

# A MOBILE ROBOT MAPPING SYSTEM WITH AN INFORMATION-BASED EXPLORATION STRATEGY

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Abstract: The availability of efficient mapping systems to produce accurate representations of initially unknown environments is undoubtedly one of the main requirements for autonomous mobile robots. This paper presents a mapping system that has been implemented on a mobile robot equipped with a laser range scanner. The system builds geometrical maps of the environment employing an exploration strategy that takes into account both the distance travelled and the information gathered to determining the observation positions. This strategy is based on stronger mathematical foundations than the exploration strategies proposed in literature: this is the distinctive feature of our approach and constitutes the main original contribution of this paper.

## 1 INTRODUCTION

The availability of the maps of the environments where they operate is undoubtedly one of the main requirements for autonomous mobile robots (Thrun, 2002). To be efficient, an autonomous robot needs an effective mapping system that incrementally builds the map of an environment by determining the most interesting observation positions in the partially known environment.

This paper presents a mapping system that has been implemented on a mobile robot equipped with a laser range scanner. The proposed system builds point-based geometrical maps of the environment employing an exploration strategy that, in the determination of the observation positions, takes into account both the distance travelled and the information gathered. This exploration strategy implements the criterion introduced in (Amigoni and Caglioti, 2003). The strong mathematical foundations of the implemented exploration strategy, based on the concept of *relative entropy* (Shore, 1984; Caglioti, 2001), are the distinctive feature of our approach and constitutes the main original contribution of this paper.

The mapping system proposed in this paper iteratively performs the three following activities:

- building a partial map that represents the portion of the environment surrounding the robot,
- updating the global map according to the newly ac-

quired partial map and, at the same time, localizing the robot within the global map, and

- determining and reaching the next observation position, according to the exploration strategy.

As a consequence, the robot reaches a sequence of observation positions: at each observation position it performs a 360 degrees scan of the environment and updates the current global map. We assume that the robot moves on a flat 2D surface and that obstacles are at the height of the laser range scanner. The sequence of the robot observation positions is the result of the exploration strategy. For simplicity, a *greedy* approach is followed: at each step, just the next observation position is planned. In correspondence to each measurement, the information about the map is updated, as well as the information about the free space. The next robot observation position is then determined within the currently known free space. We experimentally tested and validated the proposed mapping system in heterogeneous environments.

This paper is organized as follows. The next section surveys the most important robot map building results and exploration strategies discussed in literature. In Section 3 we describe the proposed mapping system, which is experimentally validated in Section 4. Finally, Section 5 concludes the paper.

## 2 ROBOT MAP BUILDING

The design of a mapping system is usually strongly influenced by the specific sensors used. In the system described in this paper we employ a laser range scanner sensor (like (González-Baños and Latombe, 2002; Kwon and S.Lee, 1999; Lu and Milios, 1997)). In general, the mapping activity is characterized by the type of the map the robot produces, namely by the way in which the acquired information about obstacles and free space is represented. One can distinguish between *geometrical* and *topological* maps. Geometrical maps can be composed of grids (Thrun, 2001; Burgard et al., 2000), points (Lu and Milios, 1997), or segments (Austin and McCarragher, 2001; González-Baños and Latombe, 2002). Point and segment maps have usually two main advantages over the grid-based approaches: firstly, they are a much more compact representation of the environment and, secondly, they are easier to use. The mapping system proposed in this paper uses a representation of the obstacles as sets of points on the obstacle contour. This approach allows a simple update of the map as new information is acquired by the laser range scanner.

The global map of an environment is incrementally built by integrating partial maps on the basis of the (probabilistic) estimated positions of the robot (Thrun et al., 2000), of the geometrical features of the maps (Lu and Milios, 1997), or of a mixture of the two approaches. Thus, in almost all cases, it is important that, during the exploration, the robot is able to localize itself not only by odometry, but also by detecting landmarks or by matching data with an existing model of the environment. Indeed, techniques for Simultaneous Localization And Mapping (SLAM) (Dissanayake et al., 2001) and for Concurrent Mapping and Localization (CML) (Thrun et al., 1998) have been extensively studied. In our approach the robot matches the geometrical features of the maps to correct the estimation of its pose provided by odometry.

