

ROBUST SENSOR BASED NAVIGATION FOR AUTONOMOUS MOBILE ROBOT

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Keywords: Interval analysis, Robotics, Navigation.

Abstract: This paper describes a new approach for mobile robot navigation using an interval analysis based adaptive mechanism for an Unscented Kalman filter. The robot is equipped with inertial sensors, encoders and ultrasonic sensors. The map used for this study is two-dimensional and it is assumed to be known a-priori. Multiple sensor fusion for robot localisation and navigation has attracted a lot of interest in recent years. An Unscented Kalman Filter (UKF) is used here to estimate the robots position using the inertial sensors and encoders. Since the UKF estimates are affected by bias, drift etc, we propose an adaptive mechanism using interval analysis with ultrasonic sensors to correct these defects in estimates. Interval analysis has been already successfully used in the past for robot localisation using time of flight sensors. But this IA algorithm has been extended to incorporate the sensor range limitation as in many real world sensors such as ultrasonic sensors. One of the problems of the use of interval analysis sensor based navigation and localisation is that it can be applicable only in the presence of land marks. This problem is overcome here using additional sensors such as encoders and inertial sensors, which gives an estimate of the robot position using an Unscented Kalman filter in the absence of land marks. In the presence of land marks the complementary robot position information from the Interval analysis algorithm using ultrasonic sensors is used to estimate and bound the errors in the UKF robot position estimate.

1 INTRODUCTION

Robot navigation is primarily about guiding a mobile robot to a desired destination or along a pre-specified path in which the robots environment consists of landmarks and obstacles. In order to achieve this objective the robot needs to be equipped with sensors suitable to localize the robot throughout the path it has to follow. Most of these sensors may give overlapping or complementary information and sometimes be redundant as well. There are many different architectures to fuse these information. Mobile robots generally carry dead reckoning sensors such as wheel encoders and inertial sensors, such as accelerometers, gyroscopes, to measure acceleration, angle rate and obstacle detecting and map making sensors such as time of flight ultrasonic sensors. All these sensor measurements can be fused to estimate the robots position by using a sensor fusion algorithm. Sensor fusion in this case is the method of integrating data from distinctly different sensors to estimate the robots position.

Classical data fusion algorithms use stochastic filters such as Kalman filters for robot position estima-

tion (Jetto et al., 1999). But one of the main disadvantages of using Kalman filters with ultrasonic sensors for robot localisation problems is that the data association step in Kalman filters is complex and also the fact that they are often affected by bias and drift. Moreover an accurate model of the robot system and accurate statistics of the sensor noises are needed, which is not available accurately in many cases.

The paper is organised as follows. This introductory section continues by presenting a background for the problem of autonomous robot localisation in section 1.1, followed by a summary of previous work in robot localisation using IA and standard stochastic filters in section 1.2. The novelty of the proposed approach is presented in section 1.3. Section 2 explains the implementation of the UKF with inertial sensors and encoders for this problem. Section 3 gives a brief explanation of the interval analysis algorithm for robot localisation and also describes how the sensor range limitation is incorporated and when multiple sets of ultrasonic sensor measurements are taken. In section 4 the implementation of the adaptive mechanism for the UKF robot position estimation using

Interval Analysis with ultrasonic sensors is described and the results are shown and finally in section 5 the conclusions are given.

1.1 Background

The problem considered here is that of robot navigation and localisation using multiple low cost sensors such as inertial sensors, encoders and ultrasonic sensors. Conventionally stochastic filters such as Extended Kalman filter or Unscented Kalman filters (UKF) are used for robot localisation (Clark et al., 2001). One of the main prerequisites for using Kalman filter is to have an accurate model of the robot and also accurate sensor noise statistics (i.e) bias, drift etc. But in practice it is difficult to have these parameters accurately, especially drift in accelerometers and gyroscopes, which affects the outcome of the UKF, there by contributing to errors in the estimated position of the robot over a period of time.

Moreover time of flight sensors such as ultrasonic sensors are used to measure the distance of land marks from the robot and to recognize the presence of any obstacles in the robots path. When the 2-D map of the environment in which the robot travels is known a priori, the distance measurements from the ultrasonic sensors can be used independently to estimate the position of the robot in the map. EKF or UKF can be used for this purpose as well (Castellanos and Tardos, 1999). But one of the main limitations encountered when using this approach is the problem of data association, as the data association problem in EKF or UKF is extremely complex and is of the third order O^3 . There are ways in which this problem can be simplified to O^2 , but the solution may be suboptimal.

