Concept Similarity under the Agent's Preferences for the Description Logic $\mathcal{FL}_0$ with Unfoldable TBox

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Abstract: Concept similarity refers to human judgment of a degree to which a pair of concepts is similar. Computational techniques attempting to imitate such judgment are called concept similarity measures. In Description Logics (DLs), we could regard them as a generalization of the classical reasoning problem of equivalence. That is, any two concepts are equivalent if and only if their similarity degree is one. When two concepts are not equivalent, the level of similarity varies depending not only on the objective factors (*e.g.* the structure of concept descriptions) but also on the subjective factors (*i.e.* the agent's preferences). The recently introduced notion called *preference profile* identified a collection of preferential elements in which any developments for concept similarity measure should consider. In this paper, we briefly review approaches of identifying the subsumption degree between $\mathcal{FL}_0$ concept descriptions and exemplify how one can adopt the viewpoint of preference profile toward the development of concept similarity measure under the agent's preferences in $\mathcal{FL}_0$. Finally, we investigate several properties of the developed measure and discuss future directions.

1 INTRODUCTION

Most Description Logics (DLs) are decidable fragments of first-order logic (FOL) (Baader et al., 2010) with clearly defined computational properties. DLs are the logical underpinnings of the DL flavor of OWL and OWL 2. The advantage of this close connection is that the extensive DLs literature and implementation experiences can be directly exploited by OWL tools. More specifically, DLs provide unambiguous semantics to the modeling constructs available in the DL flavor of OWL and OWL 2. These semantics make it possible to formalize and design algorithms for a number of reasoning services, which enable the development of ontology applications to become prominent. For instance, ontology classification (or ontology alignment) organizes concepts in an ontology into a subsumption hierarchy and assists in detecting potential errors of a modeling ontology. Though this subsumption hierarchy inevitably benefits ontology modeling, it merely gives two-valued responses, *i.e.* inferring a concept is subsumed by another concept or not. However, certain pairs of concepts may share commonality even though they are not subsumed. As a consequence, a considerable amount of research effort has been devoted to measuring similarity of two given concepts, *i.e.* a problem of *concept similarity measure* in DLs.

Intuitively, concept similarity refers to human judgment of a degree to which a pair of concepts in question is similar. Concept similarity measures are computational techniques attempting to imitate the human judgments of concept similarity. Indisputably, they often contribute to various kinds of applications. For example, they were employed in bio-medical ontology-based applications to discover functional similarities of gene such as (Ashburner et al., 2000), they are often used by ontology alignment algorithms such as (Euzenat and Valtchev, 2004), they can be employed in approximate reasoning such as (Raha et al., 2008; Sessa, 2002) and in analogical reasoning such as (Racharak et al., 2016c; Racharak et al., 2017b). Oftentimes, when similarity judgment is performed by a cognitive agent, the degree of similarity may vary w.r.t. the need and preferences of the agent. The following example illustrates such a case in which concept similarity measured not only w.r.t. objective factors (but also w.r.t. subjective factors) can give more intuitive results.

Example 1.1. An agent A is searching for a hotel

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room during his vacation. At that moment, he prefers to stay in a Japanese-style room or something similar. In the following, his desired room may be expressed as the concept $\mathsf{DesiredRoom}$. Suppose RoomA and RoomB are concepts in a room ontology as follows:

$$\begin{aligned} \mathsf{DesiredRoom} &\sqsubseteq \mathsf{Room} \sqcap \forall \mathsf{floor}.\mathsf{Tatami} \\ \mathsf{RoomA} &\sqsubseteq \mathsf{Room} \sqcap \forall \mathsf{floor}.\mathsf{Bamboo} \\ \mathsf{RoomB} &\sqsubseteq \mathsf{Room} \sqcap \forall \mathsf{floor}.\mathsf{Marble} \end{aligned}$$

Without considering his preferences, it may be understood that both RoomA and RoomB are equally similar to $\mathsf{DesiredRoom}$. However, taking into account his preferences, RoomA may appear more suitable (assuming that tatami and bamboo invoke similar feeling). In other words, he will not be happy if an intelligent system happens to choose RoomB for him.

Other examples can be found in (Tversky, 1977) where intended behaviors (desirable properties) of similarity measures were investigated. For example, people usually speak that "the portrait resembles the person" rather than "the person resembles the portrait". Also, people usually say that "the son resembles the father" rather than "the father resembles the son". These examples clearly point out that cognitive agents make similarity judgment under some subjective factors. Unfortunately, existing measures do not usually take into account subjective factors during computational procedures, though some may consider such as (Lehmann and Turhan, 2012; Tongphu and Suntisrivaraporn, 2015).

In order to develop similarity measures which can be performed under subjective factors, (Racharak et al., 2016b) has introduced a general notion called *concept similarity measure under preference profile* (and later extended in (Racharak et al., 2017a)). Instead of implicitly including preferential elements in the computational representation, (Racharak et al., 2017a) clearly separated those preferential elements from the computational procedures. Hence, the general notion makes an investigation of concept similarity measure under subjective factors more easily and provides more natural understanding when concept similarity measures are used under subjective factors.

It is worth noting that any particular DL $\mathcal{L}$ is determined by the concept constructors and the ontological constructors it provides. For instance, (Racharak et al., 2016b; Racharak et al., 2017a) concentrated on the DL $\mathcal{ELH}$, which offered the constructors conjunction ($\sqcap$), full existential quantification ($\exists r.C$), and the top concept ($\top$); and also, allowed to define role hierarchy axioms in a TBox. In this paper, we concentrate on the DL $\mathcal{FL}_0$, which provides the constructors conjunction ($\sqcap$), value restriction ($\forall r.C$), and the top concept ($\top$) (*cf.* Section 2). The main contribution of this paper is to introduce a computational technique for concept similarity measure under the agent's preferences for the DL $\mathcal{FL}_0$ (*cf.* Section 4 - 5). Finally, we relate the approach to the others (*cf.* Section 6) and discuss the future directions (*cf.* Section 7).