The exploration strategies usually aim at reducing the exploration time, by making a small number of exploration steps, while trying to reduce the error, by balancing the sensory information with the navigation cost to reach the observation positions. Since one of the main contributions of this paper is the implementation of an innovative theoretically well-founded solution to the problem of determining the next best observation position, also known as the Next Best View (NBV) problem, in the following we compare our approach with some significant techniques proposed in literature. (González-Baños et al., 2000; González-Baños and Latombe, 2002) describes an exploration method based on segment maps. Candidate NBV positions are generated across the edge of the explored regions, in which the robot is guaranteed to move

without collisions risk. To decide whether a candidate position in the free space is a good NBV position, the amount of new information about the environment that could be obtained from the candidate position is estimated. More precisely, the evaluation of a candidate position  $q$  given by

$$g(q) = A(q) \cdot \exp(-\lambda \cdot L(q))$$

where  $A(q)$  is an estimation of the unexplored area visible from  $q$ ,  $L(q)$  is the length of the path connecting the current robot position and  $q$ , and  $\lambda$  (set to  $20^{-1}$  cm) weights the new information obtainable from a candidate and the cost of travelling to reach the position. The best candidate will minimize the function  $g(q)$ . Although extensive experimental results validated the effectiveness of the above criterion, its theoretical bases are not well defined. For example, the use of the exponential function in evaluating the worthiness of a position and the value of the  $\lambda$  parameter are not theoretically justified. The criterion proposed in this paper overcomes this limitation since it has stronger mathematical bases.

The probabilistic approach proposed in (Burgard et al., 2000) coordinates the exploration activities of multiple robots in order to minimize the overall exploration time. In this case, grid maps are employed and the cell in which a robot should move next is the one that maximizes the difference between a (probabilistic-based) measure of expected utility and the path cost. Differently from our proposal, also this criterion lacks of strong theoretical foundations; moreover, it naturally applies to grid maps and its extension to the more compact point maps is not straightforward.

Finally, in (Sim and Dudek, 2003), a number of exploration policies (paths followed by the robot during mapping) are evaluated for efficiency. Although mathematically well-founded on Kalman filtering theory, this work differs from our approach since it does not evaluate only the next observation position but a whole set of pre-determined observation positions along a path.

## 3 THE PROPOSED MAPPING SYSTEM

The mapping system proposed in this paper tries to find an optimal exploration path by determining at each iteration the next best observation position. The robot performs the mapping process taking scans of the environment and using them to update a world model and to localize itself. A scan is composed of a list of points that represent the outline of the world objects within the range of the laser range scanner.

This sensor acquires a sequence of distance measurements, along directions separated by a programmable angle (one degree, in our experiments) sweeping an angle of 180 degrees; thus it returns 180 measurements per scan. A map is obtained by integrating scans and is composed of a set of points that describe the environment. In our implementation, the map is a list of points. A point is a triple  $(x, y, \sigma_p)$ , where  $x$  and  $y$  are the coordinates of the point with respect to a reference system and  $\sigma_p$  is the standard deviation that represents the uncertainty affecting the measurement of the point, due to the sensor accuracy and to the uncertainty of the robot position. The reference frame in which the coordinates of the points are expressed is centered on the laser range scanner. This means that the coordinates of the points composing the global map built so far are updated at every robot movement. After a movement, the position of the robot is estimated using odometry and then refined using a scan matching algorithm. In more detail, the mapping process goes as follows.

