ON THE DECENTRALIZED CONTROL OF LARGE DYNAMICAL COMPLEX SYSTEM

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Abstract: This paper describes a systematic procedure to build reduced order analytical models for a design of decentralized controllers for large scale interconnected dynamical systems. The design method employs Davison techniques to affect decoupling of the interconnections into its subsystems components which is done by using the most dominant eigenvalues and the most influent inputs in each subsystem. In this way, advantage can be taken of the special structural feature of a given system to devise feasible and efficient decentralized strategies for solving large control problem which are impractical to solve by one shot centralized methods.

1 INTRODUCTION

As many technological environmental or social systems have a high complexity, large scale systems became the subject of intensive research in systems and control theory. The complexity of the system leads to severe difficulties that are encountered in the tasks of analyzing the system and designing and implementing appropriate control strategies algorithms. These difficulties arise mainly from dimensionality, uncertainty and information structure constraints. For these reasons the analysis and synthesis tasks cannot be solved economically in a single step as it is possible for similar analysis and design tasks for small system. Therefore, it is common procedure in engineering practice to work with mathematical models that are simpler, but less accurate, then the best available model of a given physical process, since the amount of computation required to analyze and control large scale system grows faster than its size. It has been long recognized that it is beneficial to decompose a large scale system into subsystems, and design control for each subsystem independently on the basis of the local subsystems dynamics and the nature of their interconnections. These are two quite distinct motivations for this practice:
The first is to reduce the computational burden associated with simulation, analysis and control system design.
The second is based on the realization that a simplified model will lead to simplified control system design.

2 PROBLEM FORMULATION

Assume the large scale system is given by the following differential equation

$$\dot{x}(t) = A\,x(t) + B\,u(t) \tag{2.1a}$$

$$y(t) = D\,x(t) \tag{2.1b}$$

$$x(0) = 0 \tag{2.1c}$$

where x is an-vector of states and u is an-vector of inputs and both A and B are constant matrices of appropriate dimension, and let us assume that the system matrix A has distinct eigenvalues. Let the system described by equation (2.1) be composed of N subsystems with the i^{th} subsystem having x_i and u_i as its state and control vectors, respectively. Let the dimension of x_i and u_i be n_i and m_i respectively so that:

$$\sum_{i=1}^{N} n_i = n \quad \text{and} \quad \sum_{i=1}^{N} m_i = m$$

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The global system of (2.1) is assumed to be completely controllable and global feedback control law of the form

$u(t) = F\,x(t) + v(t)$

(2.2)

has been found using conventional state feedback control methods so that the eigenvalues of the closed loop system lie in the pre-assigned location in the s-plane, where F is an mxn constant matrix, is to be computed and v is an m-dimensional vector.

The substitution of (2.2) into the system of (2.1) yields

$\dot{x}(t) = \overline{A}x(t) + Bv(t)$

(2.3a)

where $\overline{A} = A + BF$

The decentralized control problem can now be stated as that of finding a set of decentralized controllers of the form

$u_i(t) = F_i x_i(t) + v_i(t)$

(2.4)

where $F_i\ (m_i \text{ x } n_i)$

In this paper a method is presented for the design of such controller to the turbine. The design methods employs appropriate modal and singular perturbation techniques to affect complete decoupling of the large scale system into its subsystem components. Once the decoupling process is complete, the decentralized controller design problem becomes that of finding local controllers for each of the decoupled subsystems in isolation of the rest.

3 EIGENVALUE CONTRIBUTION MEASURE

For the i^{th} subsystem, the n_i eigenvalues that contribute most to the controllability of the states of this subsystem are chosen. Let the similarity transformation

$x(t) = M\,z(t)$

(3.1)

be applied to the open loop system (2.1), where z is an n-dimensional dummy state vector.

Application of (3.1) to the system of equation (2.1) gives

$\dot{z}(t) = J\,z(t) + \Gamma u(t)$

(3.2a)

$z_0 = M^{-1}x_0$

(3.2b)

where $J = M^{-1}AM = diag(\lambda_i)$

$\Gamma = M^{-1}B$

and the system (3.2) experience step changes in all of its input variable

$u(t) = U = [\beta_1 \;\; \beta_2 \;\; \cdots\beta_m]^T$

(3.3)

where β_k are weighting factors. The steady-state response of state vectors z is calculated from (3.2) as

$z_{ss} = -J^{-1}\Gamma U = Z$

(3.4)

substituting (3.4) into (3.1) gives the following steady state response

$x_{ss} = -MZ$

(3.5)

In order to determine contribution of the j^{th} eigenvalues in the i^{th} state variable, the following measure is used:

$\omega_{ij} = |m_{ij}Z_j| \qquad i,j = 1,2,..n$

(3.6)

where m_{ij} is the element standing on the i^{th} row and j^{th} column of the transformation matrix M, and Z_j is the j^{th} element of the vector Z.