2 PRELIMINARIES

In this section, we review the basics of Description Logic $\mathcal{FL}_0$ in Subsection 2.1, particularly its syntax, semantics, and normal form which can be used for subsumption testing in $\mathcal{FL}_0$. Then, we review the notion of preference profile in Subsection 2.2.

2.1 Description Logic $\mathcal{FL}_0$

We assume finite sets CN of concept names and RN of role names that are fixed and disjoint. The set of concept descriptions, or simply concepts, for a specific DL $\mathcal{L}$ is denoted by $\mathsf{Con}(\mathcal{L})$. The set $\mathsf{Con}(\mathcal{FL}_0)$ of all $\mathcal{FL}_0$ concepts can be inductively defined by the following grammar:

$$\mathsf{Con}(\mathcal{FL}_0) ::= A \mid \top \mid C \sqcap D \mid \forall r.C$$

where $\top$ denotes the *top concept*, $A \in \mathsf{CN}$, $r \in \mathsf{RN}$, and $C, D \in \mathsf{Con}(\mathcal{FL}_0)$. Conventionally, concept names are denoted by A and B, concept descriptions are denoted by C and D, and role names are denoted by r and s, all possibly with subscripts.

A *terminology* or *TBox* $\mathcal{T}$ is a finite set of primitive concept definitions and full concept definitions, whose syntax is an expression of the form $A \sqsubseteq D$ and $A \equiv D$, respectively. A TBox is called *unfoldable* if it contains at most one concept definition for each concept name in CN and does not contain cyclic dependencies. Concept names occurring on the left-hand side of a concept definition are called defined concept names (denoted by $\mathsf{CN}^{\mathsf{def}}$), all other concept names are primitive concept names (denoted by $\mathsf{CN}^{\mathsf{pri}}$). A primitive definition $A \sqsubseteq D$ can easily be transformed into a semantically equivalent full definition $A \equiv X \sqcap D$ where X is a fresh concept name. When a TBox $\mathcal{T}$ is unfoldable, concept names can be expanded by exhaustively replacing all defined concept names by their definitions until only primitive concept names remain. Such concept names are called *fully expanded concept names*.[1]

An interpretation I is a pair $I = \langle \Delta^I, \cdot^I \rangle$, where Δ^I is a non-empty set representing the domain of the interpretation and $\cdot^I$ is an interpretation function which

[1] In this work, we assume that concept names are fully expanded and the TBox can be omitted.

assigns to every concept name A a set $A^I \subseteq \Delta^I$, and to every role name r a binary relation $r^I \subseteq \Delta^I \times \Delta^I$. The interpretation function $\cdot^I$ is inductively extended to $\mathcal{FL}_0$ concepts in the usual manner:

$$\top^I = \Delta^I; \qquad (C \sqcap D)^I = C^I \cap D^I;$$
$$(\forall r.C)^I = \{a \in \Delta^I \mid \forall b \in \Delta^I : (a,b) \in r^I \to b \in C^I\}.$$

An interpretation I is said to be a *model* of a TBox $\mathcal{T}$ (in symbols, $I \models \mathcal{T}$) if it satisfies all axioms in $\mathcal{T}$. I satisfies axioms $A \sqsubseteq C$ and $A \equiv C$, respectively, if $A^I \subseteq C^I$ and $A^I = C^I$. The main inference problem in $\mathcal{FL}_0$ is the concept subsumption problem.

Definition 2.1 (Concept Subsumption)**.** Given $C, D \in \mathsf{Con}(\mathcal{FL}_0)$ and a TBox $\mathcal{T}$, C is *subsumed* by D w.r.t. $\mathcal{T}$ (denoted by $C \sqsubseteq_{\mathcal{T}} D$) if $C^I \subseteq D^I$ for every model I of $\mathcal{T}$. Moreover, C and D are *equivalent* w.r.t. $\mathcal{T}$ (denoted by $C \equiv_{\mathcal{T}} D$) if $C \sqsubseteq_{\mathcal{T}} D$ and $D \sqsubseteq_{\mathcal{T}} C$.

When a Tbox $\mathcal{T}$ is clear from the context, we simply drop $\mathcal{T}$, *i.e.* $C \sqsubseteq D$ or $C \equiv D$.

Using the rewrite rule $\forall r.(C \sqcap D) \longrightarrow \forall r.C \sqcap \forall r.D$ together with the associativity, the commutativity, and the idempotence of $\sqcap$, any $\mathcal{FL}_0$ concepts can be transformed into an equivalent one of the form $\forall r_1 \ldots \forall r_n.A$ where $\{r_1, \ldots, r_n\} \subseteq \mathsf{RN}$ and $A \in \mathsf{CN}$. Such concepts can be abbreviated as $\forall r_1 \ldots r_n.A$ where $r_1 \ldots r_n$ is viewed as a word w over the alphabet of all role names. We note that when $n = 0$, *i.e.* the empty word ε, $\forall \varepsilon.A$ corresponds to A. Furthermore, a conjunction of the form $\forall w_1.A \sqcap \ldots \forall w_m.A$ can be abbreviated as $\forall L.A$ where $L := \{w_1, \ldots, w_m\}$ is a finite set of words over the alphabet. We also note that $\forall \emptyset.A$ corresponds to $\top$. Using these abbreviations, any concepts $C, D \in \mathsf{Con}(\mathcal{FL}_0)$ can be rewritten as:

$$C \equiv \forall U_1.A_1 \sqcap \cdots \sqcap \forall U_k.A_k \tag{1}$$
$$D \equiv \forall V_1.A_1 \sqcap \cdots \sqcap \forall V_k.A_k \tag{2}$$

where $\{A_1, \ldots, A_k\} \subseteq \mathsf{CN}$ and U_i, V_i are finite sets of words over the alphabet of role names. This normal form provides us the following characterization of subsumption in $\mathcal{FL}_0$ (Baader and Narendran, 2001):

$$C \sqsubseteq D \Leftrightarrow U_i \supseteq V_i \text{ for all } i, 1 \leq i \leq k \tag{3}$$

Theorem 2.1. Concept subsumption and concept equivalence without TBox (*i.e.* when the TBox is empty) in $\mathcal{FL}_0$ can be decided in polynomial time.

