Resonant Tunnelling and Optical-mechanical Analogy
Overcoming of Blackout Problem

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Abstract: We report on using the optical mechanical analogy to study the propagation of the electromagnetic wave in through the plasma layer surrounding the hypersonic object moving in dense gaseous medium. This analogy allows us to consider plasma sheath surrounding the object as a potential barrier and analyse the process of electromagnetic wave tunneling. The idea is to embed a dielectric layer as a «resonator» between the surface of the object and plasma sheath which is supposed to provide an effective tunneling regime. We discuss the peculiarities of optical mechanical analogy applicability and analyse the radio frequency wave tunnelling regime in detail. The cases of normal and oblique incidence of radiofrequency waves on the vehicle surface are studied. The analysis is applied for a problem of overcoming of the communication blackout during the hypersonic vehicle re-entry into the Earth's atmosphere.

1 INTRODUCTION

Analysis of the tunneling processes for electromagnetic waves in opaque media regions on the basis of the well-known optical-mechanical analogy is used in a huge number of applications. Typically, these are problems in condensed matter physics, magnetic hydrodynamics, quantum optics, physics of photonic crystals and artificial metamaterials with unique parameters (Shvartsburg, 2007; Narimanov and Kildishev, 2009; Yang et al., 2012; Bobkov et al., 2016; Jung and Keller, 2017; Razzaz and Alkanhal, 2017; Li, 2016).

We applied this approach to the solution of the urgent communication blackout problem (Shi et al., 2013; Lei et al., 2015; Xie et al., 2016). First, it allowed us to propose new methods of optimization of telecommunication systems with an ability of continuous contact with the hypersonic vehicle. Also new unobvious limitations on the region of applicability of optical-mechanical analogy in problems on the passage of electromagnetic waves through layers/structures of extremely low transparency are discussed. Aircraft, rockets, and missiles moving at supersonic speed in the atmosphere are covered with plasma sheath (thickness *d* is about 0.1–1 meter). Hence we cannot use for telemetry and control microwaves with frequencies less than the so-called plasma frequency (of about 9 GHz for object velocities in the range 8–15 Mach) and wavelengths comparable or less then *d*. We cannot ignore the presence of a plasma sheath since it is the frequency range from 100 MHz to 10 GHz that is most important for prospective telecommunication systems (Nazarenko et al., 1994; Korotkevich et al., 2007; Gillman and Foster, 2009; Wang et al., 2016; Gao and Jiang, 2015). Plasma destruction (e.g. by injecting water drops) is also quite a difficult task.

The most interesting approach is to use the features of the interaction of ionized gas with radiation. For overcoming the opaque layer one can use the nonlinear interaction of three waves in the plasma layer region: a low-frequency wave carrying a signal from the Earth, a Langmuir wave and a high-frequency wave (pump wave) generated from the onboard source. The reflected (so-called Stokes wave) carries the information encoded in the signal to the Earth.

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Significant progress can be achieved if we introduce an additional layer of double-positive (DPS) material covering the antenna and providing matching with the plasma sheath (Wang et al., 2016).

In order to upgrade this concept, we suggest the use of optical-mechanical analogy based on the mathematical similarity of the stationary Schrödinger equation with the wave Helmholtz equation. This analogy allows us to introduce the concept of tunneling of electromagnetic waves by analogy with the tunneling of a particle through a potential barriers in heterostructures (Hasbun, 2003; Kidun et al., 2005). In the considered problem plasma sheath may act as such a "barrier".

2 THE CONCEPT OF OPTICAL MECHANICAL ANALOGY

Let us consider spatially inhomogeneous nonmagnetic medium characterized by the susceptibility $\chi(\vec{r})$, or permittivity $\varepsilon(\vec{r}) = 1 + 4\pi\chi(\vec{r})$. In the case of monochromatic field $\vec{E}, \vec{H} = \vec{E}_0(\vec{r}), \vec{H}_0(\vec{r}) \exp(-i\omega t)$, ω is the radiation frequency) Maxwell equations for electric and magnetic field strength can be written as:

$$rot\,\vec{E} = \frac{1}{c} i\omega\vec{H}, \quad div\left(\varepsilon\vec{E}\right) = 0,$$
$$rot\,\vec{H} = -\frac{\varepsilon}{c} i\omega\vec{E}, \quad div\,\vec{H} = 0. \tag{1}$$