The total contribution of the j^{th} eigenvalue, λ_j in σ states, $x_{k+1}, x_{k+2}, \cdots x_{k+\sigma}$, is determined from (3.6) as

$$\omega_j = \sum_{i=k+1}^{k+\sigma} \omega_{ij}$$

(3.7)

where k+1 is the index to the σ states.

4 DECOUPLING OF THE GLOBAL SYSTEM

The decoupling procedure is based on the outcome of the previous section and on the principals of singular perturbation techniques. Let the large scale system (2.1) be written as

$$\dot{x} = \begin{pmatrix} \dot{z}_d(t) \\ \dot{z}_r(t) \end{pmatrix} = \begin{bmatrix} A_1 & A_{12} \\ A_{21} & A_2 \end{bmatrix} \begin{bmatrix} z_d(t) \\ z_r(t) \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u(t) \quad 4.1)$$

where $z_d(t)$ is (r x 1) aggregated state vector of subsystem i and $z_r(t)$ is (n – r)th order residual state.

System equations (4.1) can be transformed to its modal form,

$$\begin{bmatrix} \dot{w} \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix} \begin{bmatrix} w \\ v_2 \end{bmatrix} + \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \end{bmatrix} \quad (4.2)$$

where w is the vector of retained dominant states variables,

$$x = Mv = M\left[w \;\vdots\; v_2\right]^T$$

$$J = Block - diag\left(J_1 \quad J_2\right) = M^{-1}AM$$

$$\Gamma = \left[\Gamma_1 \;\vdots\; \Gamma_2\right]^T = M^{-1}B$$

and M is a modal matrix. The columns of this matrix are its eigenvectors, and are ordered in accordance with the total contribution of each eigenvalue in all of the states of the ith subsystem

$$M = \left[\mu_1^d \;\vdots\; \mu_2^d \;\vdots\; \cdots \;\vdots\; \mu_n^d\right] = \begin{bmatrix} M_1 & M_{12} \\ M_{21} & M_2 \end{bmatrix}$$

Where μ_i^d, i = 1, 2… n are the dominant set of eigenvectors. Assume that is desired to retain d $(d \prec n)$ modes (vector w) of Eq. (4.2), that is

$$\dot{w} = PJP^T w + P\Gamma u \quad (4.3)$$

where $P = \left[I_d \;\vdots\; 0\right]$ (4.4)

and I is an identity matrix of order d partitioned as the subsystem i; and w = Pv

Let us take the Laplace transform of the lower half of (4.2) to yield

$$V_2(s) = (sI - J_2)^{-1}\Gamma_2 U(s) \quad (4.5)$$

since J_2 represents nondominant modes, Eq. (4.5) can be approximated by

$$v_2(t) = -J_2^{-1}\Gamma_2 u(t) = Lu(t) \quad (4.6)$$

The partitioned forms of z_r and v lead to

$$\begin{bmatrix} z_d \\ z_r \end{bmatrix} = \begin{bmatrix} M_1 & M_{12} \\ M_{21} & M_2 \end{bmatrix}\begin{bmatrix} w \\ v_2 \end{bmatrix} \quad (4.7)$$

$$z_d = M_1 w + M_{12} v_2$$

$$z_r = M_{21} w + M_2 v_2$$

assuming that M_1 is nonsingular, then by using these two last equations we get

$$z_r = M_{21}M_1^{-1}z_d + (M_2 + M_{21}M_1^{-1}M_{12})Lu$$

$$z_r = N z_d + E u \quad (4.8)$$

Eliminating z_r in Eqs. (4.1)), using Eq. (4.8) leads to the aggregate decoupled model in condensed form

$$\dot{z}_d = Gz(t) + Hu(t)$$

if we set $z_d(t) \equiv \tilde{x}(t)$, then

$$\dot{\tilde{x}} = G\tilde{x}(t) + H u(t)$$

$$G = A_1 + A_{12}N$$

$$H = B_1 + A_{12}E$$

in this method, the effects of the nondominant modes have been neglected to result in the decoupled model

5 DECENTRALIZED CONTROLLER DESIGN

In this section we develop the design of decentralized controller utilizing the approach outlined in the previous section.