(1) From its current position, the robot constructs a partial map  $m_k$  of the environment, taking four consecutive scans (with headings separated by 90 degrees) of the surroundings. The four scans are aligned and their points fused. Let us illustrate this activity in more detail starting from the alignment procedure. The transformation (rotation and translation) to align two scans  $s$  and  $s'$  (and in general two maps  $m$  and  $m'$ ) is calculated in the following way. The odometry readings give a first approximation  $t_o$  of the alignment transformation;  $t_o$  is applied to  $s$  to bring its reference frame in the reference frame of  $s'$ , obtaining  $s^{t_o}$ . Then the alignment provided by  $t_o$  is refined by using the iterative algorithm described in (Lu and Milios, 1997) that matches the geometrical features of the two scans. This algorithm, called *alignment algorithm* in the following, finds pairs of corresponding points in the two partially aligned scans ( $s^{t_o}$  and  $s'$ ) and then computes a least-square solution of the relative rotation and translation. The process is iterated until it converges. A threshold is used to discard pairs of points that are too far. In our experimental activity, this threshold has been set to  $r_{max}/10$ , where  $r_{max}$  is the range of the laser range scanner. The alignment algorithm is applied to the first two scans, then to the result of their alignment and to the third scan, and, finally, to the results of this last alignment and to fourth scan (the whole process is shown in Fig. 1). We now turn to the illustration of the fusion of points. The fusion of the points of a partial map is necessary since, usually, sets of points in different scans represent the same object of the environment and thus they produce redundant information if plainly inserted in the partial map  $m_k$ . To simplify the partial map we use the clustering algorithm reported in (Dobkin and Tal, 2001; de Berg et al., 2004). In general, this algo-

rithm, called *fusion algorithm* in the following, finds the pairs of closest points belonging to a map and replaces them with their midpoints. Also in this case, a threshold (set to  $r_{max}/10$ ) discards the pairs whose points are too far. The fusion algorithm is applied to the result of the alignment of the four scans in order to obtain the partial map  $m_k$  representing the environment surrounding the current position of the robot.

(2) The current global map  $M_k$  is updated by adding  $m_k$  and obtaining  $M_{k+1}$ . A first estimation of the transformation  $t_k$  that aligns the reference frame of  $M_k$  to the reference frame of  $m_k$  (recall that the points of the global map are expressed in the reference frame centered on the robot sensor) is calculated starting from the odometry readings that give the transformation  $t_{k,o}$  between the previous observation pose and the current observation pose. This estimate is improved by applying the alignment algorithm to  $M_k^{t_{k,o}}$  (namely to  $M_k$  transformed according to  $t_{k,o}$ ) and to  $m_k$ . Then the fusion algorithm is applied to the portion of the resulting global map composed of points of  $m_k$  and  $M_k$  that represent the same objects in the environment, namely to the points of  $m_k$  and  $M_k$  are closer than  $r_{max}/10$  (see Fig. 2). The points in  $m_k$  that do not have any correspondent in  $M_k$  are simply added to  $M_{k+1}$ .

(3) The candidate observation positions are generated in the current global map. We implemented two methods to generate the candidate observation positions:

- evenly-separated candidates are generated along a circle with radius  $r = r_{max}/2$  and center in the current position of the robot,
- the candidates are randomly generated within a circular rim (with  $r_{max}/3 \leq r \leq 2 * r_{max}/3$  and center in the current position of the robot).

(4) We discard the unreachable candidates, namely the candidates that cannot be reached by a collision-free path from the current position of the robot. We consider only paths that are composed of an initial rotation and of a forward straight movement. A path is collision-free for the robot if the robot does not intersect any point of the map while moving along the path. Once the unreachable candidates have been eliminated, the remaining candidate observation positions are evaluated. The evaluation of the candidates is performed according to the criterion presented in (Amigoni and Caglioti, 2003), on which our exploration strategy is based. This criterion is briefly illustrated in the following (omitting the technical details). It refers to the concept of *relative entropy* (Shore, 1984; Caglioti, 2001):

