

A PRO-ACTIVE RESOLVER MODEL TO COPE WITH PARAMETER VARIABILITY IN THE MANUFACTURING CHAIN

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Abstract: In this paper a linearized model for Pancake resolvers is developed with the aim of compensating deviations on manufacturing inputs through computed corrections on the production controllable variables, mainly winding parameters. This model follows a two-step strategy where at the first step an accurate model computes the resolver nominal conditions and at a second step a linearized model based on production controllable variables computes the corrections on these controllable variables in order to compensate small deviations on the nominal conditions due to processes variability in the manufacturing. This model had been simulated and experimentally tested in a Siemens resolver manufacturing plant. The tests done proved the efficiency of the developed model and its usefulness in stabilizing the product specifications in a dynamic environment with high variability of manufacturing processes.

1 INTRODUCTION

Resolvers are nowadays widely spread in Industrial Applications (Logé, 1992). These electromagnetical devices which were, in the past, largely used in military applications, namely to control the position stability of heavy guns, are presently very common in industrial areas as a servomotor component (Golker, 1981). Servomotors are today widely spread in robotics, rotary machinery, aeronautics...

This continuing demand on resolvers pushes the research on new materials, new designs supported by theoretical work. Continuing research on old products is managed by the market. Similar research is done also on other old electromagnetical devices which improvements are demanded by the market (Ostolaza, 2002; Chang, 2003; Lin 2003).

The main factors that promote the widespread of the Resolvers as angular sensors in despite of optoelectronics-encoders are its robustness and stable accuracy in non-friendly environments such as mechanical vibrations and shocks, environments with dust, oil, radiations and very high stability in a

wide range of temperatures (-50° C to +150° C) and rotational velocities (1000 to 10000 turns/min.).

The main disadvantages of resolvers in relation to optoelectronic encoders are: the necessity of an AC-power source and the delivery of an analog output signal where the today's processing devices are mainly digital. However, the daily advances in the signal processing technology allows more and more speedy and cost efficient solutions to convert analog to digital signals.

The main functional characteristics of resolvers are: the angular error, the output voltage (transformation ratio $-ü$), the phase shift and the input current. All these important factors specified by customers/ applications - usually referred as Customer characteristics - are strongly influenced by constructive factors such as: magnetic properties of stators and rotors, winding geometries, manufacturing tolerances of mechanical parts. As the assembly factors change continuously in a manufacturing plant (new material charge, different thermal treatment of magnetic metals...) it means that small adjustments at the windings parameters have to be made in order to compensate the existing

variability in the manufacturing process allowed by its tolerance chain.

Facing this situation it is clear that the existence of a mathematical model at the resolver manufacturer that allows him to compensate the variability of its processes by computing the corrections at the windings parameters in order to keep the customer specifications on target, saves him, yearly, a big amount of money by drastically reducing the number of trials needed, with different windings, until the customer characteristics are met again.

In this paper an inovative linearized mathematical model for Pancake-Resolvers (fig. 1) is developed in a way that it fits the needs of a resolver manufacturer to stabilize its product specifications in an environment with high variability of manufacturing processes.

2 PANCAKE-RESOLVER MODEL

2.1 Description

The Pancake resolver is today the most common resolver design for industry and aeronautics (fig. 1).



Pancake resolver



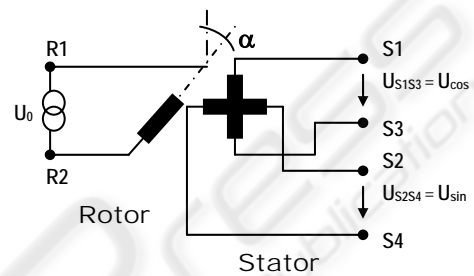
Traditional resolver with collector

Figure 1: Traditional and Pancake resolvers

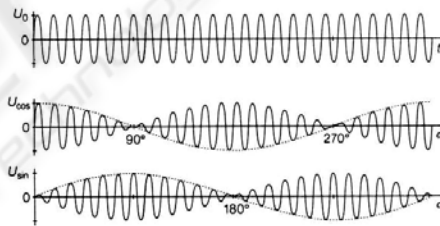
The Pancake resolver carries the current into the rotor through a transformer that is located at the stator edge. The advantage of such design, over the traditional resolver with collector system, is the absence of the relative movement between mechanical parts which causes wear, vibrations and sound.

