta_q} \lfloor (\tfrac{m_p}{M_{qp}} + \tfrac{\theta_q}{\theta_{q'}} n_{q'}(t - \tau_p)) \rfloor,$$

i.e.,

$$x_q(t) = \min_{p \in {}^\bullet q,\, q' \in {}^\bullet p} \tfrac{1}{\theta_q} \lfloor (\tfrac{m_p}{M_{qp}} + \theta_q x_{q'}(t - \tau_p)) \rfloor.$$

Because $\theta_q x_{q'}(t - \tau_p) \in \mathbb{N}$, we finally obtain

$$x_q(t) = \min_{p \in {}^\bullet q,\, q' \in {}^\bullet p} (\frac{1}{\theta_q} \lfloor \frac{m_p}{M_{qp}} \rfloor + x_{q'}(t - \tau_p)), \quad (5)$$

which corresponds to a (*min*,+) linear recurrent equation. ■

More generally, if the condition (4) is satisfied for each transition, the equation (1) can be expressed as a (*min*,+) linear recurrent equation.

Remarks:

- Let us define an equivalence class of initial markings for the equivalence relation $m' \equiv m'' \Leftrightarrow \forall q \in T, \forall p \in^\bullet q, \quad \lfloor \frac{m'_p}{M_{qp}} \rfloor = \lfloor \frac{m''_p}{M_{qp}} \rfloor$.

  We can notice that all initial markings of a same equivalence class generate the same firing times behavior of transitions and give the same (*min*,+) model.
- In (Cohen et al., 1998), the authors propose a linearization method through a similar diagonal change of counter variables for fluid TEGMs (*i.e.*, where initial marking and multipliers can take real values). Moreover, they state in Prop. IV.6 that the behavior of a TEGM coincides (in $\mathbb{N}$) with that of its fluid version if $\forall q \in T, \forall p \in^\bullet q, \frac{m_p}{M_{qp}} \in \theta_q \mathbb{N}$. Thus, under this condition, it is possible to linearize a TEGM by considering its fluid version . However, the required condition is more restrictive than the condition (4).

When the condition (4) is not satisfied, we define two linear approximated models of the TEGM by considering a greater (resp., smaller) initial marking.

Definition 1 The upper (resp., lower) linear model is obtained by a minimal addition (resp., removal) of initial tokens in the TEGM, in order to satisfy the linearization condition (4) for each initial marking.

In other words, in each place p for which $\lfloor \frac{m_p}{M_{qp}} \rfloor \notin \theta_q \mathbb{N}$, we add $\overline{m}_p$ (resp., remove $\underline{m}_p$) initial tokens until the linearization condition is checked.

We denote $\overline{x}(t)$ (resp., $\underline{x}(t)$) the state vector of the TEG obtained from the approximate linearization by addition (resp., removal) of tokens in the TEGM.

We have

$$\overline{x}_q(t) = \min_{p \in {}^\bullet q,\, q' \in {}^\bullet p} \left(\frac{1}{\theta_q} \lfloor \frac{(m_p + \overline{m}_p)}{M_{qp}} \rfloor + \overline{x}_{q'}(t - \tau_p) \right), \tag{6}$$

where $\overline{m}_p$ is the minimum number of tokens added in the place p such that $\lfloor \frac{m_p + \overline{m}_p}{M_{qp}} \rfloor \in \theta_q \mathbb{N}$.

and

$$\underline{x}_q(t) = \min_{p \in {}^\bullet q,\, q' \in {}^\bullet p} \left(\frac{1}{\theta_q} \lfloor \frac{(m_p - \underline{m}_p)}{M_{qp}} \rfloor + \underline{x}_{q'}(t - \tau_p) \right), \tag{7}$$

where $\underline{m}_p$ is the minimum number of tokens removed in the place p such that $\lfloor \frac{m_p - \underline{m}_p}{M_{qp}} \rfloor \in \theta_q \mathbb{N}$.

We have:

$$\forall q, \;\; \theta_q \underline{x}_q(t) = \underline{n}_q(t) \le n_q(t) \le \overline{n}_q(t) = \theta_q \overline{x}_q(t).$$

Example 2 The TEGM depicted in Fig.1 admits the elementary T-semiflow: $\theta = (2, 3, 3, 1)$.

For initial marking M(0)=(6,0,0,3,0), we easily verify that initial marking of each place satisfies the linearization condition (4), which means that TEGM is linearizable.

Using the change of variables $n_i(t) = \theta_i x_i(t)$ and thanks to Eq.(5), we obtain the following linear model:

$$\begin{cases} x_1(t) & = 1 + x_3(t-1), \\ x_2(t) & = \min(1 + x_4(t-3), x_1(t-2)), \\ x_3(t) & = x_2(t-1), \\ x_4(t) & = x_3(t-1). \end{cases}$$

These equations correspond to the TEG depicted in figure 2.

