Keywords: Water-resource Management, Planning, Quadratic Optimization, Predictive Optimization, Large Scale Systems, Water System.

Abstract: This paper presents a predictive optimization approach based on a quadratic minimization method to improve the water resource allocation planning of inland waterways. These networks are large scale systems composed of several interconnected reaches. Their management consists in keeping the water level of each reach close to an objective by allocating the available water resource among the network. It is particularly required in the context of global change where inland waterways should be strongly impacted by flood and drought events. The designed predictive optimization approach is achieved considering future horizons with the aim to reduce the impacts of extreme climate events thanks to anticipation of the management actions. A real part of the inland waterways in the north of France is considered in order to test the designed approach. The obtained management improvement comparing to water resource allocation planning methods that have been recently proposed in the literature is highlighted. The influence of the size of the predictive horizon is discussed.

1 INTRODUCTION

The study of the climate change impact on transport (Tafidis et al., 2017), and more specially on inland waterways is relatively recent (Koets and Rietveld, 2009; EnviCom, 2008; IWAC, 2009), with works on the inland waterways in UK (Arkell and Darch, 2006), in China (Wang et al., 2007) and on the Rhine (Jonkeren et al., 2007). The navigation is particularly vulnerable to the effects of extreme events, drought and flood which frequency and intensity are expected higher in close future (Bates et al., 2008; Boë et al., 2009; Ducharme et al., 2010). Indeed, the navigation is allowed only when the level of each canal is keeping inside a navigation rectangle that is defined by two boundaries around the setpoint: the Normal Navigation Level (NNL). Hence, an efficient management of water resource is required. It consists in allocating the available water and the water in excess (respectively) among all the waterways during drought periods and flood periods (respectively). A hierarchical management strategy has been proposed in (Duviella et al., 2013) to contribute to this objective. The water resource allocation planning is achieved in a deterministic way by defining Constraint Satisfaction Problems (Nouasse et al., 2015; Nouasse et al., 2016b), with quadratic optimization (Nouasse et al., 2016a), and with a stochastic view using Markov Decision Process in (Desquesnes et al., 2016). The quadratic approach is used considering a part of the real inland waterways of the north of France in (Duviella et al., 2018). In (Duviella et al., 2016), a water resource allocation planning over a future time horizon is proposed to reduce the pumping cost by anticipating the navigation demand. The proposed approach is based on a nonlinear programming solver that works by following an iterative process until some stopping criterion is reached. Moreover, it was tested using a fictive case-study. In this paper, the predictive water resource allocation planning is based on a quadratic programming solver that yields precise results numerically in a finite number of steps. This new approach leads to an improvement of the results that are obtained in (Duviella et al., 2018) on the part of the inland waterways of the north of France. The influence of the size of the predictive horizon is also discussed.

The paper is organized as follows: the part of the inland waterways in the north of France that is composed of three reaches is described in Section 2. The modelling methods that aim at facilitating the implementation of the allocation planning problem are described and exemplified by this case-study. They are
The studied inland waterways is composed of three Navigation Reaches (NR) that are linked to the Cuinchy-Fontinettes reach (see NR3 in Figure 1); a reach is a part of a canal between at least two locks. The NR3 is particularly important for the management of the waterways in the north of France. In effect, it is an artificial canal that can be used to dispatch water between three watersheds. The Cuinchy-Fontinettes reach is also a high water consumer. It is equipped with a lock downstream that consumes more than 25,000 m³ at each operation.

The physical characteristics of the three NR are given in Table 1. The NNL is the water level objective of each NR that corresponds to the objective depth. The boundaries, Low Navigation Level (LNL) and High Navigation Level (HNL) are given in relative according to the NNL.

Table 1: Dimensions of the NR, level objectives NNL and navigation limits.

