Reengineering of the Emergency Service System under Generalized Disutility

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Keywords: Emergency Medical Service, System Reengineering, Generalized Disutility, Radial Formulation.

Abstract: Emergency medical service system structure is defined by deployment of service providing centers, number of which is usually limited. The objective of the designer is to minimize the total discomfort of all system users. Thus, the problem often takes the form of a weighted *p*-median problem. Since population and demands for service change in time and space, current service center deployment may not meet the requirements of the users and service providers neither. In this paper, we introduce a mathematical model for system reengineering under the generalized disutility, which follows from the idea that the individual user's disutility comes from more than one located service center. At the moment of current demand occurrence, the nearest service center may be unavailable due to satisfying another arisen demand. Presented approach constitutes an extension of previously developed methods, where only the nearest center was taken as a source of individual user's demand satisfaction.

1 INTRODUCTION

The emergency service systems as the medical emergency system, system of fire brigades and police stations are designed for given geographical area to satisfy the demand of population living in the area for more secure life. The associate service is provided from a given number of service centers and the overwhelming objective used for the design evaluation is the average time necessary to deliver service from a service center to the user location, at which a demand for service has occurred.

Host of models consider that serviced population is concentrated to a finite number of dwelling places of the considered area. Frequency of the demand occurrence is proportional to the number of inhabitants of the given town or village. A finite set of possible service center locations is assumed and also, the assumption is made that a user demand is serviced from the nearest located service center. This way, the weighted *p*-median problem formulation is used to the emergency service system design and solving the underlying problem to optimality (Current et al., 2002, Doerner et al., 2005, Ingolfsson et al., 2008, Jánošíková, 2007). The original model was based on the location-allocation decision variables and constraints (Current et al., 2002), where an occurring demand is assigned to exactly one possible center location. As concerns usage of a general IP-solver, the size of the solved integer programming problem must be taken into account. In the real problems, the number of serviced users takes the value of several thousands, and the number of possible service center locations can take this value as well (Avella et al., 2007). The number of possible service center locations seriously impacts the computational time and the memory of computer due to used branch-and-bound method, which stores the unfathomed nodes of the inspected searching tree for the further processing. That is why the direct attempt at solving the problem described by a location-allocation model often fails, when larger instances are solved by a commercial IP-solver. Then another approach using so-called radial formulation was developed to avoid the particular assignment of user's locations to the located service centers. The radial approach successfulness is based on the fact that there is only finite set of radii, which must be taken into account (Elloumi et al., 2004, García et al., 2011, Janáček, 2008). To accelerate the *p*-median problem solving process, an approximate approach has been developed (Janáček and Kvet, 2013). This approach uses an approximation of a common time distance between a service center location and a user by some pre-determined time

Kvet, M. and Janacek, J.
Reengineering of the Emergency Service System under Generalized Disutility.
DOI: 10.5220/0006621400850093
In *Proceedings of the 7th International Conference on Operations Research and Enterprise Systems (ICORES 2018)*, pages 85-93
ISBN: 978-989-758-285-1

distances and gives near to optimal results in the case of integer time distances.

A bit different situation occurres, when reengineering of a current emergency service system is performed. The necessity of system updating ussualy follows from the fact that distribution of demands for service has been developping in time and space and thus, the originally determined center locations do not suit both serviced population and providers operating the service centers. Contrary to the original system design, the current service providers suggest changes in the center deployment and their suggestion may be in a conflict with public interests. That is why the system administrator permits system reengineering only subject to some formal rules, which are intented to prevent worsening the service accessibility. The considered formal rules are quantified by a maximal number of provider's centers, which are allowed to change their locations and by the maximal distance between a current center location and a possible new location. Generally, addition of constraints may considerably spoil the computational time necessary to obtain the optimal solution of the problem. The study (Kvet and Janáček, 2016) showed, that they do not impact the computational time, when a user demand is serviced from the nearest located center.