Example 2.1. (Continuation of Example 1.1) After unfolding and transforming into normal forms, each concept is represented as:[1]

$$\begin{aligned}
\mathsf{DesiredRoom} &\equiv \forall\{\varepsilon\}.X \sqcap \forall\emptyset.Y \sqcap \forall\emptyset.Z \sqcap \forall\{\varepsilon\}.\mathsf{R} \\
&\quad \sqcap \forall\{\mathsf{f}\}.\mathsf{T} \sqcap \forall\emptyset.\mathsf{B} \sqcap \forall\emptyset.\mathsf{M} \\
\mathsf{RoomA} &\equiv \forall\emptyset.X \sqcap \forall\{\varepsilon\}.Y \sqcap \forall\emptyset.Z \sqcap \forall\{\varepsilon\}.\mathsf{R} \\
&\quad \sqcap \forall\{\mathsf{f}\}.\mathsf{B} \sqcap \forall\emptyset.\mathsf{T} \sqcap \forall\emptyset.\mathsf{M} \\
\mathsf{RoomB} &\equiv \forall\emptyset.X \sqcap \forall\emptyset.Y \sqcap \forall\{\varepsilon\}.Z \sqcap \forall\{\varepsilon\}.\mathsf{R} \\
&\quad \sqcap \forall\{\mathsf{f}\}.\mathsf{M} \sqcap \forall\emptyset.\mathsf{T} \sqcap \forall\emptyset.\mathsf{B}
\end{aligned}$$

[1]Obvious abbreviations are used for succinctness.

where X, Y, and Z are fresh concept names. Using Equation 3, it yields $\mathsf{DesiredRoom} \not\sqsubseteq_\emptyset \mathsf{RoomA}$ and $\mathsf{DesiredRoom} \not\sqsubseteq_\emptyset \mathsf{RoomB}$.

2.2 Preference Profile

Preference profile was first introduced in (Racharak et al., 2016a) as a collection of preferential elements in which any developments of concept similarity measure should consider (later, it was improved in (Racharak et al., 2017a)). Its first intuition is to model different forms of preferences (of an agent) based on concept names and role names. Measures adopted this notion are flexible to be tuned by an agent and can determine the degree of similarity conformable to that agent's perception. We give its formal definition of each preferential aspect in the following definition.

Definition 2.2 (Preference Profile (Racharak et al., 2017a))**.** Let $\mathsf{CN}^{\mathsf{pri}}(\mathcal{T})$, $\mathsf{RN}^{\mathsf{pri}}(\mathcal{T})$, and $\mathsf{RN}(\mathcal{T})$ be a set of primitive concept names occurring in $\mathcal{T}$, a set of primitive role names occurring in $\mathcal{T}$, and a set of role names occurring in $\mathcal{T}$, respectively. A *preference profile* (denoted by π) is a quintuple $\langle \mathfrak{i}^{\mathfrak{c}}, \mathfrak{i}^{\mathfrak{r}}, \mathfrak{s}^{\mathfrak{c}}, \mathfrak{s}^{\mathfrak{r}}, \mathfrak{d} \rangle$ where $\mathfrak{i}^{\mathfrak{c}}, \mathfrak{i}^{\mathfrak{r}}, \mathfrak{s}^{\mathfrak{c}}, \mathfrak{s}^{\mathfrak{r}}$, and $\mathfrak{d}$ are partial functions such that:

- $\mathfrak{i}^{\mathfrak{c}} : \mathsf{CN}^{\mathsf{pri}}(\mathcal{T}) \to [0,2]$ is called a *primitive concept importance*;
- $\mathfrak{i}^{\mathfrak{r}} : \mathsf{RN}(\mathcal{T}) \to [0,2]$ is called a *role importance*;
- $\mathfrak{s}^{\mathfrak{c}} : \mathsf{CN}^{\mathsf{pri}}(\mathcal{T}) \times \mathsf{CN}^{\mathsf{pri}}(\mathcal{T}) \to [0,1]$ is called a *primitive concepts similarity*;
- $\mathfrak{s}^{\mathfrak{r}} : \mathsf{RN}^{\mathsf{pri}}(\mathcal{T}) \times \mathsf{RN}^{\mathsf{pri}}(\mathcal{T}) \to [0,1]$ is called a *primitive roles similarity*; and
- $\mathfrak{d} : \mathsf{RN}(\mathcal{T}) \to [0,1]$ is called a *role discount factor*.

We discuss the interpretation of each above function in order. Firstly, for any $A \in \mathsf{CN}^{\mathsf{pri}}(\mathcal{T})$, $\mathfrak{i}^{\mathfrak{c}}(A) = 1$ captures an expression of normal importance on A, $\mathfrak{i}^{\mathfrak{c}}(A) > 1$ and $\mathfrak{i}^{\mathfrak{c}}(A) < 1$ indicate that A has higher and lower importance, respectively, and $\mathfrak{i}^{\mathfrak{c}}(A) = 0$ indicates that A has no importance to the agent. Secondly, we define the interpretation of $\mathfrak{i}^{\mathfrak{r}}$ in the similar fashion as $\mathfrak{i}^{\mathfrak{c}}$ for any $r \in \mathsf{RN}(\mathcal{T})$. Thirdly, for any $A, B \in \mathsf{CN}^{\mathsf{pri}}(\mathcal{T})$, $\mathfrak{s}^{\mathfrak{c}}(A,B) = 1$ captures an expression of total similarity between A and B and $\mathfrak{s}^{\mathfrak{c}}(A,B) = 0$ captures an expression of total dissimilarity between A and B. Fourthly, the interpretation of $\mathfrak{s}^{\mathfrak{r}}$ is defined in the similar fashion as $\mathfrak{s}^{\mathfrak{c}}$ for any $r, s \in \mathsf{RN}^{\mathsf{pri}}(\mathcal{T})$. Lastly, for any $r \in \mathsf{RN}(\mathcal{T})$, $\mathfrak{d}(r) = 1$ captures an expression of total importance on a role (over a corresponding nested concept) and $\mathfrak{d}(r) = 0$ captures an expression of total importance on a nested concept (over a corresponding role), *e.g.* let $\mathfrak{d}(r_1) = 0.3$, then the degree of similarity under this preference between