$$H_f = - \int f(X) \ln \frac{f(X)}{f_0(X)} dX$$

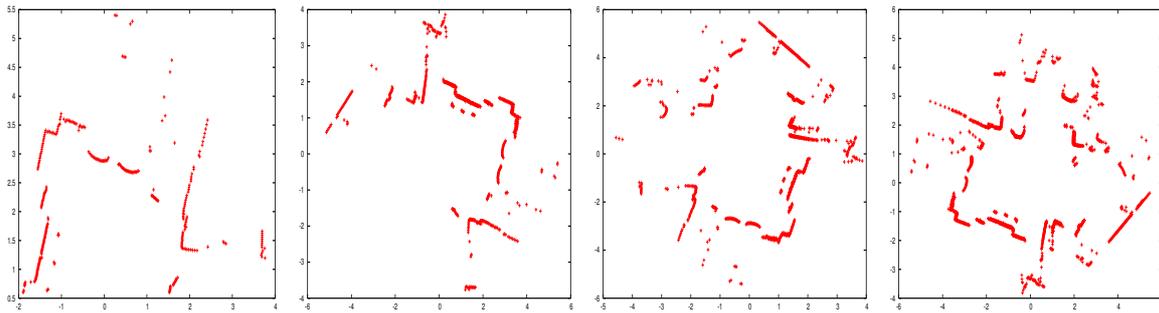


Figure 1: The construction of a partial map of a scattered environment obtained by aligning four scans (points of the map are represented as crosses, the current position of the robot is (0, 0), scales are in meters)

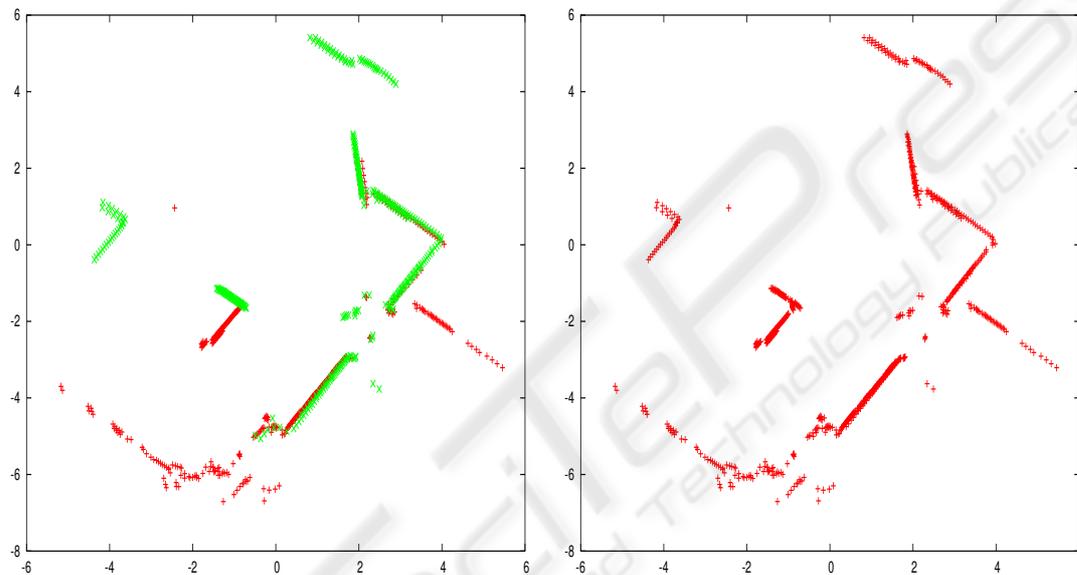


Figure 2: A global map (red) aligned with a partial map (green) before (left) and after (right) the application of the fusion algorithm

where  $X$  is a vector collecting the position parameters of the sample points of the map,  $f$  is the probability distribution function (PDF) of the values of  $X$ , and  $f_0(X)$  is the so-called non-informative PDF that can be assumed to be uniform (Tarantola, 1986). The expected increment of the relative entropy can be calculated given the current information on the map, described by a priori distribution  $f(X)$ , and given the a posteriori distribution  $f_0(X)$  after the measurement. Exploiting the additivity of relative entropy, the following formula to calculate the expected variation of relative entropy (if a measurement is performed) is obtained:

$$E[\Delta H] \approx \frac{1}{A + N} \sum_{i \in \mathcal{A} \cup \mathcal{N}} \ln \frac{\sigma_{unc,i}}{\sigma} + N \ln \frac{\sigma}{P} + \sum_{i \in \mathcal{A}} \ln \frac{\sigma}{\sigma_{p,i}}$$

