New Scenario-based Stochastic Programming Problem for Long-term Allocation of Renewable Distributed Generations

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Keywords: Stochastic Optimization, Power Systems, Renewable Energy Sources, Distributed Generations, Expansion Planning.

Abstract: Large installation of distributed generations (DGs) of renewable energy sources (RESs) on distribution network has been one of the challenging tasks in the last decade. According to the installation strategy of Japan, long-term visions for high penetration of RESs have been announced. However, specific installation plans have not been discussed and determined. In this paper, for supporting the decision-making of the investors, a new scenario-based two-stage stochastic programming problem for long-term allocation of DGs is proposed. This problem minimizes the total system cost under the power system constraints in consideration of incentives to promote DG installation. At the first stage, before realizations (scenarios) of the random variables are known, DGs' investment variables are determined. At the second stage, after scenarios become known, operation and maintenance variables that depend on scenarios are solved. Furthermore, a new scenario generation procedure with clustering algorithm is developed. This method generates many scenarios by using historical data. The uncertainties of demand, wind power, and photovoltaic (PV) are represented as scenarios, which are used in the stochastic problem. The proposed model is tested on a 34 bus radial distribution network. The results provide the optimal long-term investment of DGs and substantiate the effectiveness of DGs.

1 INTRODUCTION

1.1 Background

Large penetration of RESs-based DGs in distribution network implies that distribution companies (DISCOs) need to deal with the intermittent nature of RES such as wind speed and solar radiation in order to maintain the demand-and-supply balance continuously, and accommodate expected demand growth over the planning horizon (Eftekharnejad et al., 2013). DGs refer to small-scale energy generations and are most generally used to guarantee that sufficient energy is available to meet peak demand. Distributed generation planning (DGP), which determines the optimal siting, sizing, and timing, is modeled to tackle above problem. The objective of DGP is to ensure that the reliable power supply to the consumers is achieved at a lowest possible cost. DGP plays an important role as a strategic-level planning in modern power system planning. Commonly used approaches to solve the DGP are: sensitivity analysis-based approaches, mixed-integer linear programming, and nonlinear programming. However, the above methods can not fully handle the uncertainties. Consequently, stochastic programming and metaheuristic-based approaches have been used these days, to consider the uncertainties at the energy planning (Payasi et al., 2011; Jordehi, 2016).

1.2 Related Work

Much attention has been paid to solving several stochastic problems for one-type capacity planning. For multi-resource type, the scenario-based techniques also have been proposed to consider various uncertainties (Huang and Ahmed, 2009; Baringo and Conejo, 2013b; Munoz et al., 2016).

In power system planning on transmission and distribution network, many approaches have been developed considering some RESs, energy conversion and transmission, and the uncertainties that are caused by demand, pricing, and intermittent renewables (Verderame et al., 2010). An energy planning in individual large energy consumers was formulated as a mixed integer linear programming model by using

Tanaka I. and Ohmori H.
New Scenario-based Stochastic Programming Problem for Long-term Allocation of Renewable Distributed Generations.
DOI: 10.5220/0006189900960107
In *Proceedings of the 6th International Conference on Operations Research and Enterprise Systems (ICORES 2017)*, pages 96-107
ISBN: 978-989-758-218-9

fuzzy parameters in (Mavrotas et al., 2003). (Atwa et al., 2010) proposed a probabilistic mixed integer nonlinear problem for distribution system planning.

Several studies related to stochastic optimization of DGP have been proposed. In (Fu et al., 2015), a chance-constrained stochastic programming model was formulated for managing the uncertainty of PV, which was solved by an algorithm combining the multi-objective particle swarm optimization with support vector machines. (Abdelaziz et al., 2015) provided an energy loss minimization problem which determines the optimal location of RES-based DGs and the location and daily schedule of dispatch-able DG. In the problem, the uncertainties between wind power, PV and demand were considered using the diagonal band Copula and sequential Monte Carlo method. In (Saif et al., 2013), the uncertainties of wind energy, PV, and energy storage system were produced as chronological ones for a two-layer simulation-based allocation problem. In (Pereira et al., 2016), the allocation problem of VAR compensator and DG was formulated as a mixed-integer nonlinear problem and solved by using meta-heuristic algorithms.