<table>
<thead>
<tr>
<th>NR</th>
<th>NR1</th>
<th>NR2</th>
<th>NR3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length [km]</td>
<td>56.724</td>
<td>42.3</td>
<td>25.694</td>
</tr>
<tr>
<td>Width [m]</td>
<td>41.8</td>
<td>52</td>
<td>45.1</td>
</tr>
<tr>
<td>NNL [m]</td>
<td>3.7</td>
<td>4.3</td>
<td>3.3</td>
</tr>
<tr>
<td>LNL [m]</td>
<td>-0.05</td>
<td>-0.05</td>
<td>-0.05</td>
</tr>
<tr>
<td>HNL [m]</td>
<td>+0.1</td>
<td>+0.05</td>
<td>+0.05</td>
</tr>
</tbody>
</table>

The NR can be supplied or emptied by controlled and uncontrolled water volumes. The controlled water volumes come from gates and locks. They are expressed as:

- \( V_{c}^{i} \) (c: supply, c: controlled) is the controlled volume that supplies \( NR_{i} \) from another NR,
- \( V_{e}^{i} \) (e: empty) is the controlled volume that empties the NRs,
- \( V_{s}^{i} \) is the controlled volume from water intakes that supplies or empties the NRs. This volume is signed. Here it is negative for NR1. The uncontrolled water volumes come from water intakes or rain. They are expressed as:
- \( V_{u}^{i} \) (u: uncontrolled) is the uncontrolled volume from natural rivers, rainfall-runoff, Human uses. These volumes are signed. Here, they are positive for the three NR.

At each lock operation, an amount of water volume are exchanged between the upstream NR and the downstream NR. It is denoted \( v^{i,j} \). These water...
volume exchanges depend only to the navigation demand. The gates are controlled to deliver a discharge inside an operating range. The operating ranges of the gates are given in Table 2.

<table>
<thead>
<tr>
<th>NR</th>
<th>NR1</th>
<th>NR2</th>
<th>NR3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q^a ) ([\text{m}^3/\text{s}])</td>
<td>([-1; -1])</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( Q^m ) ([\text{m}^3/\text{s}])</td>
<td>6.56</td>
<td>0.63</td>
<td>1.2</td>
</tr>
<tr>
<td>( Q^m_{up} ) ([\text{m}^3/\text{s}])</td>
<td>-</td>
<td>([0; 6.4])</td>
<td>([0; 60])</td>
</tr>
<tr>
<td>( v^m_{up} ) ([\text{m}^3])</td>
<td>6.709</td>
<td>3.526</td>
<td>5.904</td>
</tr>
<tr>
<td>( v^m_{dw} ) ([\text{m}^3])</td>
<td>-</td>
<td>23,000</td>
<td>7,339</td>
</tr>
</tbody>
</table>

In Figure 2.c is depicted the flow-based network of the case-study. It is built according to the integrated model following the definition and the step given in (Nouasse et al., 2015). The flow-based network is composed of a set of ordered nodes (vertices) \( \mathcal{N} \) and a set of arcs (directed edges) \( \mathcal{A} \). There are nodes that correspond to the NR and two additional nodes; a common source vertex \( O \) without incoming edges, a common sink node without outgoing edges, denoted \( S \). The total number of nodes is \( \eta = \text{card}(\mathcal{N}) + 2 \). Hence, the flow-based network \( \mathcal{G} = (\mathcal{N}, \mathcal{A}) \) of the system is composed of 5 nodes, \( i.e. \eta = 5 \), whose 3 nodes correspond to the three NR.

The arcs represent the possible water exchanges between the NR, the source, \( i.e. \) water volumes that supply the waterways, and the sink, \( i.e. \) water volumes that empty the waterway. The arcs are defined as a couple \( a = (i, j) \), \( a \in \mathbb{R}^\alpha \) with \( \alpha = \text{card}(\mathcal{A}) \), where \( i \) and \( j \) are the origin and destination nodes. At each arc is associated a flow \( \phi_a(k) = \phi_{ij}(k) \) that represent the transferred water volumes between nodes \( i \) and \( j \) at time \( k \). These flows are bounded by the physical characteristics of the hydraulic devices. Thus, each flow has to respect \( l_{ij}(k) \leq \phi_{ij}(k) \leq u_{ij}(k) \), where \( l_{ij}(k) \) and \( u_{ij}(k) \) are the lower and upper bound constraints respectively.