where  $\mathcal{A}$  is the set of the already scanned points that the robot will see from a candidate position ( $A = |\mathcal{A}|$ ),  $\mathcal{N}$  is the set of new sensed points ( $N = |\mathcal{N}|$ ),  $\sigma_{p,i}$  is the prior standard deviation at the already sensed point  $i$ ,  $\sigma_{unc,i}$  is the standard deviation of the contribution to the measurement error due to the robot position error,  $\sigma$  is the sensor accuracy, and  $P$  is the (expected) total length of the map. Note that the terms of the sum in the right hand side of the above formula refer to uncertainty produced by the robot position error, to uncertainty of the new sensed points, and to

uncertainty of the already sensed points, respectively. In our experimental activity, given a candidate observation position  $q$ , the values of the parameters in the above formula are calculated as follows:

- $\mathcal{A}$  is the set of points of  $M_{k+1}$  that fall within the range of the laser sensor when the robot takes four scans in  $q$  (see step **(1)**);
- $N$  is calculated as  $N = N_k - A$ , where  $N_k$  is the number of points composing  $m_k$ ; the implicit assumption here is that the environment is enough regular so that the number of points composing the partial maps built at different iterations is similar;
- $\sigma_{unc,i}$  is equal to  $V_\theta d^2 + V_{xy}$ , where  $V_\theta$  is the expected error in the rotational position of the robot in  $q$ ,  $d$  is the distance between the current position of the robot and  $q$ , and  $V_{xy}$  is the expected error in the translational position of the robot in  $q$ ;
- $\sigma = 0.0001$ ;
- $P = 20$  m;
- $\sigma_{p,i}$  is set equal to  $2\sigma$ , assuming that the alignment algorithm works perfectly.

Once the robot has reached a new observation position, the uncertainty on its pose is subject to two opposite trends: uncertainty would increase due to odometry estimation errors; uncertainty would decrease if parts of the map already scanned are included in the set of points to be sensed. To select the next observation position, the criterion  $J$  is calculated:

$$J = E[\Delta H] + \frac{\Delta C + a}{\sigma} \ln \frac{\Delta C + a}{\sigma}$$

where  $\Delta C$  is the travelled distance to reach the candidate position and  $a$  is the length of the map covered by a single scan (note that  $a$  is related to  $r_{max}$ ). Low values of  $J$  identify the best observation positions. The value of  $J$  is calculated for every candidate observation position, and the one with the lowest value is selected.

**(5)** The robot moves to the selected observation position and the process re-starts from step **(1)** with iteration  $k + 1$ . The mapping process ends when there are no reachable observation positions, when the robot could not make a 360 turn to take the four scans needed for building the partial map, or when a given number of exploration steps has been reached.

We explicitly note that, in order to determine the next best observation position, our exploration strategy blends together the distance travelled by the robot to reach the position and the expected information gathered by the sensing activity. This information is relative both to the new points that are visible from the observation position *and* to the reduction of uncertainty on the already sensed points. This second component of information is fundamental since it enables the construction of accurate maps and improves

the localization of the robot; however, it is seldom considered in the exploration strategies presented in literature.

## 4 EXPERIMENTAL RESULTS

We have implemented the above mapping system (coded in C++) on a mobile robot based on a Robuter mobile platform equipped with a SICK LMS200 laser range scanner.

In the experimental activity we tested the correctness and the efficiency of the proposed mapping system. For example, Fig. 3 shows a mapping process that built the map of a part of an environment (with  $r_{max} = 3$  m). It can be seen that the robot follows a “reasonable” path, each time moving toward the unknown part of the environment, where it is likely to obtain more information.

Figs. 4 and 5 show that the number of exploration steps needed to map an environment with our mapping system decreases when the range of the laser range scanner increases. This means that the proposed mapping system is “sound”. Fig. 6 shows that indoor environments, like large rooms, can be mapped in few steps.

The above examples show that the robot safely moves in the already explored environment to reach new observation positions that improve the knowledge about the environment. The balance between the travelled distance and the expected information to be acquired in a candidate observation position is a distinctive feature of our approach. In Fig. 7 we show that, when two candidates are expected to provide the same amount of information ( $H$  values are similar), the one that is closest ( $C$  small) to the current position of the robot is selected (lowest value of  $J$ ). On the other hand, in Fig. 8, we show that, when a candidate position is expected to provide a greater amount of information than the other one and both candidates are at the same distance from the current position of the robot, the first one is selected.