$\forall r_1.A$ and $\forall r_1.B$ can be understood as 0.3 degree as the identical occurrence of r_1 has 0.3 importance.

It is worth noticing that role names appearing in $\mathcal{FL}_0$ are always primitive. This suggests that both $\mathsf{RN}^{\mathsf{pri}}(\mathcal{T})$ and $\mathsf{RN}(\mathcal{T})$ can be considered identically in Definition 2.2. Furthermore, due to the employed characterization, $\mathfrak{d}$ is not used in this paper.

3 FROM CONCEPT SUBSUMPTION TO SUBSUMPTION DEGREE

The idea of computing subsumption degree using the characterization of language inclusion was firstly proposed in (Racharak and Suntisrivaraporn, 2015). Two computational techniques on subsumption degree were developed, *viz.* the skeptical subsumption degree (in symbols, $\rightsquigarrow^s$) and the credulous subsumption degree (in symbols, $\rightsquigarrow^c$). The names *skeptical* and *credulous* were motivated by the fact that the degree obtained from $\rightsquigarrow^s$ is always less than or equal to the one obtained from $\rightsquigarrow^c$. Basically, the function $\rightsquigarrow^s$ checks set inclusions between sets of words whereas the $\rightsquigarrow^c$ calculates the proportion between sets of words. In the following, we rewrite their original definitions and include them here for self-containment.

Definition 3.1 (skeptical $\mathcal{FL}_0$ subsumption degree)**.** Let $C,D \in \mathsf{Con}(\mathcal{FL}_0)$ be in their normal forms and $\mathsf{W}(E,A)$ be a set of words w.r.t. the concept E and the primitive A. Then, a *skeptical $\mathcal{FL}_0$ degree* from C to D (denoted by $C \rightsquigarrow^s D$) is defined as follows:

$$C \rightsquigarrow^s D = \frac{|\{P \in \mathsf{CN}^{\mathsf{pri}} \mid \mathsf{W}(D,P) \subseteq \mathsf{W}(C,P)\}|}{|\mathsf{CN}^{\mathsf{pri}}|}, \tag{4}$$

where $|\cdot|$ denotes the set cardinality.

Definition 3.2 (credulous $\mathcal{FL}_0$ subsumption degree)**.** Let $C,D \in \mathsf{Con}(\mathcal{FL}_0)$ be in their normal forms and $\mathsf{W}(E,A)$ be a set of words w.r.t. the concept E and the primitive A. Then, a *credulous $\mathcal{FL}_0$ subsumption degree* from C to D (denoted by $C \rightsquigarrow^c D$) is defined as follows:[1]

$$C \rightsquigarrow^c D = \frac{\sum_{P \in \mathsf{CN}^{\mathsf{pri}}} \mu(D,C,P)}{|\mathsf{CN}^{\mathsf{pri}}|}, \tag{5}$$

where $|\cdot|$ denotes the set cardinality and $\mu(D,C,P) =$

$$\begin{cases} 1 & \text{if } \mathsf{W}(D,P) = \emptyset \\ \frac{|\mathsf{W}(D,P) \cap \mathsf{W}(C,P)|}{|\mathsf{W}(D,P)|} & \text{otherwise} \end{cases} \tag{6}$$

[1]We fix a minor error in (Racharak and Suntisrivaraporn, 2015) here, *i.e.* the condition $\mathsf{W}(D,P) = \emptyset$ is included.

It is worth observing that if $\frac{|\mathsf{W}(D,P) \cap \mathsf{W}(C,P)|}{|\mathsf{W}(D,P)|} = 1$, then $\mathsf{W}(D,P) \subseteq \mathsf{W}(C,P)$ holds (and vice versa). Following this observation, it is not difficult to show that $C \rightsquigarrow^s D \leq C \rightsquigarrow^c D$ for any $C,D \in \mathsf{Con}(\mathcal{FL}_0)$.

Proposition 3.1. For any $C,D \in \mathsf{Con}(\mathcal{FL}_0)$, it follows that $C \rightsquigarrow^s D \leq C \rightsquigarrow^c D$.

Proof. (Sketch) We only need to show that, for any $P \in \mathsf{CN}^{\mathsf{pri}}$, for any pair of $\mathsf{W}(D,P)$ and $\mathsf{W}(C,P)$ which share commonality without being subsumed, then it follows that $C \rightsquigarrow^s D \leq C \rightsquigarrow^c D$. Fix any $P \in \mathsf{CN}^{\mathsf{pri}}$. Also, let $\mathsf{W}(D,P) = \{r_1,\ldots,r_n,s_1,\ldots,s_m\}$ and $\mathsf{W}(C,P) = \{r_1,\ldots,r_n,t_1,\ldots,t_k\}$. Then, it is obvious that $C \rightsquigarrow^s D \leq C \rightsquigarrow^c D$ holds. ❑

Example 3.1. (Continuation of Example 2.1) Let us abbreviate the concepts $\mathsf{DesiredRoom}$, RoomA, and RoomB with DR, RA, and RB, respectively. It yields

$$\mathsf{DR} \rightsquigarrow^s \mathsf{RA} = \frac{|\{X,Z,R,T,M\}|}{|\{X,Y,Z,R,T,B,M\}|} = \frac{5}{7}$$

and $\mathsf{DR} \rightsquigarrow^c \mathsf{RA}$

$$= \frac{1 + \frac{|\{\varepsilon\} \cap \emptyset|}{1} + 1 + \frac{|\{\varepsilon\} \cap \{\varepsilon\}|}{1} + 1 + \frac{|\{f\} \cap \emptyset|}{1} + 1}{7} = \frac{5}{7}.$$

Similarly, it yields $\mathsf{DR} \rightsquigarrow^s \mathsf{RB} = \mathsf{DR} \rightsquigarrow^c \mathsf{RB} = \frac{5}{7}$.