In order to get the best estimate of the robots position, we can use different types of sensors with different algorithms which have different sources of error. In this case we use an Unscented Kalman filter for fusing the data from the accelerometers, gyroscopes and encoders and use an Interval Analysis (IA) algorithm for estimating the robots position using ultrasonic sensors. IA algorithm is used instead of EKF or UKF for the ultrasonic sensors because the IA algorithm can overcome the problems of not having an accurate model and accurate sensor noise statistics in the Kalman Filters, as the IA gives a guaranteed estimate of the robot position. It should also be noted that the IA algorithm for ultrasonic sensors bypasses the complex data association step and handles the problem in a nonlinear way even while been robust to outliers. Thus we have two independent sets of the measurements for the robot position. The estimated robot position using UKF from inertial sensors, which might be affected by bias and drift, are then fused with the estimated interval robot position using IA from ultrasonic sensors. The fused robot po-

sition estimate is much better than either one them alone since the errors in UKF estimated position are identified and corrected using the IA algorithm.

1.2 Prior work: Robot localisation with IA using range measurements

Interval analysis is basically about guaranteed numerical methods for approximating sets. Guaranteed in this context means that outer (and sometimes inner) approximations of the sets of interest are obtained, which can (at least in principle), be made as precise as desired. Thus interval computation is a special case of computation on sets, and set theory provides the foundations for interval analysis.

The localisation of an autonomous robot while navigating in a known or partially known environment is an important problem in mobile robotics. In this section an approach for the localisation of the robot using interval analysis (Kieffer et al., 2000) with sensor readings from ultrasonic sensors is described briefly. The main advantage of this method is that it bypasses the data-association step, which is very complex in other methods such as Extended Kalman Filters, and it handles the problem in a nonlinear way without any linearisation and it is very much robust to outliers.

The robot model described above moves in a known 2D environment and its motion is planned with respect to a set of obstacles and landmarks. These obstacles and landmarks define the world reference frame W and a robot frame R which is with respect to the body of the robot.

The robots position is described by the parameters x_c, y_c and θ , which form the configuration vector $\mathbf{p} = (x_c, y_c, \theta)^T$.

So the task now is to estimate the value of the configuration vector \mathbf{p} , from a map representing the environment of the robot and from distance measurements provided by a belt of n_s time of flight sensors with unlimited range present in the mobile robot. Moreover since it is assumed that the bounds on the measurement error is known, the resulting distance measurement is in terms of intervals which is stored in an interval vector

$$[d] = ([d_1], \dots, [d_n]) \quad (1)$$

If a model is available to model the ultrasonic sensor interval distance measurements represented by the interval vector $d_m(p)$, when the robot configuration is \mathbf{p} , the robot localization problem now becomes a bounded error parameter estimation problem, namely that of characterizing the set

$$P = \{p \in [p_o] \mid d_m(p) \in [d]\} \quad (2)$$

where $[p_o]$ is an initial search box, assumed to be large enough to contain all the possible robot configurations. P then contains all the configurations vectors that are consistent with the given map and measurements.

But the task is to find the configuration vector \mathbf{p} and so the equation given above can be rearranged as

$$= [p_o] \cap (d_m)^{-1}([d]) \quad (3)$$

(i.e.) for a given configuration vector \mathbf{p} the robot evaluates the measurements that its sensors would return and compares then with the actual measurements to check whether they are consistent.

The problem described by the above mentioned equation 3 could then be solved using any of the two approaches namely SIVIA (Set Inversion Via Interval Analysis) (Jaulin and Walter, 1993) and ImageSP (Kieffer et al., 1998). Both the above approaches have been described in detail in the book by Jaulin et al (Jaulin et al., 2001) and a brief introduction to both SIVIA and IMAGE SP is given in Sections 3.1, 3.2.

1.3 Novelty

This paper differs for the work in section 1.2 in three main aspects. Firstly, the above algorithm has been modified to incorporate sensor range limitation as in many real world applications. Secondly instead of using interval values in the kinematic model while tracking the robot, we use the physical constraints of the robot model to predict the robot position. Finally in order to overcome the scenario when there are no land marks in the vicinity of the robot, in which case there will be no measurements from the TOF sensors, it is proposed to use inertial sensors (INS). Thus in the presence of land marks, the fusion of the robot position from the the INS and TOF sensors is proposed.