A two-stage architecture is commonly used in stochastic programming approaches. At the first stage, DGs' investment variables are determined before realizations of random variables are known, i.e., scenarios. At the second stage, after scenarios become known, operation and maintenance variables which depend on scenarios are solved.

(Carvalho et al., 1997) modeled a two-stage scheme problem of distribution network expansion planning under uncertainty in order to minimize an expected cost along the horizon and solved by the proposed hedging algorithm in an evolutionary approach to deal with scenario representation efficiently. In (Krukanont and Tezuka, 2007), a two-stage stochastic programming for capacity expansion planning was provided in a power system of Japan. This model includes the uncertainties of the demand, carbon tax rate, operational availability. In (Wang et al., 2014), a two-stage robust optimization-based model considering uncertainties of DG outputs and demand was provided for the optimal allocation of DGs and microturbine. (Montoya-Bueno et al., 2015) proposed a stochastic two-stage multi period mixed-integer linear programming model of renewable DG allocation problem considering the uncertainties affected by demand and renewable energy production.

As an allocation problem of energy storage system (ESS), (Nick et al., 2014) formulated the optimal allocation problem as a two-stage stochastic mixed-integer second-order cone programming (SOCP) model. In (Nick et al., 2015), SOCP problem of ESS allocation was solved by using alternative direction method of multipliers. In (Asensio et al., 2016a; Asensio et al., 2016b), the allocation problem of DGs and energy storage was formulated as a stochastic programming model for maximizing the net social benefit taking account of demand response. Since the cost of ESS is very expensive and ESS seems not to be efficient at this stage, ESS is excluded from consideration in this paper.

In solving the two-stage stochastic programming, an effective methodology to create proper scenarios must be needed to represent various uncertainties because it is very difficult to realistically obtain all of the information about the uncertainty and computationally incorporate it into the model. In case some probability distributions are analytically estimated and used instead, the problem commonly becomes very complexed, even if the problem is small. Hence, when the partial information of the uncertainty is available, the stochastic programing model normally needs to be solved using scenarios. There exist many techniques of scenario generation (Dupačová et al., 2000). The uncertainty modeling such as demand and wind speed were developed to create scenarios in (Baringo and Conejo, 2011). The proposed method uses duration curves which is approximated by some demand blocks. (Baringo and Conejo, 2013a) performed the scenario reduction by using K-means clustering algorithm to arrange the historical scenarios of demand and wind into clusters according to the similarities. (Sadeghi and Kalantar, 2014) used Monte Carlo simulation and probability generation load matrix for obtaining the uncertainty of fuel and electricity price, DG outputs, and load. In (Mazidi et al., 2014), the Latin hypercube sampling was used to prepare scenarios of RESs. In (Seljom and Tomasgard, 2015), an iterative-random-sampling-based scenario generation algorithm was developed. They evaluated whether the number of scenarios is enough to obtain reliable results. In (Nojavan and allah Aalami, 2015), the normal distribution and the Weibull distribution were used for generating the scenarios of electric price, demand, and meteorological data. The created scenarios were reduced by the fast forward selection based on Kantorovich distance approach. In (Montoya-Bueno et al., 2016), a probability density function-based scenario generation method was proposed for the allocation problem of wind power and PV.

1.3 Contribution

Most of scenario generation have not considered the correlation between the uncertainties (e.g., demand and solar radiation) and usually the uncertainty

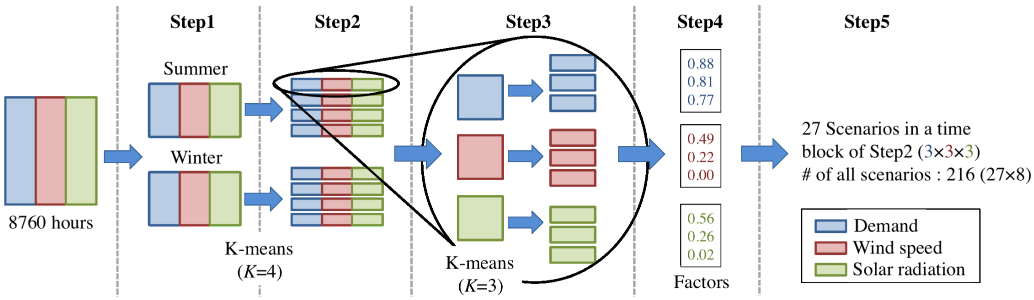

Figure 1: Outline of scenario generation. This figure shows the procedure focused on a block in Step 2.