Fig. 1 presented the two above mentioned designs.

Independently of how the energy is brought into the resolver rotor, the function of a resolver can be briefly illustrated in figure 2.



Resolver function schematics



Resolver input and output voltages

Figure 2: Resolver schematics and function

With an appropriate composition of the output voltages, the angular position of the rotor referred to the stator position can be obtained [2] as:

$$\alpha = \text{tg}^{-1} \left[\frac{\ddot{u}U_0 \sin \alpha}{\ddot{u}U_0 \cos \alpha} \right] = \text{tg}^{-1} \left[\frac{U_{S2S4}}{U_{S1S3}} \right] \quad (1)$$

Where:

\ddot{u} = transformation ratio;

α = relative angle rotor to stator;

U_0 = input voltage.

2.2 Mathematical Model

The common used mathematical model for a resolver is shown in fig. 3, and it is the typical model for a transformer.

This model is suitable to supply the usual customer demanded electrical characteristics of the resolver, namely the Rotor and Stator Impedances (open and short circuited).

According the below schematics, using the traditional circuit analysis methods, the following equations are obtained:

$$Z_{ro} = Z_1 + Z_3; \tag{2}$$

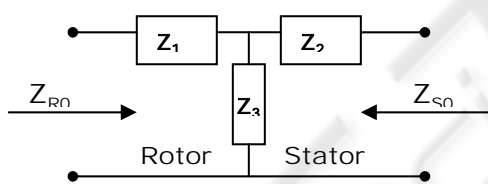
$$Z_{so} = Z_2 + Z_3; \tag{3}$$

$$Z_{rs} = Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3}; \tag{4}$$

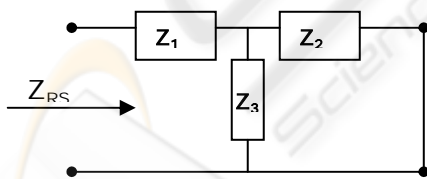
$$Z_{ss} = Z_2 + \frac{Z_1 Z_3}{Z_1 + Z_3}; \tag{5}$$

Where:

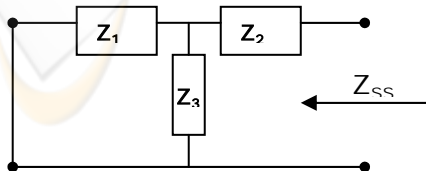
- Z_{ro} = Rotor Impedance with Stator open
- Z_{so} = Stator Impedance with Rotor open
- Z_{rs} = Rotor Impedance with Stator shorted
- Z_{ss} = Stator Impedance with Rotor shorted



Rotor Impedance with Stator open (Z_{ro}) and Stator Impedance with Rotor open (Z_{so})



Rotor Impedance with Stator shorted (Z_{rs})



Stator Impedance with Rotor shorted (Z_{ss})

Figure 3: Common used resolver models

This model although useful for computing the main electrical characteristics for customer needs is from less use for the sensors manufacturer. Actually this model doesn't copy with anyone of the directly controllable variables in a resolver manufacture.

The new model proposed in this paper is appropriate for resolver manufacturers because it deals explicitly with the actually controllable variables in a resolver production plant (mainly winding parameters).

The main variables that influence directly the customer specific electrical characteristics can be divided into 3 groups:

Group 1: Material related variables: magnetic permeability of the rotor, the stator, the rotor-transformer, the stator-transformer.

Group 2: Geometry related variables: stator dimensional tolerances, rotor dimensional tolerances, rotor/stator air-gap, transformer air-gap.

Group 3: windings related variables: windings distribution around the rotor and the stator, number of stator-windings, stator-windings wire diameter, number of rotor-windings, rotor-windings wire diameter, number of stator/transformer-windings, stator/transformer-windings wire diameter, number of rotor/transformer-windings, rotor/transformer-windings wire diameter.

From this 3 groups of variables, the resolver manufacturer can only influence on a feasible way the 3rd variables Group, since the other groups are usually fixed for the sensor manufacturer as he buys the materials and parts from external suppliers. Even if the resolver manufacturer is vertically integrated, producing also its parts, what is very unusual, the parts production pace and environment is completely apart from the resolver assembly line, this implies that, for having its parts, the assembly line has to deal always with stock management (the assembly line can never control the groups 1 and 2 related variables).