2 ROBOT LOCALISATION USING UKF WITH INERTIAL SENSORS AND ENCODERS

By fusing the measurement data from the sensors - wheel encoders, gyroscopes and accelerometers - in the mobile robot, a reliable estimation of the position and heading of the robot can be obtained. There are basically two well established approaches available in literature: one is the Kalman filter and the other is the extended Kalman filter (EKF) (Alessandri et al., 1997). The Kalman filters are well known methods used in the theory of stochastic dynamic systems, which can be used to improve the quality of estimates of unknown quantities. The difference between the two methods is that for the first one a linear kinematic

model is used, while for the second one, the EKF a nonlinear dynamic model is used. We know that, if we use the nonlinear model, it is much more difficult to tune the performances of the filter. From the theoretical point of view there are no theoretical results about the convergence properties as well. But in order to use all the available information, a nonlinear model is preferred. Most often in real world engineering applications, the most widespread and reliable state estimator for nonlinear systems is the extended Kalman filter (Sorenson, 1990).

The EKF is the Kalman filter of an approximate model of the nonlinear system, which is linearised to the first order around the most recent estimate. Assuming all the stochastic processes are Gaussian, the first order linearisation must be carried out at every iteration before applying the KF algorithms.

For the extended Kalman filter, the robot model equations can be rewritten as the state equation of the form shown below,

$$x_{k+1} = f(x_k, u_k, w_k) \quad (4)$$

which, when linearized will be of the form

$$x_{k+1} \approx \tilde{x}_{k+1} + A(x_k - \hat{x}_k) + Bu_k + Ww_{k-1}, \quad (5)$$

where, A and B are the jacobian matrix of partial derivatives.

This first order linearisation using the Taylor series expansion may introduce errors in the estimated parameters which may lead to suboptimal performance and sometimes divergence of the filter.

The above described problem can be overcome to a certain extent by using a method first described by Julier and Uhlman as the unscented transform in the Kalman filter for the linearisation process (Julier and Uhlmann, 1997). This formulation of the Kalman filter is called the Unscented Kalman Filter. The unscented transform is basically a deterministic sampling approach, where the state distribution is approximated by a gaussian random variable, but is now described using a minimal set of carefully chosen sample points. These sample points are chosen using the unscented transform method which completely describes the true mean and covariance of the gaussian random variable. When these chosen points are propagated through the true non-linear system, it can capture the posterior mean and covariance accurately up to the 3rd order for Taylor series expansion, where as in a EKF we can achieve only up to first-order accuracy. It should also be noted that the computational complexity of the UKF is the same order as that of EKF. The basic equations for the UKF has been given in detail in the book (Wan and der Merwe, 2001)

All the process noises are assumed to be zero mean, uncorrelated white random noises only.

In the measurement model for this robot there are four sources of observations that are considered:

1. velocity measurements from the wheel encoders,
2. acceleration from the accelerometers, which when integrated gives the velocity of the robot,
3. robot heading angle measurement from the encoders and
4. the angular velocity measurements from the rate gyroscope which when integrated once gives the heading angle of the robot.

Thus, for both the velocity and heading angle of the robot there are two sets of measurements from two independent sensors. These two sets of measurements are then given as input to the Kalman filter which estimates the robots speed and heading angle, from which the robots position can then be calculated.

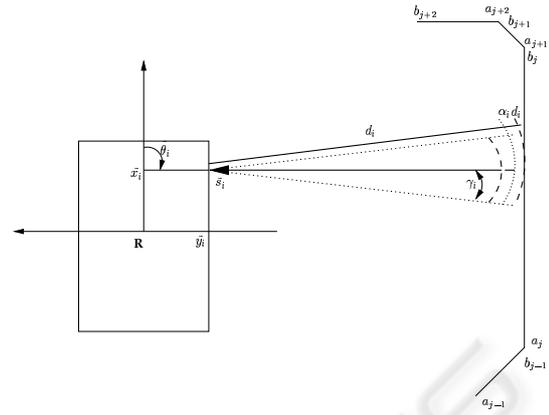


Figure 1: Robot and Sensor model

3 ROBOT LOCALISATION WITH INTERVAL ANALYSIS USING RANGE MEASUREMENTS

In this section the robot localisation using IA as first described briefly in section 3 is further improved using multiple measurements from the ultrasonic sensor measurements.