separations to the levels have been made manually (Baringo and Conejo, 2011; Montoya-Bueno et al., 2016). It is necessary, however, to create scenarios automatically in consideration of the correlations for appropriate scenarios based on data. In optimization problem mentioned above, many researches of optimal DG allocation problem that takes into account the uncertainties have been performed. Most of the studies have considered only one-year's allocation and daily/annual system operation. Realistically, in order to accomplish the optimal system operation in multi-period, obtaining the long-term optimal siting, sizing, and timing is required. Hence, this study provides the two main contributions as follows.

- A new scenario generation method with K-means is proposed to create scenario-levels automatically by using similarity measure. This procedure uses historical data and can be implemented readily. If K-means algorithm is simply applied to the available data, it is not possible to take into account the correlation between demand and meteorological data or seasonal characteristics (e.g., summer and winter). Hence, in the proposed approach K-means clustering is utilized in stages by focusing on demand and seasons. Many scenarios of demand, wind speed, and solar radiation are generated and appropriate probabilities of each scenario are calculated (not equal-probability) by use of divided time blocks.
- A new long-term allocation problem of RES-based DGs is proposed. This model is formulated as a two-stage stochastic programming problem with the objective of minimizing the total system cost. In the proposed model, some devices and constraints are integrated for improving distribution system (i.e., limitation of reverse power flow, generation of DG considering lagging/leading power factor, capacitor bank (CB)). Furthermore, the carbon emission costs and incentives are considered from the point of view of international trends and economics because the problems of carbon emissions are actively discussed at the Conference of the Parties to the UNFCCC to achieve a clean environment and the government generally, in order to reach high renewable penetration levels, subsidizes the DISCOs that invest RES to their distribution system.

1.4 Paper Organization

The reminder of this paper is organized as follows. In Section 2, the details of the proposed scenario generation procedure is described. Section 3 provides the stochastic programming model. The results of the numerical simulations are presented and discussed in Section 4. Finally, the paper is concluded providing some insights and summaries in Section 5.

2 SCENARIO GENERATION

This Section describes the proposed scenario generation method that applies K-means to historical data (i.e. load, wind speed, solar radiation) in stages. The goal is to obtain the scenario levels of demand, electricity price, wind speed, and solar radiation for creating specific scenarios. The role of K-means is to classify a original dataset into a certain number of clusters K. The centroid of each cluster is the mean value of the data allocated to each cluster. The algorithm is based on the iterative fitting process as following steps:

1. Select the number of clusters K according to the specific problem. Randomly place K points, which represent the initial cluster centroids, into the space represented by the clustered dataset.
2. Assign each data to the closest centroid base on the distances.
3. When all data have been assigned, recalculate the new cluster centroids using data allocated to each cluster.
4. Repeat Steps 2 and 3 iteratively until there are no changes in any mean, i.e. the centroids no longer

move. As a result, the clustered dataset is separated into groups minimizing an objective function, in this paper a quadratic distance is used.

Historical data need to be available for scenario creation, i.e. hourly demand, wind speed, solar radiation, and electricity price data for the 8760 hours of the year. Figure 1 shows the overview of the proposed scenario generation. The steps are described below:

Step 1) Normalize data into the [0.0,1.0] interval by dividing by the maximum value of each feature and simultaneously separate into two seasons : summer (April-September) and winter (October-March). Each seasonal group consists of 4380 hours block.

Step 2) Apply K-means (the number of clusters $K = 4$) to only the demand in each seasonal groups created in Step 1 and allocate each data into four groups. Figure 2 shows the clusters of the demand. Moreover, wind speed, solar radiation and price indexed to each demand data are also allocated to the same clusters of the demand. Each divided group is defined as a *time block* b, which is related to the representatives of demand clusters (e.g., peak-load of summer, middle-load of summer, low-load of winter). Total of the number of hours in time block b is represented as N_b^{hours}.

Step 3) Apply K-means ($K = 3$) again into the demand, wind speed, and solar radiation of the data group created in Step 2 respectively and 9 data groups are created per one block. Step 3-5 in Fig. 1 focus on the flow of the one of the data blocks in Step 2.