The examples of Figs. 3, 4, 5, and 6 have been obtained by generating the candidates along a circle (recall step **(3)** of the previous section). This method produces a regular distribution of the candidates that usually allows the robot to navigate through narrow passages (Fig. 9 (left)); it is thus appropriate for mapping scattered environments. On the other hand, the random generation of candidates works well in open environments (Fig. 9 (right)).

Finally, in Fig. 10, we schematically show a problem arising because of the greedy policy we use to select the candidate positions. In this case, the candidate on the right is selected because it is “locally” optimal. However, the candidate on the left is “glob-

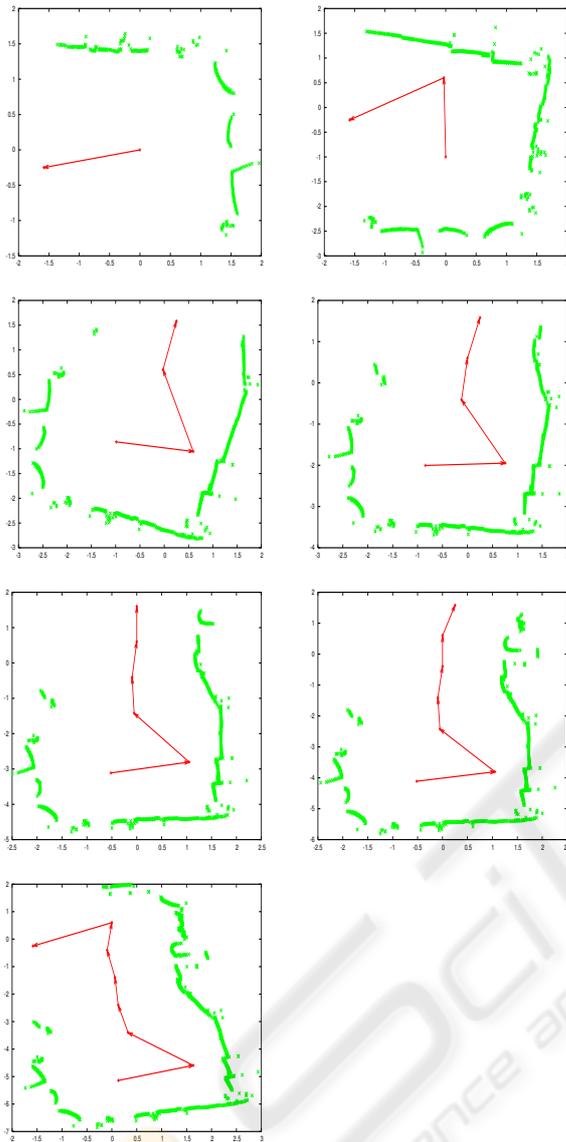


Figure 3: A sequence of observation positions reached by the robot (left to right and top to bottom)

ally” optimal (it could represent the first observation position determined by a global exploration strategy) since it provides a first view over a large unexplored area.

## 5 CONCLUSIONS

In this paper we have presented a mapping system that allows a mobile robot equipped with a laser scanner to incrementally build the map of an unknown environment. The proposed system produces point