For initial marking M(0)=(6,0,0,4,0), we can note that the place P4 does not satisfy the linearization condition.

Thanks to Eqs (6) and (7), we obtain respectively :

$$\begin{cases} \overline{x}_1(t) & = 1 + \overline{x}_3(t-1), \\ \overline{x}_2(t) & = \min(2 + \overline{x}_4(t-3), \overline{x}_1(t-2)), \\ \overline{x}_3(t) & = \overline{x}_2(t-1), \\ \overline{x}_4(t) & = \overline{x}_3(t-1), \end{cases}$$

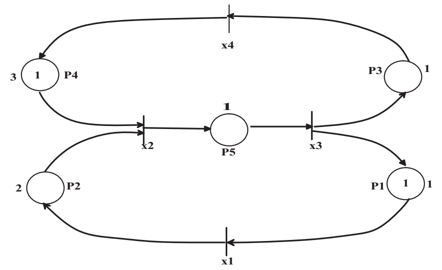

Figure 2: TEG obtained by the linearization of the TEGM of the figure 1.

and

$$\begin{cases} \underline{x}_1(t) & = 1 + \underline{x}_3(t-1), \\ \underline{x}_2(t) & = \min(1 + \underline{x}_4(t-3), \underline{x}_1(t-2)), \\ \underline{x}_3(t) & = \underline{x}_2(t-1), \\ \underline{x}_4(t) & = \underline{x}_3(t-1). \end{cases}$$

The evolution of the counter $n_2(t)$ is depicted in figure 3 and is such that $\underline{n}_2(t) \le n_2(t) \le \overline{n}_2(t)$.

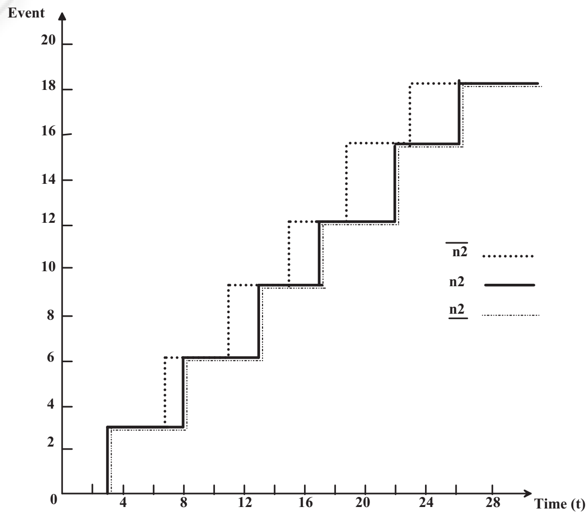

Figure 3: The evolution of the counter variables $\underline{n}_2, n_2$ and $\overline{n}_2$.

4 PERFORMANCE EVALUATION

4.1 Elements of performance evaluation for TEG

We recall main results characterizing an ordinary TEG's modelled in the dioid $\mathbb{Z}_{\min}$ (Baccelli et al., 1992), (Gaubert, 1992).

Definition 2 (Irreducible matrix) A matrix A is said *irreducible* if for any pair *(i,j)*, there is an integer m such that $(A^m)_{ij} \neq \varepsilon$.

Theorem 1 Let A be a square matrix with coefficient in $\mathbb{Z}_{\min}$. The following assertions are equivalent:

- Matrix A is irreducible,
- The TEG associated with matrix A is strongly connected.

One calls *eigenvalue* and *eigenvector* of a matrix A with coefficients in $\mathbb{Z}_{\min}$, the scalar λ and the vector v such as:

$$A \otimes v = \lambda \otimes v.$$

When the initial vector $x(0)$ of matrix equation (3) is equal to an eigenvector of matrix A, the TEG reaches a periodic regime from the initial state.

Theorem 2 Let A be a square matrix with coefficients in $\mathbb{Z}_{\min}$. If A is irreducible, or equivalently, if the associated TEG is strongly connected, then there is a single eigenvalue denoted λ. The eigenvalue can be calculated in the following way:

$$\lambda = \bigoplus_{j=1}^{n} (\bigoplus_{i=1}^{n} (A^j)_{ii})^{\frac{1}{j}}. \tag{8}$$

Regarding the TEG, λ corresponds to the firing rate identical for each transition. This eigenvalue λ can be directly deduced from the TEG by

$$\lambda = \min_{c \in C} \frac{M(c)}{T(c)}, \tag{9}$$

where:

- C is the set of elementary circuits of the TEG .
- *T(c)* is the sum of holding times in circuit c.
- *M(c)* is the number of tokens in circuit c.

It is possible that several eigenvectors can be associated with the only eigenvalue of an irreductible matrix.