From (1) one can obtain the following equation for electric field strength $\vec{E}$:

$$\Delta\vec{E} + \nabla\left(\frac{1}{\varepsilon}(\vec{E}\nabla)\varepsilon\right) + \frac{\varepsilon\omega^2}{c^2}\vec{E} = 0. \tag{2}$$

For the case when permittivity depends only on one spatial coordinate $\varepsilon = \varepsilon(z)$ and wave field propagates alone this direction the equation (2) transforms to the well-known Helmholtz equation for the spatial distribution of electric field strength E:

$$\frac{d^2E}{dz^2} + k_0^2\,(1 + 4\pi\chi(z))E = 0. \tag{3}$$

with $k_0^2 = \omega^2/c^2$. Here electric field propagates in the direction perpendicular to z-axis.

Equation (3) is mathematically equivalent to the stationary Schrödinger equation in quantum mechanics for the particle wave function $\psi(z)$ in the potential field $V(z)$:

$$\frac{d^2\psi}{dz^2} + \kappa_0^2\left(1 - \frac{V(z)}{\zeta}\right)\psi = 0. \tag{4}$$

where $\kappa_0^2 = 2m\zeta/\hbar^2$ is the wave vector of the particle with energy ζ. Direct comparison of (3) and (4) leads to the conclusion that potential function $V(z)$ in quantum mechanics is similar to the susceptibility in electromagnetic theory

$$(2m/\hbar^2)V(z) \rightarrow$$
$$(1-\varepsilon)\cdot(\omega/c)^2 = -4\pi\chi(z)\cdot(\omega/c)^2. \tag{5}$$

Thus the eigenvalue problem for the Hamiltonian in quantum theory turns out to be mathematically identical to the problem of calculating the stationary distribution of the electric field strength in a wave. The medium with $\varepsilon > 0$ can be associated with an attractive potential $V(z) < 0$ (potential well) while the medium with $\varepsilon < 0$ acts as potential barrier $V(z) > 0$.

If the potential curve $V(z)$ has the piecewise-continuous structure (fig. 1), both the ψ-function and its derivative $d\psi/dx$ should be continuous functions in the potential breaking points. Similar boundary conditions appear to exist in electromagnetic theory: the tangential components of $\vec{E}, \vec{H}$ should also be continuous functions at the interface regions. Using Maxwell equations one can rewrite the boundary conditions as the continuity of tangential components of $\vec{E}$ and its derivative. For the normal incidence when only tangential components of $\vec{E}, \vec{H}$ have the non-zero values these boundary conditions are equivalent to boundary conditions for the wave function in quantum mechanics.

The above conclusion known as an optical-mechanical analogy in quantum theory gives rise to a lot of practical applications and transfer the quantum theory problem solutions to optics and vice versa. As an example, the quantum mechanical tunnelling or the penetration of the quantum object through the barrier with a height greater than its kinetic energy is similar to the propagation of electromagnetic wave through the region with negative permittivity. It should be noted that plasma is an excellent example of the media with negative permittivity if the frequency of transmitted radiation is less than the plasma frequency. Really, for the collisionless plasma the permittivity reads

$$\varepsilon_p = 1 - \frac{\omega_p^2}{\omega^2}. \qquad (6)$$

where $\omega_p^2 = 4\pi e^2 n_e / m$ is the plasma frequency squared and n_e is the electron density. From this point of view the plasma sheath appearing around the hypersonic vehicle during the flight looks like a potential barrier for the target transmission frequencies less than plasma frequency.

3 MAIN IDEA OF OVERCOMING OF THE RADIO COMMUNICATION BLACKOUT

In this section we are going to use the above mentioned optical mechanical analogy to propose the way of overcoming the communication blackout. We consider the ideal conductive surface of vehicle covered by the dielectric layer (thickness *a*) with permittivity ε_d and plasma sheath (thickness *d*) with permittivity ε_p that corresponds to the potential well separated from the area of infinite motion by a potential barrier (see fig. 1). Let us imagine that quantum mechanical flux of particles moves towards our structure. From classical point of view this flux will be reflected from the barrier if the energy of incoming particles is less than the height of the barrier. From quantum-mechanical point of view tunneling through the barrier is possible.