In this paper, we deal with more general model of the emergency medical system design under reengineering. We assume that service of a user demand is provided from the nearest center only if the center is not occupied by servicing a former demand. Otherwise, the user's demand is serviced from the nearest unoccupied center. Initial emergency system design considering the failing centers was studied by (Snyder and Daskin, 2005) and the associated radial formulation was presented in (Kvet, 2014). Nevertheless, the reengineering of service system with failing centers has not been studied yet. Therefore, we focus on the influence of the formal rule constraints on best possible service availability in the service system and on the associated computational process convergency.

In this paper, we provide a reader with a radial model of emergency service system reengineering with failing centers under rules imposed by the system administrator. We perform a computational study to find whether real-sized instances of the problem are solvable using a common IP-solver.

The remainder of the paper is organized as follows. The next section is devoted to the radial model formulation, in which temporarily failing centers are considered. In Section 3, the administrator auxiliary rules are introduced. Section 4 contains a description of experiments. The conclusion summarizes obtained findings and contains possible directions of a further research.

2 REENGINEERING OF A SERVICE SYSTEM WITH FAILING CENTERS

To describe the problem of the users' disutility minimization by changing the deployment of centers belonging to one considered provider, we introduce J as a finite set of all users (dwelling places), where b_j denotes a volume of expected demand of user $j \in J$. Let I be a finite set of possible center locations. Symbol d_{ij} denotes the integer network time distance between locations i and j, where $i, j \in I \cup J$. The maximal relevant distance is denoted by m. The current emergency service center deployment is described by two disjoint sets of located centers $I_L \subset I$ and $I_F \subset I$, where I_L contains p centers of the considered provider, which performs updating of his part of the system and I_F is the set of centers belonging to the other providers. Locations from I_F stay unchanged. The center locations from I_L can be relocated within the set $I_R = I - I_F$.

In this paper, the generalized disutility perceived by a user is modelled by a sum of weighted time distances from the r nearest located centers. The probabilities q_k for k=1..r are positive real values, which meet the following inequalities $q_1 \geq q_2 \geq \ldots \geq q_r$ and depend only on the order of distances from the user to the r nearest centers. The k-th value can be proportional to the probability of the case that the k-1 nearest centers are occupied and the k-th nearest center is available (Jankovič, 2016, Snyder and Daskin, 2005).

We introduce coefficients a^s_{ij} for each pair i, j $i \in I$ and $j \in J$, where $a^s_{ij} = 1$ if and only if $d_{ij} \leq s$ and $a^s_{ij} = 0$ otherwise for s= 0, 1, …, m-1.

To describe decisions on new center deployment, we introduce series of decision variables, where binary variable y_i defined for each $i \in I_R$ takes the value of one, if a service center is to be located at i and it takes the value of zero otherwise. To express the total distance necessary for user demand satisfaction, we introduce auxiliary zero-one variables x_{jsk} for $j \in J$, $s \in 0, ..., m$-1, $k \in 1, ..., r$ to model the disutility contribution value of the k-th nearest service center to the user j. The variable x_{jsk} takes the value of 1 if the k-th smallest disutility contribution for the customer $j \in J$ is greater than s and it takes the value of 0 otherwise. Then the expression $x_{j0k} + x_{j1k} + \ldots + x_{jm-1k}$ constitutes the k-th smallest distance from the user j to a located center. If this k-th smallest distance is denoted by $d_{ik(j)}$, then the expression of $d_{ik(j)}$ by the auxiliary 0-1 variables x_{jsk} is clearly reported on the following figure.

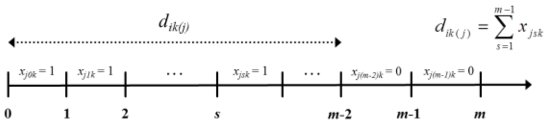

Figure 1: Expression of the k-th smallest distance from the user j to a located center by the auxiliary 0-1 variables x_{jsk} for $s \in 0, \ldots, m-1$.