In such a scenario a useful resolver mathematical model for a resolver manufacturer must deal explicitly with the Group 3 Variables.

In Figueiredo, 2004, an explicit mathematical model for Pancake resolvers was proposed. This model although very accurate has its major application on the design of new products. For manufacturing purposes where the main needs are the compensation of the processes variability that affect the product characteristics and increase the scrap, that model has less application. In fact, those model variables cannot be directly used by the resolver manufacturer, as they account for the standard physical effects of an electromagnetic device (transformer ohmic resistances, inductances, electromagnetic losses in windings and metal...). These standard electromagnetic variables are very

useful for design purposes but they are not suitable for the resolver steady production as here the controllable variables are only the winding parameters (number of windings and wire diameters).

The model developed in this paper follows a two-step strategy where at first an accurate model defines the resolver nominal conditions and at a second step an additional linearized model (with production controllable variables) compensates the product for small changes on the manufacturing processes.

Analogous to the mathematical methodology of function expansion into a Taylor series, here also the strategy adopted is to consider the model developed by Figueiredo (Figueiredo, 2004) to compute the system nominal values – $f(x_0)$ – and additionally a linearized model dependent on production controllable variables which computes the function increments. These increments are able to cancel the deviations on the standard parameters due to the variability of the production processes in a resolver manufacturer. The incremental model that is developed in this paper is an innovative approach based on experimental parameter identification.

2.2.1 Model for nominal conditions – $[f(x_0)]$

Analysing the Pancake resolver functionally we can split this device into two transformers associated in series. The first one, the transformer which carries on the energy into the rotor, which output voltage is independent from the rotor position related to the stator, and the resolver function itself that can be modeled as a transformer which output voltage is dependent on the rotor to stator position (see fig 4).

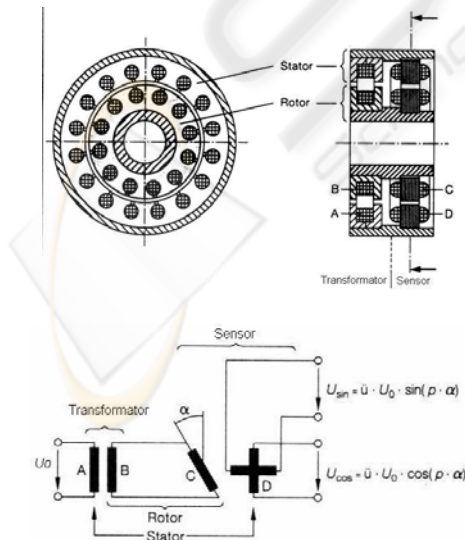


Figure 4: Pancake resolver schematics

The explicit mathematical model, proposed by Figueiredo (Figueiredo, 2004), for the main customer electrical characteristics: Output voltage for each stator winding (U_{cos} , U_{sin}) and Input current (I), is shown in the eqs. 6 and 7.

This model proved to be very accurate in the simulation of pancake resolvers (Figueiredo, 2004).

$$U_{S1S3} = \frac{1}{\ddot{u}_T \ddot{u}_D} \frac{A_3}{A_1 + A_3} U_0 \quad (6)$$

$$I = \frac{A_4}{A_1 + A_3} U_0 \quad (7)$$

where:

\ddot{u}_T = transformation ratio from Transformer
 \ddot{u}_D = transformation ratio from Sensor

$$A_3 = (R_{TFe} L_{hT} R'_{DFe} L'_{hT} S^2)$$

$$A_1 = L_{\sigma T1} a S^4 + [L_{\sigma T1} b + R_{T1} a + L'_{hD} (L_{\sigma T2} + L_{\sigma D1}) R_{TFe} L_{hT}] S^3 + [L_{\sigma T1} c + R_{T1} b + L'_{hD} R_{TFe} L_{hT} (R'_{T2} + R'_{D1}) + R'_{DFe} R_{TFe} L_{hT} (L_{\sigma T2} + L_{\sigma D1})] S^2 + [L_{\sigma T1} d + R_{T1} c + R'_{DFe} R_{TFe} L_{hT} (R'_{T2} + R'_{D1})] S + R_{T1} d$$