Definition 3 Let A be an irreducible matrix of eigenvalue λ. One defines the matrix denoted A_λ by

$$A_\lambda = \lambda^{-1} \otimes A.$$

Theorem 3 (Gondran and Minoux, 1977) Let A be an irreducible matrix of eigenvalue λ. The *j*-th column of the matrix A_λ^+, denoted $(A_\lambda^+)_j$, is an eigenvector of A if it satisfies the following equality:

$$(A_\lambda^+)_j = A_\lambda \otimes (A_\lambda^+)_j. \tag{10}$$

4.2 Elements of performance evaluation for TEGM's

In the case of TEGM's, the firing rate, denoted λ_{m_q}, is not identical for all transitions. It is defined as follows:

$$\lambda_{m_q} = \frac{\theta_q}{TC_m}, \tag{11}$$

where θ_q is the component of the T-semiflow associated with transition q, and TC_m is average the cycle time of the TEGM.

The average cycle time of a TEGM can be defined as follows :

Definition 4 (Sauer, 2003)
The average cycle time, TC_m, of a TEGM is the average time to fire once the T-semiflow under the earliest firing rule (*i.e.*, transitions are fired as soon as possible) from the initial marking.

The firing rate λ_{m_q} of a linearizable TEGM can be calculated from the (*min*, +) linear model by:

$$\lambda_{m_q} = \theta_q \lambda \tag{12}$$

where λ is the eigenvalue of the equivalent (*min*,+) linear model. This result is a direct consequence of the linearization proprety.

In the case where we have an approximate linearization, we obtain

$$\underline{\lambda}_{m_q} \leq \lambda_{m_q} \leq \overline{\lambda}_{m_q},$$

where $\overline{\lambda}_{m_q}$(resp., $\underline{\lambda}_{m_q}$) is the firing rate of the transition q obtained by using the upper (resp., lower) approximated linear model of the TEGM.

When components of the eigenvector, associated with the TEG obtained by linearization, are integer values, the initial conditions vector of TEGM, denoted v_m (which allow to reach the periodic regime from the initial state) can be deduced by:

$$v_m = (\theta_1 x_1(0), ..., \theta_n x_n(0)) \tag{13}$$

where $x(0)$ is the eigenvector of the TEG.

Example 3 One determines the firing rate for each transition of the TEGM of figure 1 from the (*min*, +) linear model.

For $M(0) = (6,0,0,3,0)$, the initial marking of each place verifies the linearization condition. This TEGM is linearizable.

Thanks to Eq.(8), the production rate of the TEG obtained after linearization is equal to $\frac{1}{5}$. From Eq.(12), we deduce the firing rate of each transition: $\lambda_{m_1} = \frac{2}{5}, \lambda_{m_2} = \frac{3}{5}, \lambda_{m_3} = \frac{3}{5}, \lambda_{m_4} = \frac{1}{5}$.

Thanks to Eq.(11), we deduce that $TC_m = 5$.

For initial marking M(0)=(6,0,0,4,0), we have two linear approximated (*min*, +) models.

In the case where we add two tokens in the place P4 in the TEGM, we obtain a TEG with λ is equal to $\frac{1}{4}$.

Thanks to Eq.(12), we deduce the firing rate of each transition :

$$\overline{\lambda}_{m_1} = \frac{3}{4}, \overline{\lambda}_{m_2} = \frac{2}{4}, \overline{\lambda}_{m_3} = \frac{2}{4}, \overline{\lambda}_{m_4} = \frac{1}{4}.$$

Thanks to equation (11), we deduce $\overline{TC}_m = 4$.

In the case where we remove one token in the place P4, we obtain a TEG with λ is equal to $\frac{1}{5}$.

Thanks to Eq.(12), we deduce the firing rate of each transition :

$$\underline{\lambda}_{m_1} = \frac{3}{5}, \underline{\lambda}_{m_2} = \frac{2}{5}, \underline{\lambda}_{m_3} = \frac{2}{5}, \underline{\lambda}_{m_4} = \frac{1}{5}.$$

Thanks to Eq.(11), we deduce $\underline{TC}_m = 5$.

Finally, for M(0)=(6,0,0,4,0), we obtain:

$$4 \leq TC_m \leq 5$$

$$\frac{3}{5} \leq \lambda_{m_1} \leq \frac{3}{4}, \quad \frac{2}{5} \leq \lambda_{m_2} \leq \frac{2}{4}, \quad \frac{2}{5} \leq \lambda_{m_3} \leq \frac{2}{4},$$

$$\frac{1}{5} \leq \lambda_{m_4} \leq \frac{1}{4}.$$

5 CONCLUSION

In order to evaluate the performances of a TEGM from an equivalent TEG, a technique of linearization has been proposed in (*min*,+) algebra. According to initial marking, a linearization condition was stated. The performance analysis of a linearizable TEGM, such as cycle times, is deduced directly from the obtained linear model.

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