Using the above introduced structures and decision variables, we suggest the following model.

$$\text{Minimize} \sum_{j \in J} b_j \sum_{s=0}^{m-1} \sum_{k=1}^{r} q_k x_{jsk} \tag{1}$$

$$\text{Subject to} \sum_{i \in I_R} y_i = p \tag{2}$$

$$\sum_{k=1}^{r} x_{jsk} + \sum_{i \in I_R} a_{ij}^{s} y_i + \sum_{i \in I_F} a_{ij}^{s} \geq r \quad \text{for } j \in J,\ s = 0, \ldots, m-1 \tag{3}$$

$$y_i \in \{0, 1\} \quad \text{for } i \in I_R \tag{4}$$

$$x_{jsk} \in \{0, 1\} \quad \text{for } j \in J,\ s = 0, \ldots, m-1,\ k = 1, \ldots, r \tag{5}$$

The objective function (1) expresses the expected volume of transportation performance denoted by generalized disutility in this paper. Constraint (2) preserves constant number of centers belonging to the considered part of the emergency service system under reengineering. For given pair of user j and a distance value s, constraints (3) assure relation between the set of location variables y_i, $i \in I_R$ and the sum of auxiliary variables x_{jsk} over range $1, \ldots, r$ of subscript k. If no center is located in the radius s, then the sum of auxiliary variables x_{jsk} equals to r. If exactly $k \leq r$ centers is located in the radius s, then the sum of variables equals to r-k due to minimization process, which presses down the values of the variables x_{jsk}. If the sum of variables x_{jsk} equals to $k<r$, then the variables $x_{js1}, \ldots, x_{jsr-k}$, equal to zero and remaining variables equal to one due to the used optimization process and decreasing values of the coefficients $q_1, \ldots, q_r$.

The objective function value of the optimal solution of the problem (1)-(5) gives expected total length or time of trips from the service centers to the demand locations necessary for satisfaction of all demands for service. This objective function value explanation holds subject to assumption that the coefficients $q_1, \ldots, q_r$ correspond to the probability values expressing that the k-th nearest center is the first available (unoccupied) service center. The next assumption is that demand volume b_j is proportional to the number of trips necessary for the demand satisfaction. The model (1)-(5) is much more realistic than the original approach based on the simple weighted p-median problem, which corresponds to the case of r=1. The bigger accuracy of the model (1)-(5) is paid for by higher complexity of the solved problem, which may issue to enormous increase of computational time. A question emerges here, which limit of accuracy presented by the value of r pays off regarding the increase of computational time. As a solution of the problem (1)-(5) is discrete and the values of probabilities q_k sharply decrease, influence of increasing value of r may appear negligible behind some limiting value r^*.

3 REENGINEERING UNDER AUXILIARY CONSTRAINTS

As mentioned in Section 1, the administrator of the system sets up parameters of rules to prevent a designer of new center deployment from increasing provider's benefit at the expense of the system users. The rules are easy to evaluate and check. That is why the studied rules have a simple form. The first rule limits the total number w of the provider's centers, which can be moved. The second rule limits the distance between original and new location of a service center.

To be able to formulate the rules in a concise way, we derive several auxiliary structures using Figure 2. We assume that all points 1-11 represent system users and the black points 2, 3, 9 and 11 represent current service center locations.

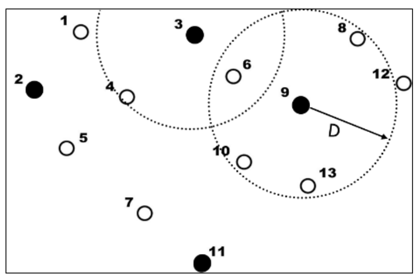

Figure 2: Simple example of reengineering restrictions.

Let $N_t=\{i \in I_R\colon\ d_{ti} \leq D\}$ denote the set of all possible center locations, to which the center $t \in I_L$ can be moved. If we consider the example depicted on Figure 2, we can observe that the center located at the point 9 can be moved to 6, 8, 10 and 13 or stay unchanged. Thus, the set N_9 = {6, 8, 9, 10, 13}. Similarly, N_3 = {3, 4, 6}. Additionally, let $S_i=\{t\in I_L: i\in N_t\}$ denote a set of all centers of the considered provider, which can be moved to $i\in I_R$. Here S_6 = {3, 9}. Realize that $t\in N_t$ and $i\in S_i$ for $t\in I_L$ and $i\in I_R$ and thus $I_L\subset I_R$.