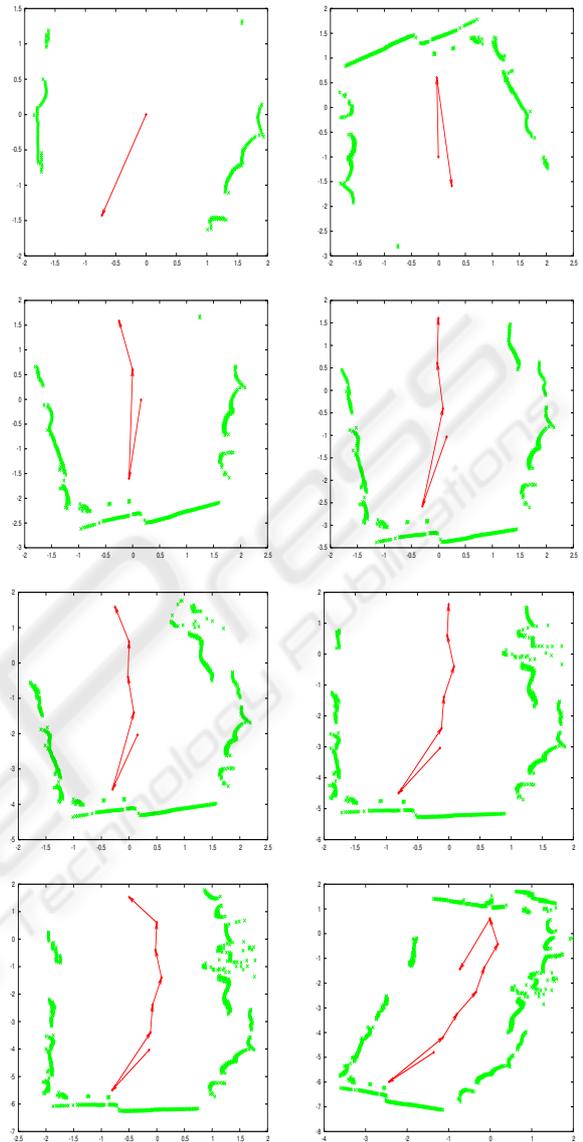


Figure 4: A sequence of observation positions reached by the robot with  $r_{max} = 3$  m

maps that are suitable for the safe navigation of the robot in the partially known environment. The employed exploration strategy blends together gathered information and cost of reaching observation positions. Experimental results show “reasonable” exploration behavior in incremental map construction.

Future research work will address the more precise quantitative evaluation of the efficiency of our approach and the extension of the system proposed in this paper to multirobot cases, in which teams of mobile robots cooperatively build the map of an environment, maybe exploiting different kinds of sensors. Fi-

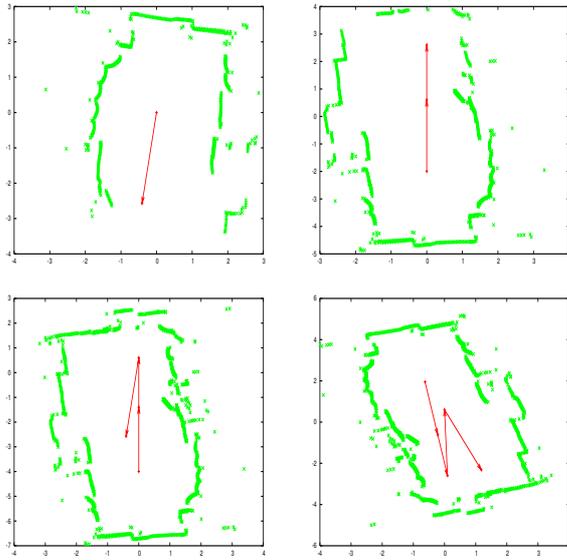


Figure 5: A sequence of observation positions reached by the robot with  $r_{max} = 4$  m

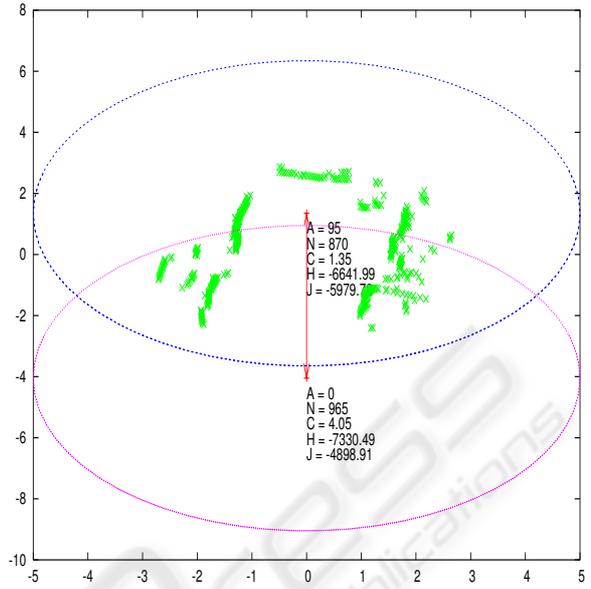


Figure 7: Candidate evaluation (the meaning of the symbols is explained in Section 3)