The following theorem shows a very nice property inherited in both functions, which have not been investigated in (Racharak and Suntisrivaraporn, 2015).

Theorem 3.1. The functions $\rightsquigarrow^s$ and $\rightsquigarrow^c$ can be computed in polynomial time.

Proof. Since the size of normal forms is polynomial in the size of the original concepts, and since the inclusion checking and the proportion checking can be also decided in polynomial time, these functions can be computed in polynomial time. ❑

The fact that there exists two concept similarity measures corresponds to an experiment in (Bernstein et al., 2005). That is, similarity measures might depend on target applications (*e.g.* target ontologies) and applicable similarity measures should be personalized to the agent's similarity judgment style. These observations were also discussed in (Racharak et al., 2017a). Now, we are ready to exemplify how the notion of preference profile can be adopted toward the development of subsumption degree under preferences in $\mathcal{FL}_0$. We continue this in the next section.

4 FROM SUBSUMPTION DEGREE TO SUBSUMPTION DEGREE UNDER PREFERENCES

To exemplify a development of subsumption degree under preferences procedure, we generalize the function $\rightsquigarrow^s$ to expose preferential elements of preference profile. As a result, the new function $\overset{\pi}{\rightsquigarrow} s$ is also driven by the structural subsumption characterization by means of language inclusion in $\mathcal{FL}_0$. As aforementioned in Subsection 2.2, some preferential aspects of preference profile might be possibly not exposed, *e.g.* the role discount factor. This is indeed dependent to an adopted characterization (*e.g.* as in $\rightsquigarrow^s$).

We start by presenting a relevant aspect of preference profile in terms of *total functions* in order to avoid computing on null values. A *total concept similarity function* is also presented as $\hat{\mathfrak{s}} : \mathsf{CN}^{\mathsf{pri}} \times \mathsf{CN}^{\mathsf{pri}} \to [0,1]$ as follows:

$$\hat{\mathfrak{s}}(x,y) = \begin{cases} 1 & \text{if } x = y \\ \mathfrak{s}^c(x,y) & \text{if } (x,y) \in \mathsf{CN}^{\mathsf{pri}} \times \mathsf{CN}^{\mathsf{pri}} \\ & \text{and } \mathfrak{s}^c \text{ is defined on } (x,y) \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

Intuitively, identical concepts are considered totally similar, *i.e.* they are set to 1. Otherwise, in case that they are not defined, different concepts are considered totally dissimilar by default.

The next step is to generalize the function $\rightsquigarrow^s$. We rewrite the numerator of $\rightsquigarrow^s$ to:

$$\sum_{P \in \mathsf{CN}^{\mathsf{pri}}} \max_{Q \in \mathsf{CN}^{\mathsf{pri}}} \{\hat{\mathfrak{s}}(P,Q) \mid \mathsf{W}(D,P) \subseteq \mathsf{W}(C,Q)\} \tag{8}$$

Basically, Equation 8 combines value of each maximal primitive concepts similarity element. Its objective is to also take into account the value of each similar concept pair, if this value is defined.

We may also put the notion of concept importance into our computational procedure. As suggested in Subsection 2.2, this results in the flexibility for weighting on primitive concepts (ranging from having no importance to having the maximum importance).

To achieve this, we continue with a similar attempt. That is, a *total concept importance function* is introduced as $\hat{\mathfrak{i}} : \mathsf{CN}^{\mathsf{pri}} \to [0,2]$ as follows:

$$\hat{\mathfrak{i}}(x) = \begin{cases} \mathfrak{i}^c(x) & \text{if } x \in \mathsf{CN}^{\mathsf{pri}} \text{and } \mathfrak{i}^c \text{ is defined on } x \\ 1 & \text{otherwise} \end{cases} \tag{9}$$

Basically, the above equation says that each concept has normal importance by default, if it is not defined.

To take these matters into account, we should rewrite both the numerator and the denominator such that they expose some rooms for tuning with the concept importance. Thus, we rewrite each part, respectively, as follows:

$$\sum_{P \in \mathsf{CN}^{\mathsf{pri}}} \hat{\mathfrak{i}}(P) \cdot \max_{Q \in \mathsf{CN}^{\mathsf{pri}}} \{\hat{\mathfrak{s}}(P,Q) \mid \mathsf{W}(D,P) \subseteq \mathsf{W}(C,Q)\} \tag{10}$$

$$\sum_{P \in \mathsf{CN}^{\mathsf{pri}}} \hat{\mathfrak{i}}(P) \tag{11}$$

Finally, putting each rewritten part together yields a concrete function for concept subsumption degree under preference profile. We denote this new function by $\overset{\pi}{\rightsquigarrow} s$ (*cf.* Definition 4.1) as it presents a generalization of $\rightsquigarrow^s$ w.r.t. preference profile.