$$A_4 = a S^3 + b S^2 + c S + d$$

$$a = L'_{hD} L_{hT} (L_{\sigma T2} + L_{\sigma D1})$$

$$b = L'_{hD} [L_{hT} (R'_{T2} + R'_{D1}) + R_{TFe} (L_{\sigma T2} + L_{\sigma D1})] + R'_{DFe} L_{hT} (L_{\sigma T2} + L_{\sigma D1}) + R_{TFe} L_{hT} L'_{hD} + R'_{DFe} L_{hT} L'_{hD}$$

$$c = L'_{hD} R_{TFe} (R'_{T2} + R'_{D1}) + R'_{DFe} R_{TFe} L_{hT} + R'_{DFe} R_{TFe} L'_{hD} +$$

$$+ R'_{DFe} [L_{hT} (R'_{T2} + R'_{D1}) + R_{TFe} (L_{\sigma T2} + L_{\sigma D1})]$$

$$d = R'_{DFe} R_{TFe} (R'_{T2} + R'_{D1})$$

where:

R_{T1} = primary winding resistance -Transformer

R'_{T2} = secondary winding resistance - Transformer

R_{TFe} = magnetic metal resistance - Transformer

$L_{\sigma T1}$ = primary winding leakage inductance - Transformer

$L_{\sigma T2}$ = secondary winding leakage inductance - Transformer

L_{hT} = common flux inductance – Transformer

R_{D1} = primary winding resistance -Sensor

R'_{D2} = secondary winding resistance - Sensor

R_{DFe} = magnetic metal resistance - Sensor

$L_{\sigma D1}$ = primary winding leakage inductance - Sensor

$L_{\sigma D2}$ = secondary winding leakage inductance - Sensor

L_{hD} = common flux inductance – Sensor

The above model will be taken to compute the resolver nominal design variables (the standards for all product variables - $f(x_0)$ -). To compute the influences on the resolver main functional characteristics: output voltage (U_{\cos} , U_{\sin}) and input current (I) caused by small changes due to production processes variability, a differential model, that copies with the marginal changes on the controllable variables, is developed, in this paper, on sec. 2.2.2.

2.2.2 Incremental Model – $[(\partial f/\partial x_i)_0(\Delta x_i)]$

Having a general function f in $\mathbb{R}^n [f(x_1, x_2, \dots, x_n)]$ this function can be linearized around the point $(x_{10}, x_{20}, \dots, x_{n0})$ by cutting its Taylor's series development after the 1st order partial derivatives:

$$f(x_1, x_2, \dots, x_n) = f(x_{10}, x_{20}, \dots, x_{n0}) + \frac{\partial f}{\partial x_1} \Big|_0 (x_1 - x_{10}) + \dots + \frac{\partial f}{\partial x_n} \Big|_0 (x_n - x_{n0}) \quad (8)$$

This approach is used to compute the influences on the resolver main functional characteristics: output voltage (U_{\cos} , U_{\sin}) and input current (I) caused by small changes on the controllable variables.

The production controllable variables for an usual resolver manufacturer are essentially the windings parameters.

In Fig. 5 the resolver controllable model for a standard manufacturer is shown, where the considered variables account for:

- U_0 = resolver input voltage;
- F = input frequency;
- n_{st} = number of windings of the stator transformer;
- n_{rt} = number of windings of the rotor transformer;
- n_{ss} = number of windings of the stator sensor;
- n_{rs} = number of windings of the rotor sensor;
- ϕ_{st} = winding wire diameter of the stator transformer;
- ϕ_{rt} = winding wire diameter of the rotor transformer;
- ϕ_{ss} = winding wire diameter of the stator sensor;
- ϕ_{rs} = winding wire diameter of the rotor sensor;