Now, we introduce series of decision reallocation variables. The variable $u_{ti}\in\{0, 1\}$ for $t\in I_L$ and $i\in N_t$ takes the value of one, if the service center at t is to be moved to i and it takes the value of zero otherwise. Using the above introduced structures and variables we suggest the following model extension.

$$\sum_{i\in I_L} y_i \geq p - w \tag{6}$$

$$\sum_{i\in N_t} u_{ti} = 1 \quad for\; t\in I_L \tag{7}$$

$$\sum_{t\in S_i} u_{ti} \leq y_i \quad for\; i\in I_R \tag{8}$$

$$u_{ti}\in\{0,1\} \quad for \quad t\in I_L,\, i\in N_t \tag{9}$$

Constraint (6) limits the number of changed center locations by the constant w. Constraints (7) allow moving the center from the current location t to at most one other possible location in the radius D. Constraints (8) enable to bring at most one center to a location i subject to condition that the original location of the brought center lies in the radius D. These constraints also assure consistency among the decisions on move and decisions on center location.

Based on our experience, we have to raise the question of technical solvability of the formulated problem (1)-(9). We ask whether a commercial solver based on the branch-and-bound technique is able to find the exact solution of a real-sized problem in acceptable time. Consequence of structural constraint addition to some model is always matter of question from the point of computational process convergence. Furthermore, we have to realize that even if the administrator's rules are established to defend users' interests, they represent further restriction of the set of feasible solutions. This phenomenon may lead to a less possible benefit (higher disutility) for the average user. That is why, the dependence of the optimal objective function value on setting of parameters w and D is worth to study.

4 NUMERICAL EXPERIMENTS

This section is devoted to the results of numerical experiments performed in the optimization software FICO Xpress 8.0, 64-bit. The experiments were run on a PC equipped with the Intel® Core™ i7 5500U 2.4 GHz processor and 16 GB RAM.

Used benchmarks were derived from real emergency health care system, which was originally implemented in eight regions of Slovak Republic. For each self-governing region, i.e. Bratislava (BA), Banská Bystrica (BB), Košice (KE), Nitra (NR), Prešov (PO), Trenčín (TN), Trnava (TT) and Žilina (ZA), all cities and villages with corresponding number of inhabitants b_j were taken into account. The coefficients b_j were rounded to hundreds. The set of communities represents both the set J of users' locations and the set I of possible center locations as well. The cardinalities of these sets vary from 87 to 664 locations. In all solved instances, the network distance from a user to the located center was taken as the user´s disutility.

An individual experiment was organized so that the optimal solution of the reengineering problem (1)-(5) was obtained first. The value of r was set to 7 and the associated coefficients q_k for k=1, …, r were set in percentage in the following way: q_1 = 77.063, q_2 = 16.476, q_3 = 4.254, q_4 = 1.593, q_5 = 0.47, q_6 = 0.126, and q_7 = 0.018. These values follow from a simulation model of existing emergency medical service system in Slovakia (Jankovič, 2016). To enrich the pool of benchmarks, for each self-governing region ten instances were created in such a way that they differ in the list of located service centers operated by the considered provider. The average results are summarized in Table 1. The left part of this table contains the basic benchmark characteristics. The total number of possible service center locations regardless the service providers is reported in the column denoted by $|I|$. The value of *TNC* represents the total number of located centers. The average percentage rate of the provider's centers is reported in the column denoted by "Prov. [%]". The right part of the table contains the results of the model (1)-(5). The average computational time in seconds of ten instances solved for each region is reported in the column denoted by "CT [s]". The last column "*ObjF*" contains the average values of the objective function (1).

Table 1: Average results of numerical experiments for each self-governing region. The value of r was set to 7.