A brief overview of the algorithm for a single measurement has been given in the *Figures 3, 4 and 5*. The main improvement in this version of the algorithm in the descriptions in the tables is that the range of the ultrasonic sensor has been limited to 3 meters, where as in (Kieffer et al., 2000) the range was unlimited. This is implemented by identifying the sensors n_i that only give readings less than 3 meters (which is done by setting all the interval ranges greater than 3 meters to infinity in *Figure 5*) and substituting them for instead of n_s (where the inclusion function was calculated for all the n_s number of sensors) in the *Figure 4*. Also it is assumed that the land marks are spaced sparsely (i.e) there is a distance of at least 3 meters between the landmarks. This is because when the land marks are close some of the land marks may not be visible to the robot sensor model in the IA algorithm. Additionally the land marks which are visible to the robot in the 3 meter radius are only given to the inclusion function in *Figure 4* instead of all the n_w segments, there by saving computational time.

This problem as given in equation 3 is then solved using any of the two approaches namely SIVIA (Set Inversion Via Interval Analysis) (Jaulin and Walter, 1993) and ImageSP. A brief introduction to both SIVIA and IMAGESP is given in the next two subsections.

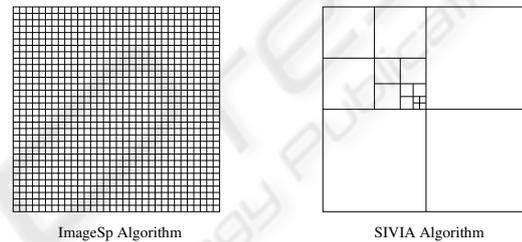


Figure 2: A schematic representation of ImageSp and SIVIA algorithm

3.1 Set Inversion Via Interval Analysis (SIVIA)

Set inversion is the computation of the reciprocal image

$$X = \{x \in R^n \mid f(x) \in Y\} = f^{-1}(Y) \quad (6)$$

of a regular subpaving Y of R^m by a possibly nonlinear function $f: R^n \rightarrow R^m$ and SIVIA is a method to compute two subpavings \underline{X} and \overline{X} of R^n such that

$$\underline{X} \subset X \subset \overline{X} \quad (7)$$

A subpaving is a finite set of non-overlapping boxes that are all included in the same root box. It is called regular if each of the boxes can be obtained by a finite succession of bisections and selections (Jaulin et al., 2001).

In this problem for robot localisation, $P = \{\mathbf{p} \in [p_o] \mid t(\mathbf{p}) = 1\}$, SIVIA can be applied (Kieffer et al., 2000). Therefore in this case, if $t_{\square}([p_o]) = 1$, p_o is in the solution set P and is stored in \hat{P} . If $t_{\square}([p_o]) = 0$ then $[p_o]$ has an empty intersection with P and is dropped from further consideration. If $t_{\square}([p_o]) = [0, 1]$ and if the width of $[p_o]$ is larger than the pre-specified precision parameter ϵ , then p_o is bisected, leading to two child sub boxes $L(p)$ and

Algorithm d_m (in: \mathbf{p} ; out: d_m)	
1	for $i := 1$ to n_s
2	$s_i := \begin{pmatrix} x_c \\ y_c \end{pmatrix} + \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \tilde{s}_i$
3	$\vec{u}_{1i} := \begin{pmatrix} \cos(\theta + \tilde{\theta}_i - \gamma) \\ \sin(\theta + \tilde{\theta}_i - \gamma) \end{pmatrix};$ $\vec{u}_{2i} := \begin{pmatrix} \cos(\theta + \tilde{\theta}_i + \gamma) \\ \sin(\theta + \tilde{\theta}_i + \gamma) \end{pmatrix};$
4	$(d_m)_i(p) := +\infty;$
5	for $j := 1$ to n_w $(d_m)_i(p) :=$ $\min((d_m)_i(p), r(s_i, \vec{u}_{1i}, \vec{u}_{2i}, a_j, b_j)).$

Figure 3: Model for calculating the distance expected from ultrasonic sensor when the configuration is \mathbf{p}

$R(p)$, and the test $t_{\square}(\cdot)$ is recursively applied to both of them. Any box with width smaller than ϵ is considered to be small enough and it is added to \hat{P} . A diagram explaining SIVIA is given in Figure 2.