For the case-study, the flows and their boundary conditions are given by:

\[
\begin{align*}
\phi_{01} & \in [v^m_{up}, \beta_{01}(k) + Q^m_1; T_M; v^m_{up}, \beta_{01}(k) + Q^m_1; T_M], \\
\phi_{02} & \in [Q^m_1; T_M; Q^m_1; T_M], \\
\phi_{03} & \in [Q^m_1; T_M; Q^m_1; T_M], \\
\phi_{15} & \in [Q^m; T_M; Q^m; T_M], \\
\phi_{25} & \in [v^m_{up}, \beta_{25}(k); v^m_{up}, \beta_{25}(k) + Q^m_2; T_M], \\
\phi_{35} & \in [v^m_{up}, \beta_{35}(k); v^m_{up}, \beta_{35}(k) + Q^m_3; T_M], \\
\phi_{12} & \in [v^m_{up}, \beta_{12}(k); v^m_{up}, \beta_{12}(k) + Q^m_2; T_M], \\
\phi_{13} & \in [v^m_{up}, \beta_{13}(k); v^m_{up}, \beta_{13}(k) + Q^m_3; T_M], \\
\end{align*}
\]

where \( \beta_{ij}(k) \in \mathbb{N} \) is the number of lock operations of the lock between nodes \( i \) and \( j \) on a given period \( T_M \), \( T_M \) is expressed in \( 10^{-3} \) s to obtain volumes in \( 10^3 \cdot [\text{m}^3] \), and \( Q \) is the upper value of the controlled discharge interval. As an example, the upper bound capacities for arcs \( \{\phi_{12}, \phi_{13}, \phi_{01}, \phi_{25}, \phi_{35}\} \) are the sum of the maximum available volumes from water intakes over \( T_M \), \( i.e. V^m = Q^m \cdot T_M \), and volumes that correspond to the lock operations \( v^m_{up}, \beta_{ij}(k) \). For more details, please refer to the rules defined in (Nouasse et al., 2015).

### 2.3 Water Resource Planning Objective

The dynamics of the waterways is modelled by considering the dynamics of each NR according to the integrated model. The dynamics of the NR is given by:

\[
V_i(k) = V_i(k - 1) + V^m_i(k) - V^c_i(k) + V^0_i(k) + V^{up}_i(k) - V^{down}_i(k),
\]

where \( k \) corresponds to the current period of time and \( k - 1 \) the last one.

This equation can be easily applied to the flow-based network by considering a relative volume dynamics and by giving a capacity of each node with the exception of the source and sink nodes. The dynamic capacity of each node is expressed as:

\[
d_i(k) = d_i(k - 1) + \phi_{ai}(k) - \phi_{ia}(k) \quad \text{for} \quad i \in \mathcal{N} \setminus \{O, S\},
\]

where \( a^+ \) is the set of arcs entering the node \( i \), \( a^- \) the set of arcs leaving the node \( i \), and \( d_i(k - 1) \) the capacity of the node \( i \) in the last period. Then, a relative volume objective that corresponds to the NNL is introduced. It is denoted \( D_i(k) \), with \( i \in \mathcal{N} \setminus \{O, S\} \), and such as \( D_i(k) = 0 \). To keep this objective, the water volume that supplies each node has to be equal to the water volume that empties it, at each step time. For real systems, this condition can not be guaranteed at each step time. Thus, an interval around the objective \( D_i(k) \) is allowed. It corresponds to the limits LNL and HNL and leads to \( d_i \leq D_i(k) \leq d_i \), with \( d_i \) and \( d_i \) the lower and upper bounds. The capacity \( d_i(k) \) can be negative or positive.