After identifying the n_i eigenvalues which make the largest contribution to the dynamics of the i^{th} subsystem, we use the procedure outlined in § 4 to obtain an approximate model of the i^{th} subsystem given by:

$$\dot{\tilde{x}}_i(t) = G_i\tilde{x}_i(t) + H_i u(t) \qquad i = 1, 2, \ldots N \quad (5.1)$$

Thus the overall system can be approximated by

$$\dot{\tilde{x}}(t) = G\,\tilde{x}(t) + Hu(t) \quad (5.2)$$

where

$$\tilde{x}(t) = \begin{bmatrix} \tilde{x}_1(t) \\ \vdots \\ \tilde{x}_N(t) \end{bmatrix};\; G = \begin{bmatrix} G_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & G_N \end{bmatrix};\; H = \begin{bmatrix} H_1 \\ \vdots \\ H_N \end{bmatrix}$$

we set $u_j = \beta_j$ and $u_k = 0$, k = 1, 2…m; $k \neq j$ to calculate the controllability measure of each of the n_i eigenvalues of the i^{th} subsystem from the j^{th} input. We retain those columns of H_i that correspond to the input u_j that has the largest controllability measure, which gives the following approximate models of the i^{th} subsystem:

$$\dot{\tilde{x}}_i(t) = G_i\,\tilde{x}_i(t) + \hat{H}_i u \quad (5.3)$$

where

$$\hat{H} = \left[h_{ij}\right] = \begin{cases} \text{finite} & j = k \\ \text{null} & j \neq k \end{cases} \quad (5.4)$$

The global approximate model takes the following form:

$$\dot{\tilde{x}}(t) = Gx(t) + \hat{H}u(t) \quad (5.5)$$

where k is the index to those inputs that exert large influence on the behavior of subsystem i.

Let us assume that the global system described by (2.1) is completely controllable and a satisfactory global state feedback control law of the form

$$u(t) = Fx + v \quad (5.6)$$

has been found using existing state feedback control methods, so that the eigenvalues of the closed loop system lie in pre-assigned locations in the s-plane. This gives

$$\dot{\mathrm{x}}(\mathrm{t}) = \overline{\mathrm{A}}\mathrm{x}(\mathrm{t}) + \mathrm{Bv}(\mathrm{t})$$

(5.7)

where $\overline{\mathrm{A}} = \mathrm{A} + \mathrm{BF}$ is the closed loop system matrix

Next, we design a state feedback controller $\mathrm{u} = \hat{\mathrm{F}}\tilde{\mathrm{x}} + \mathrm{v}$ for the decoupled system (5.5), so that the closed-loop eigenvalues are the same or close to those of the original global closed-loop system (5.7). This yield

$$\dot{\tilde{\mathrm{x}}}(\mathrm{t}) = \hat{\mathrm{G}}\tilde{\mathrm{x}}(\mathrm{t}) + \hat{\mathrm{H}}\mathrm{v}$$

(5.8)

6 EXAMPLE

In this example a four interconnected power system (TAIPS) will be considered for the application of the proposed decentralized control approach.

The following state vectors are defined with respect to [10] as:

$\dot{\mathrm{x}}(\mathrm{t}) = \mathrm{Ax}(\mathrm{t}) + \mathrm{Bu}(\mathrm{t})$ where

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix} \text{ such that}$$

Subsystem 1

$$\dot{x}_1(t) = \begin{bmatrix} 0 & 0 \\ -2 & -4 \end{bmatrix} x_1(t) + \begin{bmatrix} 0 & 0.4 & 0.5 \\ 0 & 0 & -0.6 \end{bmatrix} x_2(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_1(t) =$$

$$A_{11} x_1(t) + A_{12} x_2(t) + A_{13} x_3(t) + A_{14} x_4 + B_1 u(t)$$

where $[A_{13}] = [A_{14}] = [0]$

$$y_1(t) = [1 \quad 0] x_1(t)$$

Subsystem 2

$$\dot{x}_2(t) = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ -17 & -8 & -5 \end{bmatrix} x_2(t) + \begin{bmatrix} 0 & 0 \\ -0.4 & 0 \\ -0.5 & 0.6 \end{bmatrix} x_1(t) + \begin{bmatrix} 0 & -0.5 \\ 0 & 0 \\ 0.1 & 0 \end{bmatrix} x_3(t) + \begin{bmatrix} 0.3 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} x_4(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_2(t) =$$