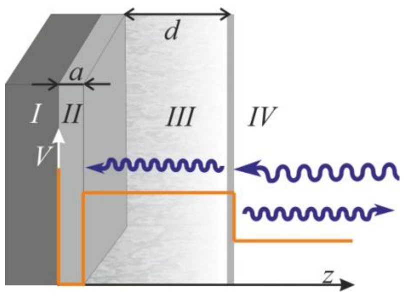

Figure 1: The concept of overcoming of radio communication blackout: profile of the "potential barrier" $V(z) \leftrightarrow (1-\varepsilon(z))$ containing a vehicle (antenna) surface (I), dielectric layer (II) which covers the antenna and plasma sheath (III). (IV) corresponds to the region of infinite motion of the electromagnetic wave (atmospheric air).

If energy of particles coincides with the position of one of the energy levels in the well the tunneling will have resonant character. This will result in effective filling of the potential well by the particle wave function. Then using the analogy between optics and quantum theory we can state that if the frequency of incident radiation coincides with the eigen-frequency of resonator (dielectric layer) the wave-field will penetrate through the plasma sheath and fill the resonator even in the case when $\omega < \omega_p$. In nonresonant case the wave field is rejected from the plasma layer and filling is negligible.

3.1 Normal Incidence

We will start our consideration of overcoming the radio communication blackout problem from the case of normal incidence of the electromagnetic wave on the hypersonic vehicle. Moving at hypersonic speed through the Earth atmosphere it creates around itself a layer of air plasma of $d \approx 5-10$ cm thickness with electron concentration about $n_e \sim 10^{10} - 10^{11}$ cm^{-3} (Nusca, 1997). For such values of concentration one obtains $\omega_p \sim 1-2\times10^{10}$ s^{-1}. If we take into account the collisions of electrons plasma permittivity becomes complex:

$$\varepsilon_p = 1 - \frac{\omega_p^2}{\omega^2+\nu^2} + i\frac{\omega_p^2 \nu}{(\omega^2+\nu^2)\omega}. \qquad (7)$$

Here ν is the transport frequency. For the gas concentration $N \approx 10^{17}$ cm^{-3} (we consider the atmospheric air at heights of several dozens of kilometers) we assume that $\nu \approx 10^9$ s^{-1}. Imaginary part of permittivity leads to absorption of the radiation in plasma. Similarly, the imaginary part of potential function $V(z)$ in quantum theory provides the possibility to put in the absorption or birth of the particles in the flux.

As it has been already noted to provide the effective tunneling of electromagnetic wave through the plasma barrier one should embed the dielectric layer ε_d between antenna and plasma sheath which will act as a resonator. We assume that the plasma layer is characterized by rectangular profile of the electron density. Than in our calculations we have the following permittivity profile:

$$\varepsilon(z)=\begin{cases}\varepsilon_d\,,\ 0\le z\le a\\ \varepsilon_p\,, a<z\le a+d\\ \varepsilon_{air}\,, z>a+d\end{cases}. \qquad (8)$$

Here a is the thickness of dielectric layer with permittivity ε_d, $\varepsilon_{air}=1$ is the permittivity of the atmospheric air. Spectrum of standing waves in dielectric layer is determined by $k_n \approx (\pi/a)n,\ n=1,2,3\ldots,$ therefore resonant frequencies are

$$\omega_n \approx \frac{\pi c}{a\sqrt{\varepsilon_d}}\,n\,. \qquad (9)$$

For example, for dielectric layer with $a=1$ cm and $\varepsilon_d=150$ (this corresponds to novel ferroelectric polymer composites (Dang et al, 2003) we obtain $\omega_1 \approx 7{,}7\times10^9$ s^{-1} ($f\equiv\omega/2\pi=1.2$ GHz). In particular for $n_e=10^{11}$ cm^{-3} ($\omega_p\sim1.8\times10^{10}$ s^{-1}) one obtains two stationary states in resonator (n =1,2) for frequencies $\omega_n<\omega_p$.