3.2 IMAGE SubPaving (IMAGESP)

When \mathbf{f} is not invertible, a specific and computationally more demanding procedure is used. The basic idea of IMAGESP algorithm is to describe the initial search box p_o using a subpaving consisting of p boxes whose width are less than ϵ . Then IMAGESP evaluates the image of each of these p boxes using an inclusion function f_{\square} of f and stores them on a list. Therefore we will be getting p image boxes, each of which contains the true image set of the associated initial box. At last, IMAGESP merges all these image boxes into a subpaving to allow further processing (Kieffer et al., 1999). A diagrammatic representation of IMAGESP is given in Figure 2.

Thus the actual position of the robot represented here by the configuration vector \mathbf{p} at any given instant of time can be found using SIVIA or IMAGESP algorithms.

The Figure 1 gives a brief description of a map that represents the surroundings in which the robot travels (which is assumed to be known in order to calculate $d_m(p)$) and also the measurement process.

A detailed description of the above inclusion functions has been provided by (Kieffer et al., 2000). Also a better version of the above algorithm in terms of computational time, incorporating the interval elementary tests to eliminate some of the infeasible configurations in the configuration vector has been described in (Kieffer et al., 2000), in which the problem is reformulated as $P = \{\mathbf{p} \in [p_o] \mid t(p) \text{ holdstrue}\}$ (i.e.) $P = \{\mathbf{p} \in [p_o] \mid t(p) = 1\}$, where $t(p)$ is a global test. The global test $t(p)$ consists of various elementary tests (three tests (Kieffer et al., 2000)) and they are robust to outliers as well. Also the

Algorithm $[d_m]$ (in: $[\mathbf{p}];$ out: $[d_m]$)	
1	for $i := 1$ to n_s
2	$[s_i] := \begin{pmatrix} x_c \\ y_c \end{pmatrix} + \begin{pmatrix} \cos[\theta] & -\sin[\theta] \\ \sin[\theta] & \cos[\theta] \end{pmatrix} \tilde{s}_i$
3	$[\vec{u}_{1i}] := \begin{pmatrix} \cos([\theta] + \tilde{\theta}_i - \gamma) \\ \sin([\theta] + \tilde{\theta}_i - \gamma) \end{pmatrix};$ $[\vec{u}_{2i}] := \begin{pmatrix} \cos([\theta] + \tilde{\theta}_i + \gamma) \\ \sin([\theta] + \tilde{\theta}_i + \gamma) \end{pmatrix};$
4	$[d_m]_i([\mathbf{p}]) := +\infty;$
5	for $j := 1$ to n_w $[d_m]_i([\mathbf{p}]) :=$ $\min([d_m]_i([\mathbf{p}]), [r]([s_i], [\vec{u}_{1i}], [\vec{u}_{2i}], a_j, b_j)).$

Figure 4: Inclusion function for the measurement model

Algorithm $[r]$ (in: $[s], [\vec{u}_1], [\vec{u}_2], \mathbf{a}, \mathbf{b};$ out: $[r]$);	
1.	$[t_r] := (\det(\vec{ab}, \vec{as}) \geq 0)$ if $[t_r] = 0$ then $[r] := +\infty$; return;
2.	$[t_h] := (\langle \vec{ab}, \vec{a}[s] \rangle \geq 0) \wedge (\langle \vec{ba}, \vec{b}[s] \rangle \geq 0)$ $\wedge (\langle \vec{ab}, [\vec{u}_1] \rangle \leq 0) \wedge (\langle \vec{ab}, [\vec{u}_2] \rangle \geq 0);$
3.	$[r_h] := [\chi]([t_h], [t]([s], (a, b)), +\infty);$
4.	$[t_a] := (\det([\vec{u}_1], [s] \vec{a}) \geq 0) \wedge$ $(\det([\vec{u}_2], [s] \vec{a}) \leq 0);$
5.	$[r_a] := [\chi]([t_a], \ [s] \vec{a}\ , +\infty);$
6.	$[t_b] := (\det([\vec{u}_1], [s] \vec{b}) \geq 0) \wedge$ $(\det([\vec{u}_2], [s] \vec{b}) \leq 0);$
7.	$[r_b] := [\chi]([t_b], \ [s] \vec{b}\ , +\infty);$
8.	for $i := 1$ to 2
9.	$[t_{h_i}] := (\det([\vec{s}] \vec{a}, [\vec{u}_i]) \geq 0) \wedge$ $(\det([\vec{s}] \vec{b}, [\vec{u}_i]) \leq 0);$
10.	$[r_{h_i}] := [\chi]([t_{h_i}], [t_{[u_i]}]([s], (a, b)), +\infty);$
11.	$[r] := \min([r_h], [r_a], [r_b], [r_{h_1}], [r_{h_2}]);$
12.	$[r] := [\chi]([t_r], [r], +\infty).$