Step 4) The mean values of each data block in Step 3 are used as a block representative to create the factors of demand, wind speed, and solar radiation. Note that the price levels are determined by the mean values of the price within each demand block. Renewable production models in (Eduardo, 1994) and (Atwa et al., 2010) are used in this paper so that renewable observation data are transformed into power output (i.e., wind generation factor and PV generation factor)

Step 5) Considering the combination of each factor made in Step 4, 27 scenarios are obtained for each time block. Therefore, 216 scenarios are obtained as the total number of scenarios. The probabilities of the factors within each time block, $\text{Pr}_{b,s}^{\text{load}}, \text{Pr}_{b,s}^{\text{WD}}, \text{Pr}_{b,s}^{\text{PV}}$,are defined by the ratio of the number of hours of the blocks divided in Step3 to the corresponding block in Step2, i.e., N_b^{hours}. Hence, the scenario probabilities $\text{Pr}_{b,s}$ are calculated as:

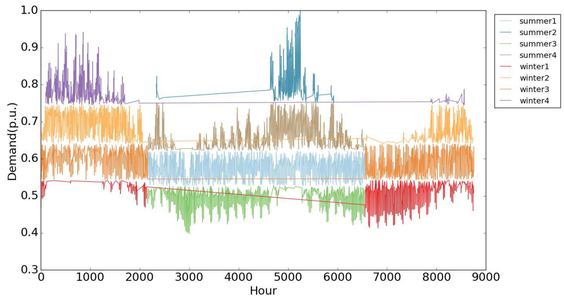

Figure 2: The clusters of demand in Step 2 (time blocks).

$$\text{Pr}_{b,s} = \text{Pr}_{b,s}^{\text{load}} \times \text{Pr}_{b,s}^{\text{WD}} \times \text{Pr}_{b,s}^{\text{PV}}.$$

Note that the time block b represents the demand periods related to season (e.g., high-demand in summer, low-demand in winter) and the index s represents the scenarios in the time block b (e.g., (high demand, large wind, large PV), (low-demand, middle wind, small PV)).

3 OPTIMAL LONG-TERM ALLOCATION PROBLEM OF DISTRIBUTED GENERATION

Two-stage stochastic linear programming is used as a formulation of the long-term allocation problem of DGs. The model uses the scenarios and provides the optimal siting, sizing, and timing of RES-based DGs to be installed (wind power and PV). The nomenclature related to the problem formulation described in Appendix.

3.1 Objective Function

This model minimizes the total system cost consisting of the investment cost π_t^{inv} and operation & maintenance cost in consideration of the incentive μ_t^{inc}. The expected value of the O&M cost in year t is shown as:

$$\sum_{b \in \Omega_{B_t}} N_{t,b}^{\text{hours}} \sum_{s \in \Omega_{S_{t,b}}} \text{Pr}_{t,b,s} \pi_{t,b,s}^{\text{om}} \qquad ,t \in \Omega_T \tag{1}$$

where, Ω_{B_t} is the set of time blocks in year t, $N_{t,b}^{\text{hours}}$ is the total hours of time block b in t, $\Omega_{S_{t,b}}$ is the set of the scenarios in t and b, $\text{Pr}_{t,b,s}$ is the probability of the scenario s in t and b, and $\pi_{t,b,s}^{\text{om}}$ is the O&M cost per unit time in t, b, and s. In this paper, it is assumed that the time blocks and scenarios are the same every year,

$$\Omega_{B_t} = \Omega_B, N_{t,b}^{\text{hours}} = N_b^{\text{hours}}, \Omega_{S_{t,b}} = \Omega_{S_b}, \text{Pr}_{t,b,s} = \text{Pr}_{b,s},$$

because, in the same region, the trend of the demand profile and the average of the weather data are considered not to change significantly. It is important to note that the operational environment of the power system is different in each year since the time-dependent parameters exist, such as demand growth factor, discount rate, and price increasing factor, although the scenarios do not change.

Therefore, the aim of the model is minimizing the total system cost over the planning horizon T:

Minimize:

$$\sum_{t\in\Omega_T}\alpha_t\left(\pi_t^{\text{inv}}+\sum_{b\in\Omega_B}N_b^{\text{hours}}\sum_{s\in\Omega_{S_b}}\text{Pr}_{b,s}\pi_{t,b,s}^{\text{om}}\right)-\sum_{t\in\Omega_T}\mu_t^{inc} \tag{2}$$

where $\alpha_t=\frac{1}{(1+d)^t}$ is the present value factor.