Reg.	$\|I\|$	*TNC*	Prov. [%]	CT [s]	*ObjF*
BA	87	14	55.1	0.5	28087.8
BB	515	36	44.9	43.6	47706.5
KE	460	32	46.0	30.4	48490.9
NR	350	27	50.7	10.8	52024.6
PO	664	32	44.3	50.5	61070.2
TN	276	21	52.9	5.1	36800.9
TT	249	18	49.6	6.1	43986.1
ZA	315	29	46.8	6.1	45341.2

The results indicate that the reengineering of the emergency service system under generalized disutility for r=7 from the point of service provider does not represent a hard solvable problem. It can be observed that the radial formulation enables to get the optimal solution within 1 minute. In spite of this useful feature, the second portion of experiments was performed to find out, whether a lower value of r will have significant influence on the resulting solution from the point of the objective function value. As we have mentioned in Section 2, we assume that the influence of increasing value of r may appear negligible behind some limiting value r^*. To verify this hypothesis and to find a suitable value of r^*, we have solved all instances with different values of r. If r<7, then the coefficient q_r was computed according to (10) as a complement of the coefficients q_k for k=1, …, r-1 to the value of 100, i.e. the sum of q_k for k=1, …, r must equal 100.

$$q_r = 100 - \sum_{k=1}^{r-1} q_k \tag{10}$$

The dependency of average computational time on the value of r was studied first. We assume that the computational time grows with increasing value of r, because it affects the number of variables and the model size as well. Our expectation has been confirmed by the results summarized in Table 2. Each row represents the average results of ten instances for each region and the columns are used for different setting of parameter r. The last row contains the average values of all instances. The dependency of average computational time on the value of r is shown also on Figure 3.

Table 2: Average computational time in seconds of the solving process depending on r for each region.

Reg.	r = 1	r = 2	r = 3	r = 4	r = 5	r = 6
BA	0.1	0.1	0.2	0.2	0.3	0.3
BB	6.5	8.3	11.5	19.3	27.1	33.7
KE	6.0	7.0	8.9	11.6	15.7	18.3
NR	2.1	2.6	3.6	6.7	6.8	8.4
PO	20.6	22.8	26.1	31.8	38.8	47.0
TN	1.4	1.8	3.2	2.8	3.4	4.3
TT	1.2	1.7	2.3	3.1	4.2	5.2
ZA	1.7	2.0	2.6	3.3	4.4	5.2
AVG	**4.96**	**5.79**	**7.29**	**9.87**	**12.58**	**15.29**

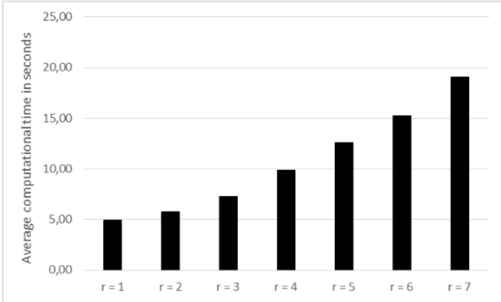

Figure 3: Dependency of average computational time in seconds on the number r.

When studying the impact of r on the resulting system design, we have evaluated Hamming distance of the vectors of resulting location variables obtained for various values of parameter r. Generally, Hamming distance of two 0-1 vectors $\boldsymbol{y}$ and $\boldsymbol{z}$ is defined by the expression (11). The average results are reported in Table 3.

$$HD(\boldsymbol{y},\boldsymbol{z}) = \sum_{i \in I} \left(y_i - z_i \right)^2 \tag{11}$$

Table 3: Average Hamming distance from the optimal solution obtained for r=7 computed for each region.

Reg.	r = 1	r = 2	r = 3	r = 4	r = 5	r = 6
BA	5.2	2.0	0.6	0.4	0.0	0.0
BB	12.6	11.0	3.6	0.6	0.4	0.0
KE	11.8	6.0	3.4	1.4	0.6	0.6
NR	10.2	6.6	2.0	0.4	0.0	0.0
PO	11.8	7.4	4.0	0.0	0.6	0.0
TN	8.0	2.6	0.6	0.2	0.0	0.0
TT	8.4	3.8	0.4	1.4	0.0	0.0
ZA	12.4	4.0	3.2	1.0	0.2	0.0
AVG	**10.05**	**5.43**	**2.23**	**0.68**	**0.23**	**0.08**

The dependency of average Hamming distance from the optimal solution obtained for r=7 on the number r of service providing centers for each system user is shown also on Figure 4.