Even if an interval around the objective \( D_i(k) \) is allowed, the capacity \( d_i(k) \) has to be closest as possible to the objective. To this aim, a dynamical cost function \( W_i((D_i(k) - d_i(k))^2) \), \( i \in \mathcal{N} \setminus \{O, S\} \) is associated to each capacity \( d_i(k) \). This function aims at penalizing the gap between the current capacity \( d_i(k) \) and the objective \( D_i(k) \). It is expressed as:

\[\text{Improvement of Water Resource Allocation Planning of Inland Waterways based on Predictive Optimization Approach}\]
\[ W_i((D_i - d_i(k))^2) = \begin{cases} C_{\text{max}}(d_i)^2 (D_i - d_i(k))^2, & \text{if } d_i(k) \leq 0, \\ C_{\text{max}}(d_i)^2 (D_i - d_i(k))^2, & \text{if } d_i(k) > 0, \end{cases} \tag{4} \]

with \( C_{\text{max}} \) the maximal cost, assuming that \( D_i \) and \( d_i \) correspond to the lower and upper boundaries respectively.

Moreover, the way to supply or empty one NR has not the same cost. As example, the volume of water from natural river may be more expensive than the volume of water from upstream NR, i.e. water volume already inside the systems. Thus, a dynamical cost \( \omega_{ij}(k) \in \mathbb{R}^{IN} \) is associated to each arc \( a \). All these elements are used to define the criteria to optimize.

3  PREDICTIVE ALLOCATION PLANNING

The optimal water resource allocation consists in satisfying the objects of each node, i.e. \( D_i(k) \), by optimizing the flows \( \Phi(k) \) in terms of minimal cost. Two vectors \( \Phi(k) \) and \( \Delta(k) \) are introduced to gather the set of flows \( \phi_{ij}(k) \) and of capacities \( d_i(k) \) at time \( k \) respectively. The optimal sequence of flows \( \Phi(k) \) is determined by considering a management horizon \( H = n \times T_M \) with \( n \in \mathbb{N} \) and \( T_M \) the management period (in hours) to guarantee the objectives \( D_i(k) \) over the horizon \( H \) (Duvicella et al., 2016). The objective criterion to minimize is:

\[ f^H(x) = \sum_{i=1}^{\eta} \left[ \sum_{k} W_i (D_i(k) - d_i(k)) + \sum_{a} \omega_{ia} (k) \times \phi_{ia}(k) \right], \tag{5} \]

with \( \eta \) the number of nodes without nodes \( O \) and \( S \), and \( \alpha \) the number of arcs. The initial conditions are \( d_i(k-1) = 0 \) for \( i \in [1, \eta] \).

The quadratic programming method quadprog in Matlab is used to minimize \( f^H(x) \) under the equality constraints defined for each flow and each capacity:

\[
\min f^H(x) \text{ such that } \begin{cases} L^H(k) \leq x^H(k) \leq U^H(k), \\ A^H_{eq} \cdot x^H(k) = b^H_{eq}(k), \\ A^H \cdot x^H(k) \leq b^H(k) \end{cases} \tag{6} \]

where \( x^H(k) \) is the vector at time \( k \) gathering \( \Phi^H(k) \) and \( \Delta^H(k) \). The \( L^H(k) \) and \( U^H(k) \) are vectors that comprises all boundaries of \( \phi^H(k) \) and \( d^H(k) \) and have to be computed according to the boundary constraints on \( H \). The second condition is the dynamic relation of each node, where \( b^H_{eq} \) contains the values of \( d_i \) at the previous period, and \( A^H_{eq} \) is a vector composed of 0 or 1 following the structure of the network. Finally, the last condition is the representations of the linear coefficients, so \( A^H \) and \( b^H \) are equal to zero.