3.1.1 Investment Costs

The following equations show the investment costs of the substation, wind turbine, PV, and CB. The costs are, respectively, annualized by using the interest rate and lifetime of the devices. Therefore, the previous year's investment cost is added to the next one except for the first year.

$$\pi_t^{\text{inv}}=\sum_{n\in\Omega_{SS}}\pi_{\text{anu}}^{\text{SS}}X_t^{\text{SS},n}+\sum_{n\in\Omega_L}(\pi_{\text{anu}}^{\text{PV}}X_t^{\text{PV},n}+\pi_{\text{anu}}^{\text{WD}}X_t^{\text{WD},n}+\pi_{\text{anu}}^{\text{CB}}X_t^{\text{CB},n})+\pi_{t-1}^{\text{inv}};t>1, \tag{3}$$

$$\pi_t^{\text{inv}}=\sum_{n\in\Omega_{SS}}\pi_{\text{anu}}^{\text{SS}}X_t^{\text{SS},n}+\sum_{n\in\Omega_L}(\pi_{\text{anu}}^{\text{PV}}X_t^{\text{PV},n}+\pi_{\text{anu}}^{\text{WD}}X_t^{\text{WD},n}+\pi_{\text{anu}}^{\text{CB}}X_t^{\text{CB},n})\quad;t=1, \tag{4}$$

$$\pi_{\text{anu}}^{\text{SS}}=\frac{\pi_{\text{inv}}^{\text{SS}}i(1+i)^{L^{\text{SS}}}}{(1+i)^{L^{\text{SS}}}-1}, \tag{5}$$

$$\pi_{\text{anu}}^{\text{WD}}=\frac{\pi_{\text{inv}}^{\text{WD}}i(1+i)^{L^{\text{WD}}}}{(1+i)^{L^{\text{WD}}}-1}, \tag{6}$$

$$\pi_{\text{anu}}^{\text{PV}}=\frac{\pi_{\text{inv}}^{\text{PV}}i(1+i)^{L^{\text{PV}}}}{(1+i)^{L^{\text{PV}}}-1}, \tag{7}$$

$$\pi_{\text{anu}}^{\text{CB}}=\frac{\pi_{\text{inv}}^{\text{CB}}i(1+i)^{L^{\text{CB}}}}{(1+i)^{L^{\text{CB}}}-1}. \tag{8}$$

3.1.2 Operation and Maintenance Costs

O&M costs are shown in the following equations. Total O&M cost includes the power loss cost, unserved energy cost, purchased energy cost, O&M cost of DGs and CB, and CO_2 emission cost.

$$\pi_{t,b,s}^{\text{om}}=\pi_{t,b,s}^{\text{loss}}+\pi_{t,b,s}^{\text{ENS}}+\pi_{t,b,s}^{\text{SS}}+\pi_{t,b,s}^{\text{new}}+\pi_{t,b,s}^{\text{CB}}+\pi_{t,b,s}^{\text{emi}}, \tag{9}$$

$$\pi_{t,b,s}^{\text{loss}}=\pi^{\text{loss}}\sum_{n,m\in\Omega_N}S^{\text{base}}r^{n,m}I_{t,b,s}^{\text{sqr},n,m}, \tag{10}$$

$$\pi_{t,b,s}^{\text{ENS}}=\pi^{\text{ENS}}\sum_{n\in\Omega_L}S^{\text{base}}P_{t,b,s}^{\text{ENS},n}, \tag{11}$$

$$\pi_{t,b,s}^{\text{SS}}=\pi_{b,s}^{\text{SS}}\eta_t^{\text{SS}}\sum_{n\in\Omega_{SS}}S^{\text{base}}P_{t,b,s}^{\text{SS},n}, \tag{12}$$

$$\pi_{t,b,s}^{\text{new}}=S^{\text{base}}\sum_{n\in\Omega_L}(\pi_{\text{om}}^{\text{PV}}P_{t,b,s}^{\text{PV},n}+\pi_{\text{om}}^{\text{WD}}P_{t,b,s}^{\text{WD},n}), \tag{13}$$