Figure 5: Inclusion function for the remoteness of the cone from a segment

purpose of the first two tests is to eliminate some infeasible configurations there by saving computational time. But if all the three tests are used when the range of the sensor is limited, it leads to scenarios in which some feasible configurations are ignored prematurely. Therefore only two tests were used (inroom test and the data test in (Kieffer et al., 2000)). The main consequence of not using the leg-in test when the sensor range is limited is that it may increase the computation time.

In the case when the robot is moving the robots position needs to be tracked, in which case the robots position needs to be predicted at the next instant to estimate the robots position at that instant. This is done by using the physical limitations of the robot based on the maximum speed and heading angle rate of the robot, instead of using a kinematic model of the robot. Thus we obtain an independent position of the robot from the ultrasonic sensors.

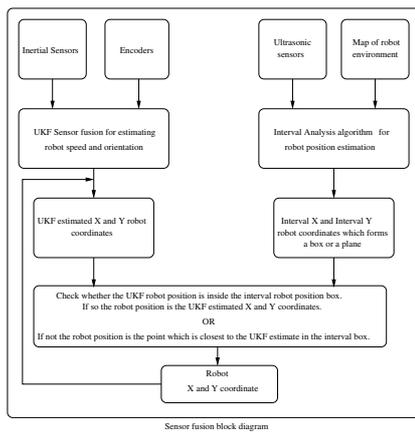


Figure 6: Block diagram of IA based adaptive mechanism for UKF

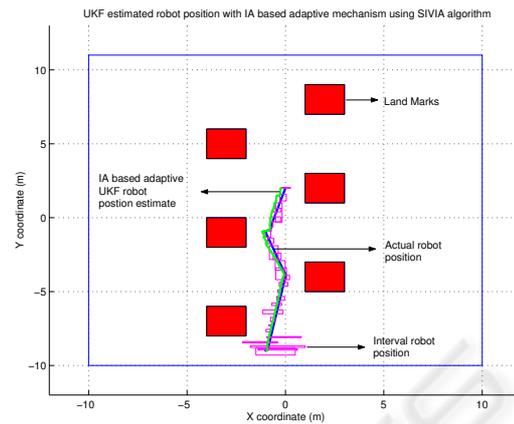


Figure 8: Fused robot position using SIVIA algorithm as adaptive mechanism in UKF

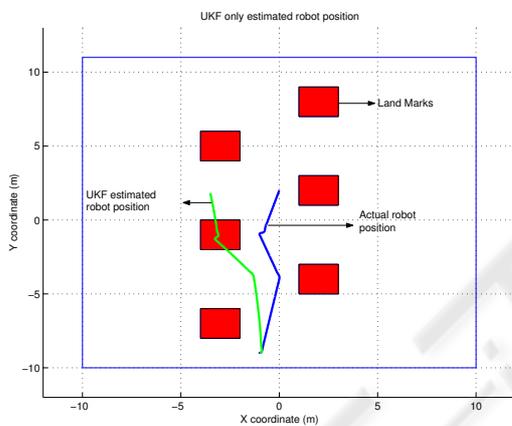


Figure 7: UKF only estimated robot position

4 INTERVAL ANALYSIS BASED ADAPTIVE MECHANISM FOR UKF

Sensor fusion is a very important and keenly researched topic in the domain of mobile robotics. This is due to the fact that, instead of using bespoke expensive sensors for estimating the robots position, multiple low cost sensors can be used, thereby reducing the cost of developing the robot. Moreover these same sensors can be used to do other tasks other than estimating the robots position such as building the map of the robots environment using ultrasonic sensors etc. Also the source of errors in one sensor may be different from another one and this fact can be exploited to eliminate the errors in the measurements.