$$A_{21} x_1(t) + A_{22} x_2(t) + A_{23} x_3(t) + A_{24} x_4(t) + B_2 u_2(t)$$

$$y_2(t) = [0 \quad 0 \quad 1] x_2(t)$$

Subsystem 3

$$\dot{x}_3(t) = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} x_3(t) + \begin{bmatrix} 0 & 0 & -0.1 \\ 0.5 & 0 & 0 \end{bmatrix} x_2(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_3(t) =$$

$$A_{33} x_3(t) + A_{32} x_2(t) + A_{31} x_1(t) + A_{34} x_4(t) + B_3 u_3(t)$$

where $[A_{31}] = [A_{34}] = [0]$

$$y_3(t) = [1 \quad 0] x_3(t)$$

Subsystem 4

$$\dot{x}_4(t) = \begin{bmatrix} 0 & 1 \\ -9 & -20 \end{bmatrix} x_4(t) + \begin{bmatrix} -0.3 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} x_2(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_4(t) =$$

$$A_{44} x_4(t) + A_{42} x_2(t) + B_4 u_4(t);$$

$$A_{41} = A_{43} = [0]$$

$$y_4(t) = [1 \quad 0] x_4(t)$$

Following the procedure given in the previous section we get:

Eigenvalues contributions

Table 1: Eigenvalue contribution measures

	$X_{1,2}$	$X_{3,4,5}$	$X_{6,7}$	$X_{8,9}$
λ_1=-19.53	0	0.001	0	0.05
$\lambda_2 = -2.05 + j4.2$	0.016	0.106	0.004	0.001
$\lambda_3 = -2.05 - j4.2$	0.016	0.106	0.004	0.001
$\lambda_4 = -2.003$	0.477	0.115	0.026	0.003
$\lambda_5 = -0.061$	1.095	0.355	0.018	0.063
$\lambda_6 = -0.57 + j0.01$	7.897	10.47	4.383	7.153
$\lambda_7 = -0.57 - j0.01$	7.897	10.47	4.383	7.153
$\lambda_8 = -1.06 + j0.03$	3.022	30.61	17.98	2.057
$\lambda_9 = -1.06 - j0.03$	3.022	30.61	17.98	2.057

From the table, we see that

Subsystem 1. The eigenvalues that contribute most in its states x_1, x_2 are $\lambda_{6,7}$

Subsystem 2. The eigenvalues that contribute most in its states are x_3, x_4, x_5 □ eigenvalues $\lambda_{8,9}$

Subsystem 3. States are x_6 , x_7 □ eigevalues $\lambda_{8,9}$

Subsystem 4. States are x_8 , x_9 □ eigenvalues $\lambda_{6,7}$

It is clear from these results that, the relative contribution measures are satisfactorily high.

6.1 System decoupling

Application of the decoupling procedure may now be carried out, incorporating the results of the previous section. As a result each subsystem is represented by an approximate model having the same states as the original subsystem, but with the input to the global system.

To determine the relative importance of each input to each subsystem, the controllability measure of the state of each subsystem from each input must be evaluated.

Table 2: Controllability measures for subsystem 1

states	$\lambda_{6,7}$			
	U_1	U_2	U_3	U_4
X_1	0.112	0.268	3.453	0.284
X_2	0.388	0.923	11.89	0.978

Table 3: Controllability measures for subsystem 2

states	$\lambda_{6,8,9}$			
	U_1	U_2	U_3	U_4
X_1	0.100	0.161	5.256	0.037
X_2	0.476	0.755	25.104	0.095
X_3	0.536	0.850	28.248	0.107

Table 3: Controllability for subsystem 3

states	$\lambda_{8,9}$			
	U_1	U_2	U_3	U_4
X_1	0.286	0.448	15.271	0.003
X_2	0.357	0.559	19.051	0.004

Table 4: Controllability measures for subsystem 4

states	$\lambda_{6,7}$			
	U_1	U_2	U_3	U_4
X_1	0.310	0.737	9.499	0.782
X_2	0.143	0.341	4.401	0.362

From the tables, the following conclusion with regard to the four subsystems can be easily made. For example; subsystem 1 is most influenced by u_3, subsystem 2 is influenced by u_3 and so on.