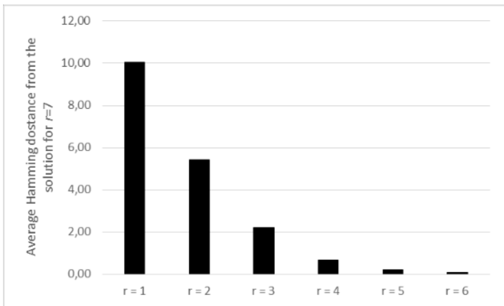

Figure 4: Dependency of Hamming distance from the optimal solution obtained for r=7 on the number r of service providing centers for each system user.

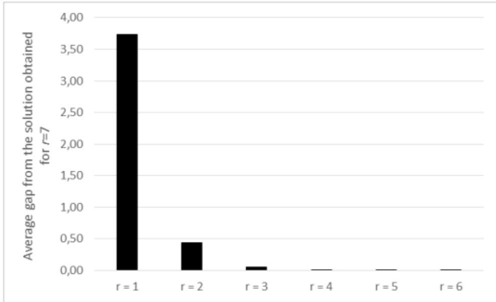

Figure 5: Dependency of average gap from the optimal solution obtained for r=7 on the number r of service providing centers for each system user.

The reported results show that the suitable value of r^* is 3. Thus, 3 nearest located service centers are enough to be taken into account when emergency system reengineering under generalized disutility. As shown, the service center deployment for r=3 differs from the service center deployment obtained for r=7 only in one center on the average.

The last characteristics studied in this portion of experiments consists in the objective function value. For each system design obtained for particular value of r=1, 2, …, 6, the objective function (1) with r=7 and the full set of coefficients q_k was computed. This value was compared to the objective function value obtained for r=7 and the gap was evaluated. Here, the gap is defined as a percentage difference of two objective function values, where the objective function value for r=7 was taken as the base. The average values of gaps of ten instances computed for each self-governing region are reported in Table 4, which follows the structure of previous tables. To find a suitable value of r^*, the gaps lower than 0.1 percent are marked.

Table 4: Average gap from the optimal solution obtained for r=7.

Reg.	r = 1	r = 2	r = 3	r = 4	r = 5	r = 6
BA	2.52	0.23	0.02	0.01	0.00	0.00
BB	6.19	0.43	0.07	0.00	0.00	0.00
KE	2.88	0.21	0.11	0.01	0.00	0.00
NR	2.31	0.55	0.06	0.00	0.00	0.00
PO	5.19	0.62	0.04	0.00	0.00	0.00
TN	2.81	0.24	0.04	0.01	0.00	0.00
TT	2.60	0.32	0.01	0.02	0.00	0.00
ZA	4.33	0.69	0.05	0.00	0.00	0.00
AVG	**3.73**	**0.43**	**0.05**	**0.01**	**0.00**	**0.00**

The dependency of average gap from the optimal solution obtained for r=7 on the number r of service providing centers for each system user is shown also on Figure 5.

The detailed analysis of presented results shows that if we use three nearest service providing centers for each system user instead of seven, we can obtain very similar results and save more than one half of computational time. That is why, the next portion of experiments was performed with r=3. This new portion of experiments was aimed at the additional constraints (6)-(9) and their impact on the solving process characteristics, mainly computational time.

This new set of experiments was organized so that the maximal radius D, in which a center can be moved, was fixed at the value of 15 and the maximal number w of centers allowed to change their locations was set to p/4, p/2, 3p/4, and p respectively. It must be realized, that the parameter p represents the total number of centers operated by considered provider, who performs reengineering. Dependency of average computational time in seconds computed for 10 instances of each region is reported in Table 5. Each row of the table represents one region and the columns are devoted to different settings of w.

Table 5: Average computational time in seconds for each region and different values of w. Parameter D was 15.