To calculate the matrix \( L^H(k) \) (respectively \( U^H(k) \)) is necessary know the lower boundaries (resp. high boundaries) on \( \Phi^H(k) \) for all the horizon time period. The matrix \( \Omega^H \) is composed of the weights of all the arcs for each step time \( k \in [1, n] \) of the management horizon \( H \). These vectors are obtained with the concatenation of each line of the matrices. For example for \( H = 3 \), if:

\[ L^H_0 = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, \tag{7} \]

the vector \( L^H_0(k) \) is equal to:

\[ L^H_0(k) = \begin{bmatrix} 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \end{bmatrix}. \tag{8} \]

The vector \( b^H(k) \) is computed of \( \eta \times n \) elements \( b^H_{ij}(k) \) with \( i \in [1, \eta] \) and \( j \in [1, n] \). The index \( i \) represents the node number at the time step \( l \). By considering only one node over the time horizon \( H = n \times T_M \) and the relation (3), it is possible to write the relations that give the value of the capacity \( d_1(l) \) at each time step:

\[
\begin{align*}
d_1(1) &= d_1(0) + \phi_{a_1}(1) - \phi_{a_1}(0) \\
d_1(2) &= d_1(1) + \phi_{a_1}(2) - \phi_{a_1}(1) \\
&\vdots \\
d_1(n-1) &= d_1(n-2) + \phi_{a_1}(n) - \phi_{a_1}(n-1) \\
d_1(n) &= d_1(n-1) + \phi_{a_1}(n) - \phi_{a_1}(n) 
\end{align*} \tag{9} \]

where \( d_1(0) \) corresponds to the initial capacity of the node, \( \phi_{a_1}(l) \) (resp. \( \phi_{a_2}(l) \)) are the arcs leaving (resp. entering) the node \( n \) at the time step \( l \). Thus, it is possible to express the value of \( d_1(n-1) \) at time \( n-1 \) such:

\[ d_1(n-1) = d_1(0) + \sum_{m=1}^{n-1} [\phi_{a_1}(m) - \phi_{a_1}(m)]. \tag{10} \]

Hence, the components of the vector \( b^H(k) \) can be computed. Its first elements \( b^H_{ij}(k), i \in [1, \eta] \) are equal to 0. The following elements are computed according to relations (3) and (11) such as:

\[ b^H_{ij}(k) = b^H_{ij}(k) + \sum_{m=1}^{l-1} [\phi_{a_1}(m) - \phi_{a_1}(m)], \tag{11} \]

with \( i \in [1, \eta], l \in [2, n] \), where \( \phi_{a_1}(m) \) (resp. \( \phi_{a_1}(m) \)) are the arcs leaving (resp. entering) the node \( n \) at the time step \( m \).

It is also necessary create the matrix \( \Omega^H \) that is composed of the weights of all the arcs for each step.

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time \( k \in [1, n] \). Then, the algorithm 1 is proposed to obtain the sequence of optimal flows \( \Phi_H^0 \).

That optimization approach leads to the determination of the optimal sequence of flows on the horizon \( H \). The proposed approaches are used in next sections to optimize the water dispatching on an inland navigation network.

Input: \( L_H^0, U_H, Q_H, W \)
Output: \( \Phi_H^0 \)
Build \( L_H^0(k) \) and \( U_H^0(k) \)
Build \( A_H^0 \) with \( G \)
Build \( b_H^0(k) \) with \( G \) following relation 11
Build \( f_H^0(k) \) with \( \Omega_H, W \) and \( G \)
\[
\begin{align*}
\min f(i) & \text{ such that} \\
L_H^0(i) & \leq x(i) \leq U_H^0(i) \\
A_{eq}x(i) & = b(i) \\
A_{x}x(i) & = b(i) \\
\end{align*}
\]
Return \( x_H^0(k) \)
\( \Phi_H^0 = x_H^0(k) \)
Algorithm 1: Time horizon optimization algorithm.

4 SIMULATION RESULTS

The Cuinchy-Fontinettes system is considered over two weeks, starting by Monday. The navigation demand is given and the daily lock operations \( B_{ij} \) are depicted in Figure 3. The navigation scheduled time is 14 hours, that means that the remaining 10 hours (night period) are not allowed the navigation. The navigation is also reduced the 7th day that corresponds to Sunday.