Accordingly the following approximate representations for each subsystem are obtained:

$$\dot{\tilde{x}}_1(t)=\begin{bmatrix}0.09 & 1.38\\ -0.32 & -1.24\end{bmatrix}\tilde{x}_1(t)+\begin{bmatrix}0.70\\ 0.37\end{bmatrix}u_3(t)$$

$$\dot{\tilde{x}}_2(t)=\begin{bmatrix}19.52 & 43.38 & 0.00\\ -9 & -20 & 0\\ -250 & -540.8 & -1.71\end{bmatrix}\tilde{x}_2(t)+\begin{bmatrix}-0.41\\ 0.19\\ 0.23\end{bmatrix}u_3(t)$$

$$\dot{\tilde{x}}_3(t)=\begin{bmatrix}17.87 & 39.84\\ -9 & -20\end{bmatrix}\tilde{x}_3(t)+\begin{bmatrix}-0.41\\ -0.30\end{bmatrix}u_3(t)$$

$$\dot{\tilde{x}}_4(t)=\begin{bmatrix}0.09 & 1.38\\ -0.32 & -1.24\end{bmatrix}\tilde{x}_4(t)+\begin{bmatrix}0.70\\ -0.37\end{bmatrix}u_3(t)$$

6.2 Design of optimal controller

The optimal control problem may be stated as that of finding the control input u(t) which, subject to the constraints given by the system dynamical equations, minimizes the following cost function:

$$J=\int_0^\infty\left[x^T\,'t)Qx(t)+u^T(t)Ru(t)\right]dt$$

where Q and R are the state and control weighting matrices, respectively. The solution to this is given by

u(t) = F x(t) where F is the state feedback optimal control matrix. If Q and R are chosen as:

Q = diag(0,2,2,0,0,0,2,0,2) and R = (1,16,4,1) then

$$F=\begin{bmatrix}-0.65 & 0.44 & 0.23 & 0.09 & -0.03\\ 0.01 & -0.002 & 0 & -0.001 & 0.001\\ 0.103 & -0.006 & 0.035 & -0.01 & 0\\ -0.42 & 0 & 0.156 & 0.058 & 0\end{bmatrix}$$

$$\begin{matrix}0 & -0.024 & 0.018 & 0\\ 0 & 0 & -0.001 & 0\\ -0.001 & 0.129 & -0.003 & 0\\ 0.078 & 0 & 0.104 & 0.0551\end{matrix}\Bigg]$$

$\overline{A}=A+BF$ have the following set of eigenvalues:

$\lambda_1=-19.48;\ \lambda_{2,3}=-2.05\pm j4.23;\ \lambda_4=-1.56$

$\lambda_5=-0.05;\ \lambda_{6,7}=-0.98\pm j0.301;\ \lambda_8=-0.54$

$\lambda_9=-0.63$

$$\tilde{F}=\begin{bmatrix}0.123 & -0.275 & 0 & 0 & 0\\ 0 & 0 & -0.838 & -1.72 & 0.008\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

$$\begin{matrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ -0.237 & -0.448 & 0 & 0\\ 0 & 0 & 0.123 & -0.275\end{matrix}\Bigg]$$

$\widetilde{A} = A + B\widetilde{F}$ has the following set of eigenvalues

$\lambda_1 = -19.82;\ \lambda_{2,3} = -2.09 \pm j4.53;\ \lambda_4 = -2.27$

$\lambda_5 = -1.75;\ \lambda_{6,7} = -0.54 \pm j0.07;\ \lambda_8 = -0.81$

$\lambda_9 = -0.063$

These eigenvalues are close to those of the closed-loop matrix $\overline{A}$.

7 SIMULATION RESULTS

Extensive simulation studies on the four subsystem interconnection have been carried out under both the decentralized and global optimal controllers. To test the effectiveness of the decentralized controller, the closed loop system performance was tested when multiple changes in the reference settings at different time intervals were introduced. Figures 1-4 show the two set of responses overlaid on each other.

8 CONCLUSION

An interconnected dynamical system comprising four subsystems has been considered as a study case. Based on the example studied the proposed design method appears to be quite attractive. A satisfactory global optimal controller was designed for the system. It was shown that the performance of the decentralized controller designed by using the method presented is satisfactorily close to that of the global optimal one.