Reg.	w = p/4	w = p/2	w = 3p/4	w = p
BA	0.11	0.11	0.11	0.12
BB	4.36	6.40	5.36	5.32
KE	4.44	5.95	5.77	5.09
NR	2.02	2.20	2.75	2.78
PO	9.78	9.76	9.79	9.83
TN	1.55	1.64	1.68	1.73
TT	1.30	2.30	1.47	1.52
ZA	1.74	1.65	1.63	1.66
AVG	3.16	3.75	3.57	3.50

The reported results show that different settings of w do not significantly affect the computational process, because the value of w limits only the number of possible service center location changes and thus, the number of variables and constraints is independent on w.

The objective function values can be compared in Table 6. Even if parameter r was set to 3 in all solved models, the objective function values were computed for r=7 based on the resulting service center deployment.

Table 6: Average objective function values for each region and different values of w. Parameter D was 15. The objective function value was recomputed for r=7 and the whole set of probability coefficients q_k.

Reg.	$w = p/4$	$w = p/2$	$w = 3p/4$	$w = p$
BA	28607.0	28334.8	28334.8	28334.8
BB	50676.2	50433.7	50430.4	50430.4
KE	51141.3	50916.9	50913.4	50913.4
NR	53995.8	53482.7	53471.5	53471.5
PO	63791.3	63532.1	63526.0	63526.0
TN	37286.6	37225.5	37225.5	37225.5
TT	45670.3	44915.7	44733.6	44733.6
ZA	47278.1	46673.0	46634.3	46634.3

The last portion of experiments was aimed at exploration of the impact of parameter D on the solving process complexity. Here, the parameter w was set to its maximal value p, i.e. all centers operated by the provider could change their current locations. The average computational times in seconds computed for each self-governing region and given values of D are reported in Table 7, which has the same structure as previous tables.

Table 7: Average computational time in seconds for each region and different values of D. Parameter w was set to its maximal value, i.e. w=p.

Reg.	$D = 5$	$D = 10$	$D = 15$	$D = 20$	$D = 25$
BA	0.04	0.08	0.12	0.16	0.17
BB	0.90	3.13	5.32	10.54	15.38
KE	1.02	2.61	5.09	7.41	8.81
NR	0.42	1.11	2.78	6.91	5.31
PO	1.91	4.93	9.83	15.85	19.06
TN	0.40	1.05	1.73	2.16	2.96
TT	0.28	0.79	1.52	1.99	2.44
ZA	0.45	0.97	1.66	2.14	2.81
AVG	0.68	1.83	3.50	5.89	7.12

The results reported in Table 7 have confirmed our expectation that the parameter D has a direct impact on the computational process. As it can be observed, the average computational time grows with increasing value of D, i.e. with increasing radius, in which current center can be removed. This phenomenon has a simple explanation. The bigger is the radius for center location change, the higher is the number of its possible new locations. As we can see in constraints (6)-(9), this parameter defines the number of decision variables and it directly affects the model size. Therefore, the solving process for higher distance D takes longer time. Finally, the dependency of objective function value on the parameter D is shown in Table 8.

Table 8: Average objective function values for each region and different values of D. Parameter w=p. The objective function value was recomputed for r=7 and the whole set of probability coefficients q_k.

Reg.	$D = 5$	$D = 10$	$D = 15$	$D = 20$	$D = 25$
BA	29563.0	28798.3	28334.8	28255.0	28136.1
BB	52115.0	50635.4	50430.4	49429.6	49130.0
KE	52111.9	51398.4	50913.4	50412.6	49959.5
NR	56153.7	54360.2	53471.5	52674.5	52422.9
PO	66115.8	65081.5	63526.0	63070.1	62444.1
TN	37714.0	37320.5	37225.5	37148.4	37009.5
TT	46162.7	45395.9	44733.6	44114.7	44078.5
ZA	48712.7	47763.3	46634.3	46230.4	46115.0

As far as the objective function value expressed by generalized disutility is concerned, the achieved results indicate that the higher is the value of D, the better solution can be obtained. The radius D defines the set of all new possible locations of a center and thus, it affects the possibility for obtaining better results. More elements in the set N_t for each $t \in I_L$ mean more candidates for new center locations and bigger possible change of current center deployment, which can bring better service accessibility for system users.