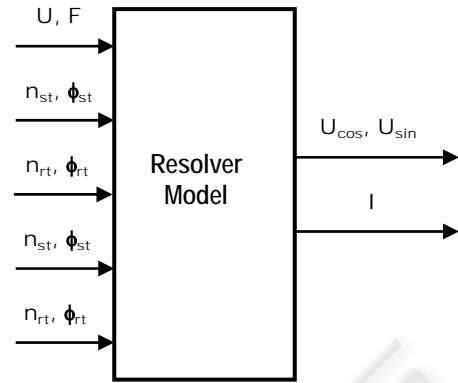


Figure 5: Resolver complete controllable Model

The differential model for the resolver output voltage (U_{\cos}) that copies with the marginal changes on the manufacturer controllable variables is:

$$\begin{aligned} U_{\cos}(U, F, n_{st}, n_{rt}, n_{ss}, n_{rs}, \phi_{st}, \phi_{rt}, \phi_{ss}, \phi_{rs}) = \\ = U_{\cos}(U_0, F_0, n_{st0}, n_{rt0}, n_{ss0}, n_{rs0}, \phi_{st0}, \phi_{rt0}, \phi_{ss0}, \phi_{rs0}) \Big|_0 + \\ + \frac{\partial U_{\cos}}{\partial U} \Big|_0 (U - U_0) + \frac{\partial U_{\cos}}{\partial F} \Big|_0 (F - F_0) + \\ + \frac{\partial U_{\cos}}{\partial n_{st}} \Big|_0 (n_{st} - n_{st0}) + \frac{\partial U_{\cos}}{\partial n_{rt}} \Big|_0 (n_{rt} - n_{rt0}) + \\ + \frac{\partial U_{\cos}}{\partial n_{ss}} \Big|_0 (n_{ss} - n_{ss0}) + \frac{\partial U_{\cos}}{\partial n_{rs}} \Big|_0 (n_{rs} - n_{rs0}) + \\ + \frac{\partial U_{\cos}}{\partial \phi_{st}} \Big|_0 (\phi_{st} - \phi_{st0}) + \frac{\partial U_{\cos}}{\partial \phi_{rt}} \Big|_0 (\phi_{rt} - \phi_{rt0}) + \\ + \frac{\partial U_{\cos}}{\partial \phi_{ss}} \Big|_0 (\phi_{ss} - \phi_{ss0}) + \frac{\partial U_{\cos}}{\partial \phi_{rs}} \Big|_0 (\phi_{rs} - \phi_{rs0}) \quad (9) \end{aligned}$$

Using the same approach, the influences on the input current (I) caused by small changes in the controllable variables can be computed as:

$$\begin{aligned} I(U, F, n_{st}, n_{rt}, n_{ss}, n_{rs}, \phi_{st}, \phi_{rt}, \phi_{ss}, \phi_{rs}) = \\ = I(U_0, F_0, n_{st0}, n_{rt0}, n_{ss0}, n_{rs0}, \phi_{st0}, \phi_{rt0}, \phi_{ss0}, \phi_{rs0}) \Big|_0 + \\ + \frac{\partial I}{\partial n_{st}} \Big|_0 (n_{st} - n_{st0}) + \frac{\partial I}{\partial n_{rt}} \Big|_0 (n_{rt} - n_{rt0}) + \end{aligned}$$

$$\begin{aligned}
 & + \frac{\partial I}{\partial U} \Big|_0 (U - U_0) + \frac{\partial I}{\partial F} \Big|_0 (F - F_0) + \\
 & + \frac{\partial I}{\partial n_{ss}} \Big|_0 (n_{ss} - n_{ss0}) + \frac{\partial I}{\partial n_{rs}} \Big|_0 (n_{rs} - n_{rs0}) + \\
 & + \frac{\partial I}{\partial \phi_{st}} \Big|_0 (\phi_{st} - \phi_{st0}) + \frac{\partial I}{\partial \phi_{rt}} \Big|_0 (\phi_{rt} - \phi_{rt0}) + \\
 & + \frac{\partial I}{\partial \phi_{ss}} \Big|_0 (\phi_{ss} - \phi_{ss0}) + \frac{\partial I}{\partial \phi_{rs}} \Big|_0 (\phi_{rs} - \phi_{rs0}) \quad (10)
 \end{aligned}$$

The several partial diferencial functions stated on both models (eqs. 9 and 10) have been experimentally evaluated, with a set of measuring points, which were fitted by 2nd order polynomials. This method proved to be very suitable for this purpose (Cruz, 1997).