Definition 4.1 (skeptical $\mathcal{FL}_0$ subsumption degree under π)**.** Let $C, D \in \mathsf{Con}(\mathcal{FL}_0)$ be in their normal forms and $\mathsf{W}(E,A)$ be a set of words w.r.t. the concept E and the primitive A. Then, a *skeptical $\mathcal{FL}_0$ subsumption degree under* π from C to D (denoted by $C \overset{\pi}{\rightsquigarrow} s\ D$) is defined as follows:

$$C \overset{\pi}{\rightsquigarrow} s\ D = \frac{\sum_{P \in \mathsf{CN}^{\mathsf{pri}}} \hat{\mathfrak{i}}(P) \cdot \max_{Q \in \mathsf{CN}^{\mathsf{pri}}} \{\hat{\mathfrak{s}}(P,Q) | \mathsf{W}(D,P) \subseteq \mathsf{W}(C,Q)\}}{\sum_{P \in \mathsf{CN}^{\mathsf{pri}}} \hat{\mathfrak{i}}(P)} \tag{12}$$

Example 4.1. (Continuation of Example 2.1) Suppose that Bamboo is quite similar to Tatami. Then, ones may express the agent A's preferences as: $\mathfrak{s}^c(\mathsf{Bamboo}, \mathsf{Tatami}) = 0.8$. Following Definition 4.1, it yields that

$$\mathsf{DR} \overset{\pi}{\rightsquigarrow} s\ \mathsf{RA} = \frac{1+0+1+1+1+0.8+1}{|X,Y,Z,R,T,B,M|} = \frac{5.8}{7}$$

and

$$\mathsf{RA} \overset{\pi}{\rightsquigarrow} s\ \mathsf{DR} = \frac{0+1+1+1+0.8+1+1}{7} = \frac{5.8}{7}.$$

Similarly, it yields $\mathsf{DR} \overset{\pi}{\rightsquigarrow} s\ \mathsf{RB} = \mathsf{RB} \overset{\pi}{\rightsquigarrow} s\ \mathsf{DR} = \frac{5}{7}$.

Ones may also observe that this function has a nice property, *i.e.* there exists an algorithmic procedure whose execution time is upper bound by a polynomial expression. We state this in the following theorem.

Theorem 4.1. The function $\overset{\pi}{\rightsquigarrow} s$ can be computed in polynomial time.

Proof. Since the size of normal forms is polynomial in the size of the original concepts and since the maximum function and the inclusion checking can be decided in polynomial time, this function can be computed in polynomial time. ❏

It is worth noticing that subsumption degree under preferences for the credulous subsumption degree can also be developed by incorporating with the notions role importance ($\mathfrak{i}^{\mathfrak{r}}$) and primitive roles similarity ($\mathfrak{s}^{\mathfrak{r}}$) (*cf.* Definition 2.2). However, it requires us to well investigate how those elements should be incorporated and we leave this as a future task.

4.1 Backward Compatibility with $\rightsquigarrow^{s}$

Under a special setting of preference profile, the function $\overset{\pi}{\rightsquigarrow}s$ can be reduced backward to $\rightsquigarrow^{s}$. This means that $\overset{\pi}{\rightsquigarrow}s$ can be also used for a situation when preferences are not given. Following the convention introduced in (Racharak et al., 2016a; Racharak et al., 2017a), let us call this special setting the *default preference profile* (denoted by π_0). We give its formal definition as follows:

Definition 4.2 (Default Preference Profile)**.** Let $\mathsf{CN}^{\mathsf{pri}}(\mathcal{T})$ be a set of primitive concept names occurring in $\mathcal{T}$. The *default preference profile*, in symbol π_0, is the pair $\langle \mathfrak{i}^{\mathfrak{c}}_0, \mathfrak{s}^{\mathfrak{c}}_0 \rangle$ where

$$\mathfrak{i}^{\mathfrak{c}}_0(A) = 1 \text{ for all } A \in \mathsf{CN}^{\mathsf{pri}}(\mathcal{T}) \text{ and}$$

$$\mathfrak{s}^{\mathfrak{c}}_0(A,B) = 0 \text{ for all } (A,B) \in \mathsf{CN}^{\mathsf{pri}}(\mathcal{T}) \times \mathsf{CN}^{\mathsf{pri}}(\mathcal{T}).$$

As for its syntactic sugar, let us denote a setting on $\overset{\pi}{\rightsquigarrow}s$ by replacing the setting with π. For instance, we may write the setting with π_0 as $\overset{\pi_0}{\rightsquigarrow}s$. Next, we show that, under this special setting on $\overset{\pi}{\rightsquigarrow}s$, the computation produces the same outcome as $\rightsquigarrow^{s}$.

Proposition 4.1. For any $C,D \in \mathsf{Con}(\mathcal{FL}_0)$, $C \overset{\pi_0}{\rightsquigarrow}s\ D = C \rightsquigarrow^{s} D$.

Proof. (Sketch) We know that, for any $P \in \mathsf{CN}^{\mathsf{pri}}$, if $\mathsf{W}(D,P) \subseteq \mathsf{W}(C,P)$, the cardinality is increased by one. This is indeed equivalent to the value of $\mathfrak{s}^{\mathfrak{c}}$ w.r.t. the identical concepts (*i.e.* $\mathfrak{s}^{\mathfrak{c}}(A,A) = 1$ for any $A \in \mathsf{CN}^{\mathsf{pri}}(\mathcal{T})$). This is obvious. ❏

5 CONCEPT SIMILARITY UNDER PREFERENCE PROFILE IN $\mathcal{FL}_0$

A general notion of *concept similarity measure under preference profile* was originally proposed in (Racharak et al., 2016b) and was later extended in (Racharak et al., 2017a). It is worth noting that the general notion makes a clear distinction between the core computational procedure and the preference setting, rather than implicitly using or omitting the preferences. Thus, it provides us capabilities to study and understand intended behaviors on concrete measures of the concept similarity measure under preference profile *e.g.* when they are used under the agent's preferences. For instance, rather than saying that "the son resembles the father", we would say "if certain preferences or perspectives are fixed, the son and the father are similar to each other". This viewpoint is more natural to use and gives more intuitive computational understanding.