The solutions of wave equation (3) with permittivity (8) in each spatial region are:

$$\begin{aligned} &II:\quad E=E_d\sin(k_1 z),\\ &III:\quad E=E_{p+}\exp(-\kappa z)+E_{p-}\exp(\kappa z),\\ &IV:\quad E=E_{air+}\exp(-ik_3 z)+E_{air-}\exp(ik_3 z);\end{aligned} \qquad (10)$$

where $k_1=\frac{\omega}{c}\sqrt{\varepsilon_d}$, $\kappa=\frac{\omega}{c}\sqrt{|\varepsilon_p|}$, $k_3=k_0$, E_{air+} is the given amplitude of the incident wave field, E_p and E_d are the amplitudes of wave field in plasma and dielectric correspondingly.

Provided that the function (10) and its derivative are regular at points of permittivity discontinuity we obtain the results for the filling factor $F(f)$ in dependence on radiation frequency for given dielectric layer parameters (fig. 2) which represents the degree of resonator filling by the incoming radiation flux:

$$F(f)=\max\left\{|E_d|^2/|E_{air+}|^2\right\}. \qquad (11)$$

where $\left|\vec{E}_d\right|^2$ and $\left|\vec{E}_{air+}\right|^2$ are the squared absolute values of electric field strength in the dielectric layer and air correspondingly. The introduced filling factor characterizes the effectiveness of the interaction of antenna with incoming flux. We would like to note that the filling factor can be even greater than unity (see fig. 2a).

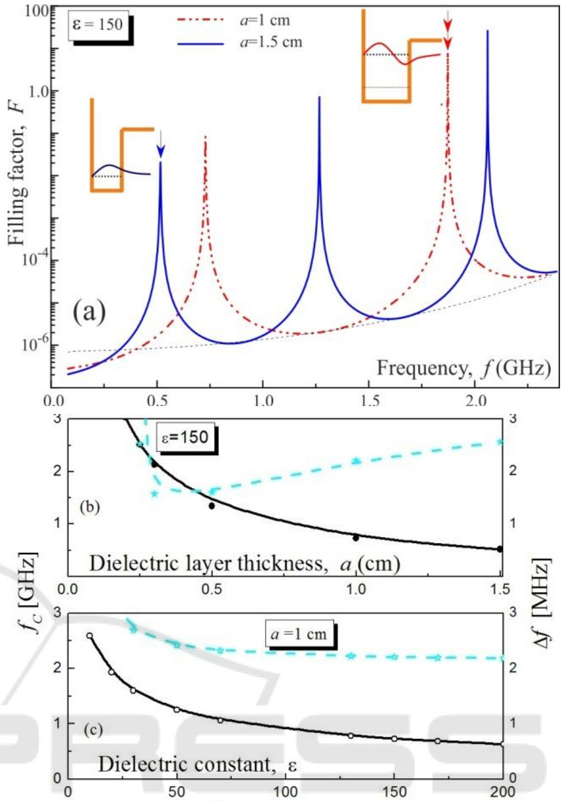

Figure 2: The filling factor $F(f)$ in dependence on the transmitted signal frequency (a) and the position of first resonance and its FWHM in dependence on the dielectric layer thickness (b) and dielectric constant (c). Calculations are made for $d=10$ cm.

As it can be seen from fig. 2(a) the position and number of resonances essentially depends on thickness of dielectric layer: to shift the resonance to the lower frequencies one should increase the width of dielectric. There is the difference between resonant frequencies obtained by expression (9) and in numerical calculations which is related to the finite height of the plasma barrier. To transfer information to antenna through a plasma sheath, the positions f_C and the widths Δf of the transparency peaks are of critical importance. Figure 2(b) presents the resonant peaks behavior (the carrier frequency of the order of 1 ... 2 GHz, the bandwidth of a few MHz) in dependence on dielectric properties. It is easy to see that tracking telemetry and command (TT&C) signals in the presence of plasma sheath are possible within the framework of our concept with reasonable parameters of the resonator used.

The dependence of filling factor on the electronic density in the plasma layer is displayed at fig. 3. We see that with the increase of n_e the position of resonances slightly shifts towards higher values of frequency, the limit value of this shifting is determined by the expression (9), that corresponds to the resonator with ideally conducted walls. Also one should mention the decrease of the filling factor value with increase of the electronic density both for resonant and nonresonant tunneling. This fact results from the decreasing of the tunnel transparency of the plasma layer for higher electronic densities.