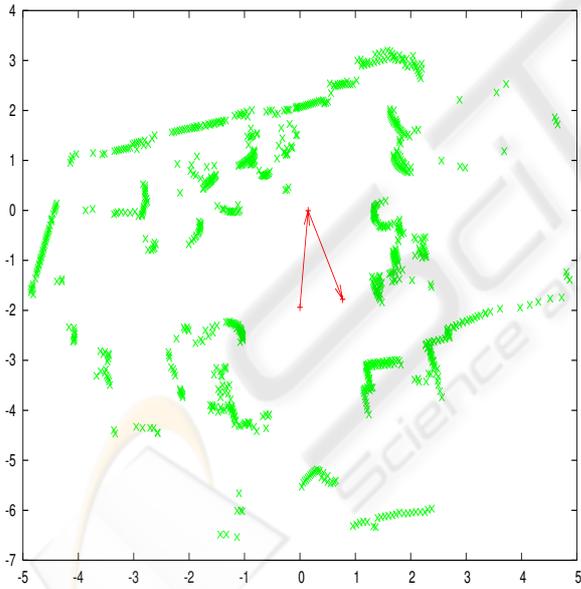


Figure 6: A sequence of observation positions reached by the robot with  $r_{max} = 5$  m

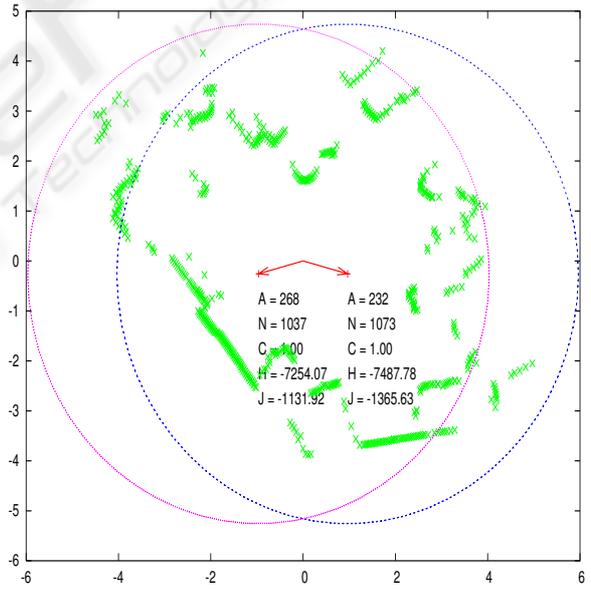


Figure 8: Another candidate evaluation

nally, we aim to extend the information-based exploration strategy – employed in this paper for mapping unknown environments – to other perception tasks, such as the monitoring of the electro-magnetic fields over an area.

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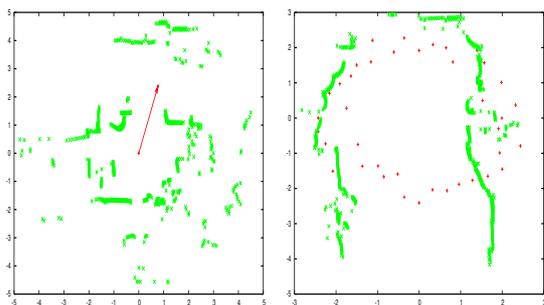


Figure 9: Generation of candidates

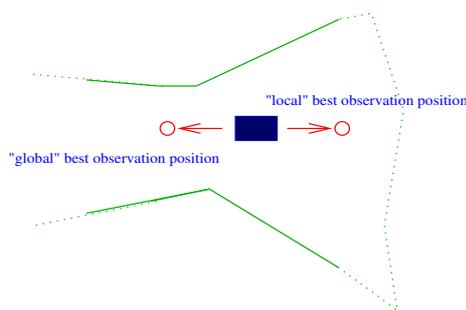


Figure 10: Greedy selection of candidates, dashed lines are walls not yet perceived by the robot

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