Definition 5.1 ((Racharak et al., 2017a))**.** Given a preference profile π, two concepts $C,D \in \mathsf{Con}(\mathcal{L})$, and a TBox $\mathcal{T}$, a *concept similarity measure under preference profile* w.r.t. a TBox $\mathcal{T}$ is a function $\overset{\pi}{\sim}_{\mathcal{T}} : \mathsf{Con}(\mathcal{L}) \times \mathsf{Con}(\mathcal{L}) \to [0,1]$.

When a TBox $\mathcal{T}$ is clear from the context, we simply write $\overset{\pi}{\sim}$. Furthermore, to avoid confusion on the symbols, $\overset{\pi}{\sim}_{\mathcal{T}}$ is used when referring to arbitrary measures of the notion.

To develop a concrete measure as an instance of $\overset{\pi}{\sim}$, we make a similar attempt as in (Racharak et al., 2017a). That is, we take a look on the logical notion of concept equivalence. Informally, ones may view the logical notion of concept equivalence as an operator for comparing two concepts, *i.e.* it returns true (1) if both are equivalent concepts; or it returns false (0) otherwise.

We recall that the logical notion of concept equivalence can be computed from subsumption checking w.r.t. two corresponding directions (*cf.* Definition 2.1). Analogously, an instance of concept similarity measure under preference profile should also be computed from a corresponding subsumption degree under preferences function w.r.t. two corresponding directions. Following this observation, we introduce a new measure in Definition 5.2. This measure is denoted by $\overset{\pi}{\sim}s$ as it is motivated from the fact that $\overset{\pi}{\rightsquigarrow}s$ is used for computing subsumption degree under preferences w.r.t. the two directions.

Definition 5.2. Let $C,D \in \mathsf{Con}(\mathcal{FL}_0)$ be in their normal forms and π be a preference profile as a preference setting from the agent. Then, the *skeptical $\mathcal{FL}_0$ similarity measure under preference profile* π between C and D (denoted by $C \overset{\pi}{\sim}s\ D$) is defined as follows:

$$C \overset{\pi}{\sim}s\ D = \frac{C \overset{\pi}{\rightsquigarrow}s\ D + D \overset{\pi}{\rightsquigarrow}s\ C}{2} \tag{13}$$

We may also argue to calculate the value by using alternative binary operators accepting the unit interval, *e.g.* based on the multiplication (in symbols, $\overset{\pi}{\sim}_{\times}$) on both directions or the root mean square (in symbols, $\overset{\pi}{\sim}_{rms}$) on values of both directions. Unfortunately, those give unsatisfactory values for the extreme cases. For example, $A \overset{\pi}{\sim}_{\times} \top = 0 \times 1 = 0$ and $A \overset{\pi}{\sim}_{rms} \top = \sqrt{\frac{0^2+1^2}{2}} = 0.707$, whereas $A \overset{\pi}{\sim}_{s} \top = \frac{0+1}{2} = 0.5$. Since $C \overset{\pi}{\sim}_{\times} D \leq C \overset{\pi}{\sim}_{s} D \leq C \overset{\pi}{\sim}_{rms} D$ for any concepts C and D, we agree with (Suntisrivaraporn, 2013; Racharak et al., 2017a) that the average-based definition given above is the most appropriate method.[1] Based on this form, $\overset{\pi}{\sim}_{s}$ basically determines the degree of concept similarity under preference profile, *i.e.* behavioral expectation of the measure will conform to the agent's perception.

Example 5.1. (Continuation of Example 4.1) It yields that

$$\mathsf{DR} \overset{\pi}{\sim}_{s} \mathsf{RA} = \frac{\frac{5.8}{7} + \frac{5.8}{7}}{2} = \frac{5.8}{7}.$$

Similarly, it yields that $\mathsf{DR} \overset{\pi}{\sim}_{s} \mathsf{RB} = \frac{5}{7}$. Since $\mathsf{DR} \overset{\pi}{\sim}_{s} \mathsf{RA} > \mathsf{DR} \overset{\pi}{\sim}_{s} \mathsf{RB}$, it corresponds to the agent A's perception that he may decide to stay in RoomA when his $\mathsf{DesiredRoom}$ is not available.

It is also worth noting that $\overset{\pi}{\sim}_{s}$ comes up with a very nice property as follows:

Theorem 5.1. The measure $\overset{\pi}{\sim}_{s}$ can be computed in polynomial time.

Proof. Since $\overset{\pi}{\rightsquigarrow}_{s}$ can be computed in polynomial time (by Theorem 4.1), this is trivial. ❑

Finally, it is worth stating that the measure $\overset{\pi}{\sim}_{s}$ can be also used in the case that a preference profile is not defined by the agent. In such a case, we can tune the profile setting to π_0. This property is immediately followed from Proposition 4.1.

Remark 5.1. Computing $\overset{\pi_0}{\sim}_{s}$ yields the degree of concept similarity merely w.r.t. the structure of concept descriptions in question.

6 RELATED WORK

Concept similarity has been widely studied in various fields, *e.g.* psychological science, computer science, artificial intelligence, and linguistic literature. Roughly, they can be classified into four ways, *viz.* path finding, information content, context vector, and semantic similarity. We review each way as follows.

The path finding approach requires to firstly construct the concept hierarchy. That is, the more general concepts they are, the more they are closer to the root of the hierarchy. Also, the more specific they are, the more they are closer to the leaves of the hierarchy. Once the hierarchy is constructed, the degree of concept similarity is computed from paths between concepts. Indeed, there are various ways for determining the degree. For instance, (Rada et al., 1989) used a path length between concepts according to successively either more specific concepts or less specific concepts. A similar approach was introduced in (Caviedes and Cimino, 2004) where the degree was computed based on the shortest path between concepts. Ones may also assign different weights to the role depth as in (Ge and Qiu, 2008). Unfortunately, this approach fully relies on the concept hierarchy and ignores constraints defined in the ontology.