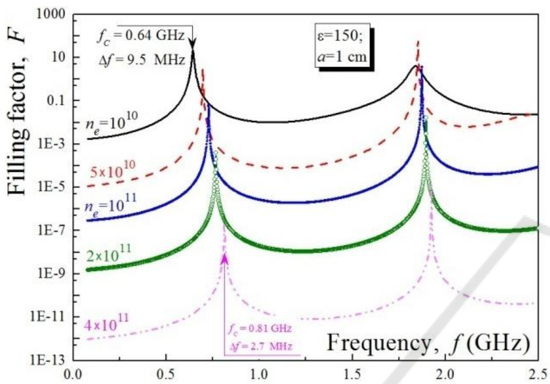

Figure 3: The filling factor $F(f)$ in dependence on the transmitted signal frequency for different values of electronic density in the plasma layer.

To give more insight to the process of electromagnetic field penetration through the plasma sheath we perform the data for spatial distribution of the absolute value of the electric field strength corresponding for two lower resonances (fig. 4). The resonant distribution corresponds to the frequencies $f_1 = 0.73$ GHz, and $f_2 = 1.87$ GHz.

3.2 Oblique Incidence

Here we are going to move up to the case of the oblique incidence of the electromagnetic wave on the above discussed structure. It is important to notice that in this case one can distinguish two types of electromagnetic waves: TE and TM. Let us remind that in the TE wave vector $\vec{E}$ is perpendicular to the plane of incidence (TE - Transverse Electric) while in TM wave $\vec{E}$ belongs to it (TM - Transverse Magnetic). Propagation of these two waves through the potential structure differs from each other.

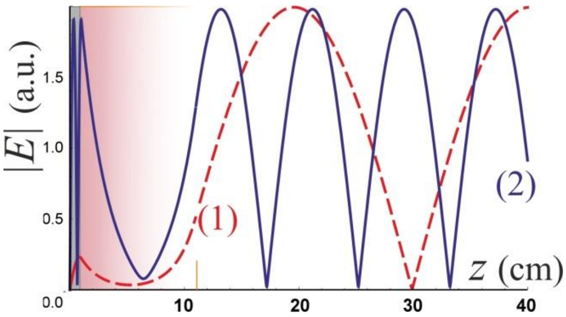

Figure 4: Spatial distribution of the absolute value of the electric field strength in resonant cases (the distribution is normalized to the incoming flux). Curves (1) and (2) correspond to the first and second eigen-frequencies of the resonator. Calculations are performed for $a = 1$ cm, $\varepsilon_d = 150$, $d = 10$ cm, $n_e = 10^{11}$ cm^{-3}.

Let us first consider the case of TE wave. Here we suppose that the wave vector lies in the xz-plane, θ is the angle of incidence of electromagnetic wave counted off from the z-axis. In this case electric field has the only tangential x - component, while for magnetic field both y and z components are non-zero. Then the wave equation for electric field reads:

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial z^2} + \varepsilon(z)\frac{\omega^2}{c^2} E = 0. \tag{12}$$

One can write its solution in each spatial region

$$\begin{aligned} II &: E = E_d \sin(k_{1z} z)\exp(ik_{1x} x), \\ III &: E = \left(E_{p+} \exp(-\kappa_{2z} z) + E_{p-} \exp(\kappa_{2z} z)\right)\times \\ &\qquad \exp(ik_{2x} x), \\ IV &: E = \left(E_{a+} \exp(-ik_{3z} z) + E_{a-} \exp(ik_{3z} z)\right)\times \\ &\qquad \exp(ik_{3x} x); \end{aligned} \tag{13}$$

here due to the boundary conditions $k_{1x} = k_{2x} = k_{3x} = k_0 \sin\theta$; $k_{1z} = \sqrt{k_1^2 - k_{1x}^2}$, $\kappa_{2z} = \sqrt{\kappa^2 + k_{2x}^2}$, $k_{3z} = k_0 \cos\theta$. The incoming flux is normalized to unity. Boundary conditions represent the continuity of the tangential components of electric field and its derivative at the interfaces.