3 SIMULATION AND EXPERIMENTAL RESULTS

3.1 Simulation Results

The nominal model shown in 2.2.1 was numerically evaluated with the software Matlab (Mathworks) for different resolver winding designs. All the winding designs were configured for the Siemens 1-speed resolver H2109 which has the following main characteristics:

$$\begin{aligned}
 U &= 5V; \\
 \text{Freq.} &= 4\text{kHz} \\
 I_{\text{max.}} &= 50 \text{ mA}
 \end{aligned}$$

The parameters referred on the nominal model (eqs. 6 and 7) were experimentally evaluated following the methodology proposed by Figueiredo (Figueiredo, 2004). Applying this methodology for each one of the possible combinations of the 10 controllable variables, it resulted on 10 different sets of parameters needed. These results account only for a single change in each one of the 10 controllable variables. Actually the experiments have been repeated, at least, for 5 different values for each variable, also it resulted finally on a total of 50 sets of parameters evaluated.

The simulated values for both customer main specifications: U_{cos} and I (according eqs. 6 and 7) are shown in figs. 6 to 13.

These figures show the model ability to deliver very good results when compared with the

experimental measurements for a broad configuration of windings.

The results shown here were selected from a huge quantity of computed data according to the following main criterion: - selection from the 8 defined controllable variables (number of windings and wire diameters) those that are, from manufacturer side, easier to change, and that produce effectiver results on the customer main specifications (U_{cos} and I).

Concerning the Output voltage (U_{cos}) as it depends on the rotor relative position to the stator, the values shown, concern the zero electrical angle where the stator and rotor are align at the null value. This relative position rotor to stator is referred as $U_{\text{cos}(0)}$.

According the above criterion the following studies are presented in the below figures: $U_{\text{cos}(0)}(n_{st})$; $U_{\text{cos}(0)}(n_{rt})$; $U_{\text{cos}(0)}(\phi_r)$; $U_{\text{cos}(0)}(\phi_s)$; $I_{(0)}(n_{st})$; $I_{(0)}(n_{rt})$; $I_{(0)}(\phi_r)$; $I_{(0)}(\phi_s)$.

3.2 Experimental Results

The experimental results have been taken from the Siemens 1-speed Resolver H2109 which electrical main specifications had already been shown in 3.1.

As it had been also related in 3.1, in order to evaluate the model parameters, a huge amount of measurements had been carried on.

The experimental results shown here correspond to the simulated values shown in figs. 6 to 13. The plotting of the experimental results side by side with the simulated ones displays clearly the quality of the model developed in this paper.

The knowledge of the experimental curves that reflect the sensitivity of the resolver to each one of the production controlable variables proved to be a strong valuable tool to the manufacturer. Actually this knowledge allows the manufacturer to react to product deviations due to unknown changes in the production processes.

The production variables, selected by the manufacturer, to serve as the most suitable ones to react quickly to undesirable changes in the assembly processes have been already referred in 3.1. These variables are: n_{st} , n_{rt} , ϕ_r and ϕ_s .

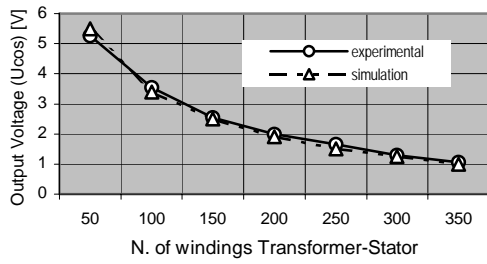


Figure 6: Output Voltage $U_{cos(0)}$ vs number of windings of the stator transformer (n_{st})

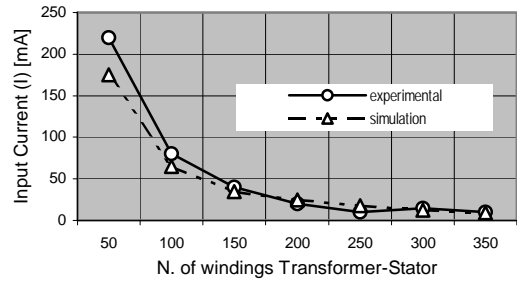


Figure 10: Input Current ($I_{(0)}$) vs number of windings of the stator transformer (n_{st})