All the experiments presented above were aimed primarily at studying the model solvability and the sensitivity of the associated computational process on different model parameters. Besides some interesting findings and suitable settings of parameters, we also present the emergency system characteristics in the next paragraphs. The following table contains the comparison of current service center deployment to the results of suggested reengineering model, which was configured as follows. Based on the above presented results, the parameter r was set to 1 (simple disutility) and 3 (generalized disutility). In the experiments with the generalized disutility, the associated probability coefficients q_1 = 77.063, q_2 = 16.476 and q_3 = 6.461 were used. It must be noted that the objective function reported in the table was recomputed for r=7 and the whole set of probability values reported at the beginning of this section. The maximal number w of centers operated by the considered service provider, which are allowed to change their current location, was set to the cardinality of the set I_L, i.e. all considered provider's centers could be moved. The value 15 limited the radius D, in which a center could be relocated. This initial value of D corresponds to the rule applied in the emergency health care system of the Slovak Republic (Kvet and Janáček, 2016). Table 9 contains the average results of 10 instances solved for each self-governing region. The objective function value corresponding to the current service center deployment is reported in the column denoted by "Current *ObjF*". The right part of the table is dedicated to the results of suggested reengineering problems. The abbreviation "*ObjF*" denotes the objective function value of the emergency system design obtained by solving the reengineering model. Finally, the value of *Imp* was computed to show possible improvement of the objective function value expressed by the generalized disutility, which can be achieved by relocating of some service centers. Its value was computed as a percentage difference between objective function values of the current service center deployment and the new system design resulting from the model. The objective function value of current deployment was taken as the base.

Table 9: Comparison of current service center deployment to the results of reengineering model for r=1 (simple disutility) and r=3 (generalized disutility). The reengineering parameters were set at $w = p$ and $D = 15$.

Reg.	Current *ObjF*	r=1		r=3	
		ObjF	*Imp.* [%]	*ObjF*	*Imp.* [%]
BA	29792	28810	3.30	28335	4.89
BB	52510	51094	2.70	50430	3.96
KE	52786	51894	1.69	50913	3.55
NR	56759	54440	4.09	53472	5.79
PO	67037	65807	1.83	63526	5.24
TN	38625	38091	1.38	37226	3.62
TT	472163	45569	3.48	44734	5.25
ZA	49324	47566	3.56	46634	5.45

The reported results show that the emergency system reengineering may bring a considerable improvement of service accessibility for system users expressed by general disutility. The average values of *Imp* indicate that the objective function value corresponding to the system design can be reduced up to 6 percent. The achieved results also confirm the usefulness of suggested reengineering model, because it enables us to obtain better system design from the point of service accessibility. It is obvious from the comparison of the case r=1 to r=3 that the usage of generalized disutility leads to such solutions, which are approximately by 2 percent better than those, which can be obtained by usage of simple disutility model.

5 CONCLUSIONS

This paper was focused on emergency medical system reengineering under generalized disutility, which follows the idea that the associated service can be provided from more than one nearest located centers. Presented generalized disutility makes the model more realistic by taking into account possible temporarily unavailability of service centers. In our computational study we have found, that three nearest located centers are enough to be considered in the objective function value, because the accuracy of the result is satisfactory. The second part of experiments was aimed at additional constraints, which define some new restrictions to service center location changes. Based on reported results we can conclude that we have constructed a very useful tool for emergency medical system reengineering under generalized disutility from the point of service provider. Presented model is easy to be implemented and solved in common optimization environment equipped with the branch-and-bound method or other technique to integer programming problems.

Future research in this field may be aimed at using of the suggested modelling technique in such situations, where the time distances are influenced by randomly occurring failures in the underlying transportation network.

ACKNOWLEDGEMENTS

This work was supported by the research grants VEGA 1/0518/15 "Resilient rescue systems with uncertain accessibility of service", VEGA 1/0463/16 "Economically efficient charging infrastructure deployment for electric vehicles in smart cities and communities", and APVV-15-0179 "Reliability of emergency systems on infrastructure with uncertain functionality of critical elements".

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