The information content approach augments each concept with a corpus-based statistics. Generally, the information content of each concept in a hierarchy is calculated based on the frequency of occurrence of that concept in a corpus. The more specific concepts they are, the higher information content values of them will be. For instance, (Resnik, 1995) defined the degree of similarity between concepts as the information content of the least common subsumer of them. Intuitively, this measure was defined to calculate the degree of the shared information between concepts. However, this approach requires a set of world descriptions such as a text corpus; and also, may be not sufficient since many concept pairs may share the same least common subsumer.

On the one hand, the first two approaches may utilize the concept hierarchy to compute the degree of concept similarity. On the other hand, the context vector totally relies on the vector representation. Roughly speaking, each concept is represented by a context vector and the cosine of the angle between vectors is used to determine the degree of similarity between related concepts. Work which employs this approach includes (Pedersen et al., 2007; Patwardhan, 2006; Schütze, 1998).

The semantic similarity approach basically uses the syntax and semantics of DLs for the development of measures. A simple approach was proposed in (Jaccard, 1901) for the basic DL $\mathcal{L}_0$ (*i.e.* no use of roles). Later, the idea was extended in (Lehmann and Turhan, 2012) for the DL $\mathcal{ELH}$. The extended work suggested a new framework that satisfied several properties for similarity measure. The framework was defined in general; thus, functions and operators were parameterized and were left to be specified. A different approach for the same $\mathcal{ELH}$ was proposed in (Tongphu and Suntisrivaraporn, 2015) in which the measure was developed based on the structural subsumption characterization of tree homomorphism.

[1] Though we recommend to use the average, its choice of operators may be changed and it may produce a different behavior as discussed.

Indeed, this approach had its root in the study of similarity measure for the DL $\mathcal{EL}$ (Suntisrivaraporn, 2013). Later, it was extended to the DL $\mathcal{ALEH}$ in (Suntisrivaraporn and Tongphu, 2016). Furthermore, (Racharak and Suntisrivaraporn, 2015) suggested two measures for the DL $\mathcal{FL}_0$ based on the structural subsumption characterization of language inclusion. It is worth observing that these measures calculated the degree of concept similarity according to the structure of concept descriptions in question.

Instead of using the structure of concept descriptions, ones may try to compute the degree based on an interpretation of concepts for semantic similarity. These measures often employ the canonical interpretation and the set cardinality such as work in (D'Amato et al., 2009; D'Amato et al., 2008). Unfortunately, these measures strictly require an ABox.

Another approach for semantic similarity was proposed in (Alsubait et al., 2014). This work introduced a family of similarity measures in which a classical subsumption reasoner was used to determine features for calculating the degree based on the feature model.

While many similarity measures exist, a few of them utilize the agent's preferences for calculating the degree of concept similarity. In addition, existing approaches may implicitly use the agent's preferences in their computational procedures; thus, it is not easy to investigate intended behaviors of similarity measures if they are used under the agent's preferences. An experiment in (Bernstein et al., 2005) also suggests that measures should be made personalized to the target application (*e.g.* the agent). To help such investigation, a general notion called concept similarity measure under preference profile was introduced in (Racharak et al., 2016b) with the developed measure sim^π for the DL $\mathcal{ELH}$. This work was continuously studied in (Racharak et al., 2017a).

7 DISCUSSION AND FUTURE RESEARCH

This paper introduces a measure for identifying the degree of concept similarity under preferences in the DL $\mathcal{FL}_0$ w.r.t. an unfoldable TBox. This introduced measure is developed based on the calculation of subsumption degree under preferences w.r.t. two corresponding directions. To achieve this desire, we first review approaches of identifying the subsumption degree between $\mathcal{FL}_0$ concepts and generalize the approach based on the recently introduced notion called preference profile. As a result, the proposed measure can be regarded as an instance of concept similarity measure under preference profile (*cf.* Definition 5.1 and Definition 5.2). We have also investigated several properties of the measure, *viz.* its computational complexity and its backward compatibility. That is, when the TBox is unfoldable, computing the degree of concept similarity under preferences can be done in polynomial time. Furthermore, employing the default preference profile as its setting yields the degree w.r.t. the structure of concept descriptions.

In (Racharak et al., 2016b; Racharak et al., 2017a), the measure sim^π was introduced as a concrete measure of concept similarity measure under preference profile for the DL $\mathcal{ELH}$[1]. On the one hand, sim^π allowed to fully define preferential expressions over all types of preference profile. On the other hand, ones might still want to understand how concrete measures of $\overset{\pi}{\sim}$ for other DLs should be developed. This work provides an answer to that question. In this work, we concentrate on another sub-Boolean DL, *i.e.* $\mathcal{FL}_0$. We recall that $\mathcal{FL}_0$ offers the constructors conjunction ($\sqcap$), value restriction ($\forall r.C$), and the top concept ($\top$) (*cf.* Subsection 2.1). The approach presented in this paper also differs to (Racharak et al., 2017a) on the adopted characterization, *i.e.* the language inclusion. This work has potential use in developing knowledge-based systems in which their ontologies can be represented in $\mathcal{FL}_0$, such as the development of recommendation systems based on the agent's preferences with $\mathcal{FL}_0$-based knowledge base.

There are several possible directions for its future work. Firstly, we may try to conduct an experiment on an appropriate ontology of real-world domains. Similar experiments as conducted in (Racharak et al., 2017a) can be carried out. Secondly, it is an obvious work to investigate its intended behaviors regarding the properties introduced in (Racharak et al., 2017a). Thirdly, we are also interested to explore similarity measures for the more expressive DLs. Lastly, as reported in (Bernstein et al., 2005) about the need of having multiple measures, it would be interesting to investigate the possible classes of similarity measures w.r.t. their potential use cases and applications.

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[1] We recall that $\mathcal{ELH}$ offers the constructors conjunction ($\sqcap$), full existential quantification ($\exists r.C$), and the top concept ($\top$); also, the TBox can contain (possibly primitive) concept definitions and role hierarchy axioms.

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