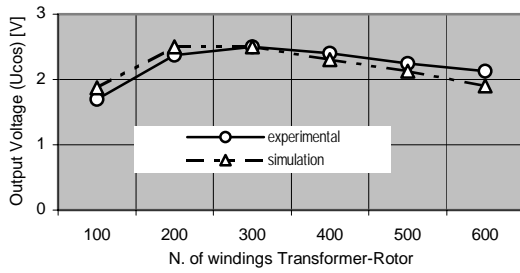


Figure 7: Output Voltage ($U_{cos(0)}$) vs number of windings of the rotor transformer (n_{rt})

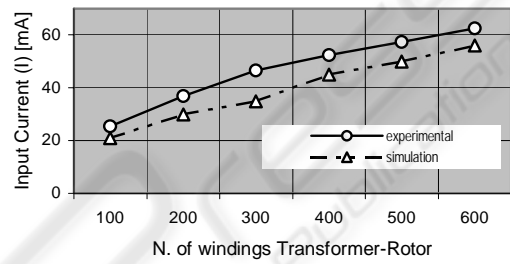


Figure 11: Input Current ($I_{(0)}$) vs number of windings of the rotor transformer (n_{rt})

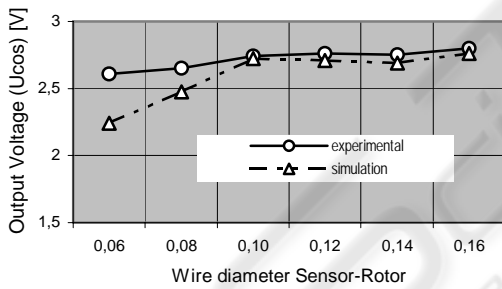


Figure 8: Output Voltage ($U_{cos(0)}$) vs winding wire diameter of the rotor sensor (ϕ_r)

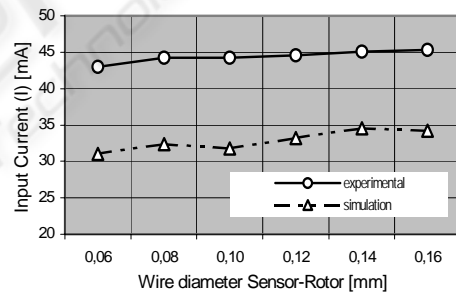


Figure 12: Input Current ($I_{(0)}$) vs winding wire diameter of the rotor sensor (ϕ_{rd})

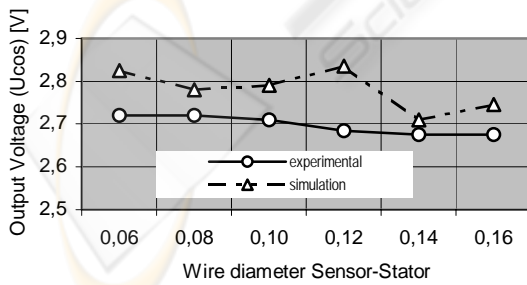


Figure 9: Output Voltage ($U_{cos(0)}$) vs winding wire diameter of the stator sensor (ϕ_s)

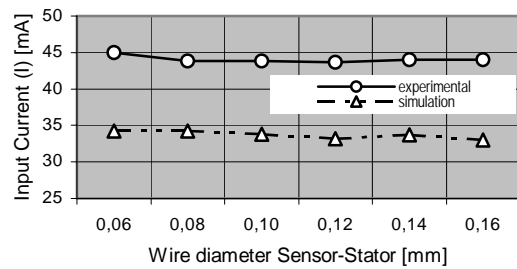


Figure 13: Input Current ($I_{(0)}$) vs winding wire diameter of the stator sensor (ϕ_s)

4 CONCLUSIONS

From a practical point of view the results from this research proved to be very valuable for the Siemens resolver manufacturer. In fact the knowledge of the experimental curves that reflect the sensitivity of the resolver to each one of the production controllable variables proved to be a strong valuable tool to the manufacturer. Actually this knowledge allows the manufacturer to react quickly to product deviations due to unknown changes in the production processes.

From a scientific point of view the accuracy of the combined model strategy (nominal and incremental models) delivers results with an average error, in the worst case, of 22%.

This error is still substantial and it denotes that there are some physical effects that should be better accounted on the nominal model, specially when it concerns the wire diameters. The incremental model must remain as it was presented, as long as the winding parameters remain the only controllable variables for the resolver manufacturer.

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