For the problem of sensor fusion stochastic filters, such as Kalman filters, are commonly used. But they suffer from the same disadvantages described before

(i.e.) an accurate model of the system and statics are needed. In order to overcome these difficulties while using the UKF, a new approach has been introduced in this paper. As described above the robots position are estimated using two independent sources namely, the robot position from inertial sensors and the interval robot position from ultrasonic sensors.

The position obtained using interval analysis is updated only once every second, whereas the position from inertial sensors and encoders are updated 100 times per second as it has a sampling time of 0.01 seconds. Moreover we know that the robots interval position to be guaranteed (even though in a few cases it may not be due to the way the boxes are subdivided but still they are very close to the actual position). The interval position thus obtained will therefore be like a plane or rectangle. Then the estimated position using the inertial sensors is checked whether they are inside this rectangle. In case they are present inside the rectangle then they are trusted to be accurate and used. Alternatively if they are not then both the measurements are fused by selecting the point on the rectangle (box) (i.e.) the interval robot position, which is geometrically closest to the robot position estimated using UKF with inertial sensors, thereby bounding the error in the UKF estimates. A block diagram of the sensor fusion method is given in *Figure 6*.

But since the UKF estimates of inertial sensors is not accurate as they may be affected by bias, drift etc and also as the robot is slow moving, instead of using a dynamic interval model of the robot to predict the next step as in (Jaulin et al., 2001) we use the physical limitations of the robot to predict the next step in the Interval Analysis algorithm.

The above described algorithm was successfully implemented and tested in simulation using MATLAB and C++ software. The *Figure 8* shows the robot position after using the adaptive UKF robot position

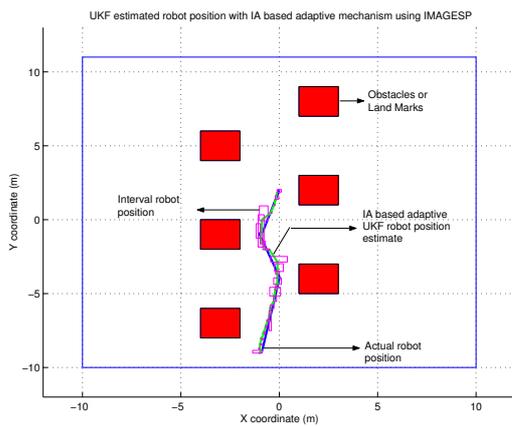


Figure 9: Fused robot position using IMAGESP algorithm as adaptive mechanism in UKF

estimate using the SIVIA interval robot position estimate for adaptation. It can be seen that the interval position estimate is very conservative when there are no land marks in one coordinate, for example at the beginning of the robot path in *Figure 8*. Moreover it can be observed that the UKF estimate with the interval analysis based adaptive mechanism has significantly improved the UKF robot position estimate when compared with the UKF position estimate without the IA adaptive mechanism shown in *Figure 7*. Similarly the *Figure 9* shows the UKF robot position estimation using the IMAGESP algorithm for the adaptive mechanism. It should be noted that the SIVIA interval position uncertainty is more when compared with the IMAGESP algorithm, but the computational complexity for the IMAGESP algorithm is more when compared with the SIVIA algorithm.

5 CONCLUSION

An Unscented Kalman filter (UKF) using an Interval Analysis based adaptive mechanism has been described. The UKF uses accelerometers, gyroscopes and encoders to measure the robots speed and heading angle, so that the robots position can be estimated. But the UKF robot position estimate is affected by errors in robot model, sensor bias, drift etc. The Interval Analysis (IA) is a deterministic approach to estimating the robots position without using a model of the robot system, thereby minimizing errors due to robot model. The interval analysis algorithm with ultrasonic sensor measurement to estimate robot position has been described. Additionally previous work on robot localisation and navigation using interval analysis has been extended so to incorporate sensor range limitation. Moreover instead of using dynamic interval model of the robot to predict the next step of the

interval robot position while tracking the robot position, the physical limitations of the robot are used to predict the next step in the interval analysis algorithm.

The IA robot position estimate is then used for an adaptive mechanism to correct the errors in the UKF robot position estimate. Then the newly implemented approach to fuse both these robot position estimation has been described and it can be observed that the UKF with the IA based adaptive mechanism gives a much accurate estimate when compared to estimates with UKF without the adaptive mechanism.

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