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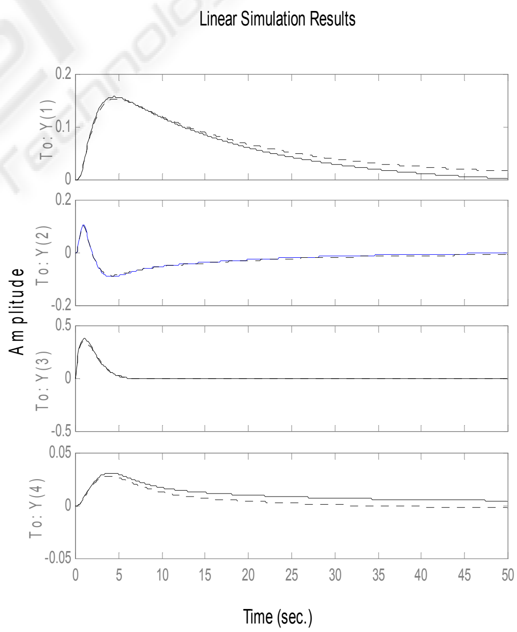

Figure 1: Responses to a step change in υ_3 at t = 0

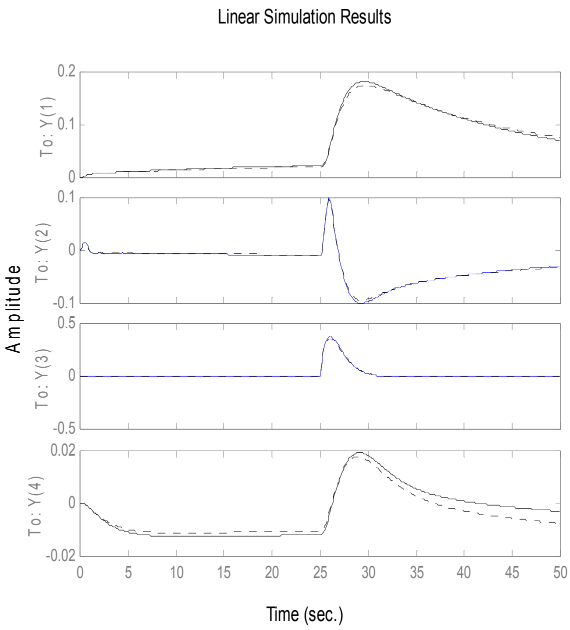

Figure 2: Responses to a step change in υ_1 at t = 0; υ_3 at t = 25s

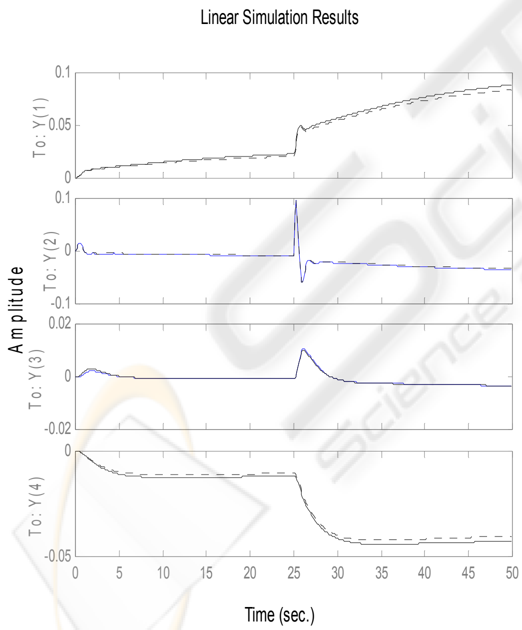

Figure 3: Responses to a step change in υ_1 at t = 0; υ_2 at t = 25s

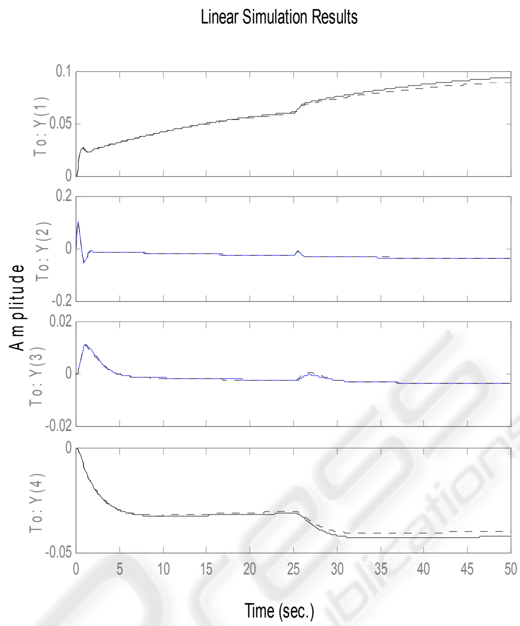

Figure 4: Responses to a step change in υ_2 at t = 0; υ_1 at t = 25s