The filling factor dependences (2nd resonance) on the angle of incidence for TE wave are performed at fig. 5. With the increase of the angle one can see shifting of the peak to higher frequencies and decreasing of its maximum value. The last circumstance appears due to the decrease of the normal component of the incoming electromagnetic wave flux. Actually, in the case of oblique incidence standing wave frequencies in dielectric layer are

$$\omega_n \approx \frac{\pi c}{a\sqrt{\varepsilon_d}\cos\phi} n . \qquad (14)$$

where ϕ is angle of slope of electromagnetic wave in dielectric layer measured from the z axis: $\sin\phi = \sin\theta\sqrt{\varepsilon_{air}/\varepsilon_d}$. As the result the more is the angle of incidence the more is the resonance shifting. In this case dielectric layer with high value of permittivity plays the role of «stabilizer» preventing significant displacement of resonant peaks. To demonstrate this fact we present the filling factor for two values of dielectric layer permittivity, $\varepsilon_d = 10$ and $\varepsilon_d = 150$ (see fig. 5). Actually, for $\varepsilon_d = 150$ which is within the range of particular interest for the problem of overcoming of the communication blackout the resonance shifting is negligible (fig. 5(a)).

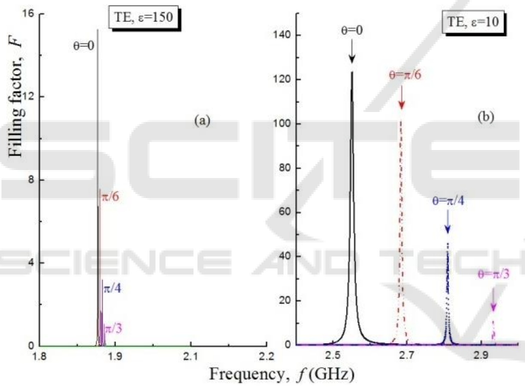

Figure 5: Filling factor $F(f)$ for the piecewise continuous dielectric permittivity profile (7) for TE wave in dependence on incidence angle. (a) corresponds to the high value of dielectric constant $\varepsilon_d = 150$, (b) is for the dielectric constant $\varepsilon_d = 10$. Plasma sheath parameters are the same as at fig. 2.

For the TM wave magnetic field has only the tangential x-component. Hence, it is handier to solve the wave equation for the magnetic field:

$$\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial z^2} + \varepsilon(z)\frac{\omega^2}{c^2} H = 0. \qquad (15)$$

Provided that permittivity is piecewise continuous function in space equation (15) is identical to eq. (12). Thus the solutions for magnetic field in each spatial region will be determined by expressions similar to (13). The difference appears when writing the condition for continuity of the tangential component of field E :

$$i\frac{\omega}{c}E_x = \frac{1}{\varepsilon}\frac{\partial H}{\partial z}. \qquad (16)$$

It means that instead the continuity of H and its derivative we have the continuity of H and $\frac{1}{\varepsilon}\frac{\partial H}{\partial z}$. This circumstance leads to some peculiarities of the process of TM wave propagation through the plasma barrier: optical mechanical analogy will work only for TE wave propagation, TM wave propagation is beyond this analogy. Physical reason for this is directly associated with the induced oscillations of the plasma barrier resulting from the existence of z-component of field *E*. We plan to study this phenomenon in more detail in further publications. Actually, the phenomenon of electromagnetic wave tunnelling is more complicated that the quantum-mechanical tunnelling due to the vector nature of the electromagnetic field.

4 CONCLUSIONS

Thus, the new approach is proposed to overcome the communication blackout during the hypersonic vehicle movement through the Earth atmosphere. The approach is based on the optical-mechanical analogy which allows to consider plasma sheath surrounding the vehicle as a potential barrier and analyse the process of electromagnetic wave tunnelling. It is demonstrated that dielectric layer covering of the antenna surface can act as the resonator providing resonance tunneling at definite frequencies of the electromagnetic wave. Some peculiarities of optical-mechanical analogy applicability for the analysis of the radio frequency wave tunnelling regime are studied. It is shown that this analogy can be applied only for the case of TE wave.

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