![Figure 3: Navigation demand over 15 days.](image)

It is assumed that water volumes that supply or empty the network from natural rivers \( \{\Phi_{O2}, \Phi_{O3}, \Phi_{S1}\} \) have less priority than the others \( \{\Phi_{O1}, \Phi_{O2}, \Phi_{O3}, \Phi_{S3}\} \). Thus, two different costs are chosen such as \( \{\omega_{O1}, \omega_{O2}, \omega_{O3}, \omega_{S25}, \omega_{O35}\} = 0 \) and \( \{\omega_{O2}, \omega_{O3}, \omega_{S1}\} = 1 \). In addition, the cost is tune as \( C_{max} = 2000 \), a big arbitrary value.

The proposed integrated model of the Cuinchy-Fontinettes systems has been implemented in Matlab/Simulink. A Matlab function is defined to use the proposed optimization approach.

Then, three simulated scenarios have been defined to estimate the impacts of extreme events on the system and to study the improvement of the predictive management strategy. It is supposed that these extreme events have only impacts on uncontrolled discharges from natural rivers. The first scenario is based on a normal period of navigation. There is no modification on \( Q_u(i) \) as it is depicted in Figure 4.a. The second scenario aims at simulating a rainy period with strong intensity, starting on day 3 and stopping only on day 12 (see Figure 4.b). The third scenario corresponds to a period of drought (see Figure 4.c).

![Figure 4: Climatic event impacts in percentage on uncontrolled discharges \( Q_u \) for the three scenarios.](image)
The second scenario highlights the impacts of strong rain on the Cuinchy-Fontinettes system for $H = 1$ (see Figure 7.b). The combination of the strong rain intensity and the no navigation day leads to an overflow on $NR_2$ in days 7 and 8. This rain creates flood. The impact of rain is highly reduced when the horizon $H = 5$ is used as it is shown for $NR_2$ in Figure 8.b. Here again, there is an anticipation in the setpoint determination that allows to empty more the $NR_2$ before the no navigation day. Even if the water level is close to the HNL on Saturday, the water level is kept inside the defined boundaries.

The drought scenario effects on the Cuinchy-Fontinettes system are shown in Figure 9 for $H = 1$. In this scenario, the most impacted reach is $NR_1$. $NR_1$ is the upstream NR that supplies the two other NR. Moreover, it has to supply a natural river with a constant control discharge of $1 \, m^3/s$. Thus, the effect of the strongest drought periods (day 7) has impacts on the $NR_1$ water level in days 8 and 9 (see Figure 9.a). At the opposite, the water level in $NR_2$ is close to the NNL during drought period (see Figure 9.b).

Figure 10 shows that the water resource allocation planning is improved when $H = 5$. The water level of the most impacted $NR_1$ is kept to the objective NNL. The water level of $NR_2$ oscillates around the NNL leading to the optimal global cost of the management strategy (see Figure 10.b).

These results show that optimizing the water allocation problem by considering the operating horizon leads to better performance. It remains one question concerning the size of the prediction horizon. To determine the best value of the prediction horizon, the
three scenarios are considered by increasing the value of $H$ from 1 to 10. Then, the global cost of the management strategy is computed and depicted in Figure 11 for the three scenarios. It is shown that the global cost decreases between $H = 1$ to $H = 4$ and remains stable for higher values. This is mainly due to the navigation scheduling of the Cuinchy-Fontinettes systems and to the fact that the effects of extreme events are known \textit{a priori}. That confirms that the consideration of $H = 5$ was well adapted to the management of the Cuinchy-Fontinettes systems.

5 CONCLUSIONS

In this paper, a predictive optimization approach based on a quadratic minimization method is proposed to improve the water resource allocation planning of inland waterways. A realistic case study, the Cuinchy-Fontinettes system is considered to evaluate these improvements by considering drought and rainy scenarios. The simulation results show that the anticipation of extreme events leads to an efficient management of inland waterways. However, even if some improvements are obtained, uncertainties on the impacts of extreme climate events have not been taken into account. It will be the main concern of future works. Moreover, it will be also possible to design a predictive water allocation planning by considering a sliding windows.

REFERENCES