$$\pi_{t,b,s}^{\text{CB}}=S^{\text{base}}\sum_{n\in\Omega_L}\pi_{\text{om}}^{\text{CB}}Q_{t,b,s}^{\text{CB},n}, \tag{14}$$

$$\pi_{t,b,s}^{\text{emi}}=\pi_{t,b,s}^{\text{emi,SS}}+\pi_{t,b,s}^{\text{emi,DG}}, \tag{15}$$

$$\pi_{t,b,s}^{\text{emi,SS}}=\eta_t^{\text{emi}}S^{\text{base}}\sum_{n\in\Omega_{SS}}\pi^{\text{CO}_2}\nu_{\text{emi}}^{\text{SS}}P_{t,b,s}^{\text{SS},n}, \tag{16}$$

$$\pi_{t,b,s}^{\text{emi,DG}}=\eta_t^{\text{emi}}S^{\text{base}}\sum_{n\in\Omega_L}\pi^{\text{CO}_2}\left(\nu_{\text{emi}}^{\text{WD}}P_{t,b,s}^{\text{WD},n}+\nu_{\text{emi}}^{\text{PV}}P_{t,b,s}^{\text{PV},n}\right). \tag{17}$$

3.1.3 Incentive

Incentive will be paid for the new investment of DGs by using the subsidy rare.

$$\mu_t^{inc}=\alpha_t\sum_{n\in\Omega_L}\left(\gamma_{\text{sub}}^{\text{WD}}\pi_{\text{inv}}^{\text{WD}}X_t^{\text{WD},n}+\gamma_{\text{sub}}^{\text{PV}}\pi_{\text{inv}}^{\text{PV}}X_t^{\text{PV},n}\right). \tag{18}$$

3.2 Constraints

3.2.1 Power Balance Constraints

The following constraints describe the active and reactive power balance of the load and substation buses. It should be mentioned that the scenario of demand, $\eta_{b,s}^{\text{load}}$, is used by multiplying the peak load of each bus.

$$\sum_{n,m\in\Omega_N}\left(P_{t,b,s}^{n,m}-r^{n,m}I_{t,b,s}^{\text{sqr},n,m}\right)-\sum_{m,n\in\Omega_N}\left(P_{t,b,s}^{m,n}\right)+P_{t,b,s}^{\text{ENS},m}+P_{t,b,s}^{\text{WD},m}+P_{t,b,s}^{\text{PV},m}=\eta_t\eta_{b,s}^{\text{load}}P^{\text{load},m}, \tag{19}$$

$$\sum_{n,m\in\Omega_N}\left(P_{t,b,s}^{n,m}-r^{n,m}I_{t,b,s}^{\text{sqr},n,m}\right)-\sum_{m,n\in\Omega_N}\left(P_{t,b,s}^{m,n}\right)+P_{t,b,s}^{\text{SS},m}=\eta_t\eta_{b,s}^{\text{load}}P^{\text{load},m}, \tag{20}$$

$$\sum_{n,m\in\Omega_N}\left(Q_{t,b,s}^{n,m}-x^{n,m}I_{t,b,s}^{\text{sqr},n,m}\right)-\sum_{m,n\in\Omega_N}\left(Q_{t,b,s}^{m,n}\right)+Q_{t,b,s}^{\text{WD},m}+Q_{t,b,s}^{\text{PV},m}+Q_{t,b,s}^{\text{CB},m}=\eta_t\eta_{b,s}^{\text{load}}Q^{\text{load},m}, \tag{21}$$

$$\sum_{n,m\in\Omega_N}\left(Q_{t,b,s}^{n,m}-x^{n,m}I_{t,b,s}^{\text{sqr},n,m}\right)-\sum_{m,n\in\Omega_N}\left(Q_{t,b,s}^{m,n}\right)+Q_{t,b,s}^{\text{SS},m}=\eta_t\eta_{b,s}^{\text{load}}Q^{\text{load},m}. \tag{22}$$

3.2.2 Voltage and Current Equations

The nodal voltage equation and power flow equation are shown as follows:

$$\begin{aligned} V^{\mathrm{sqr},m}_{t,b,s} - 2(r^{m,n}P^{m,n}_{t,b,s} + x^{m,n}Q^{m,n}_{t,b,s}) \\ + |z^{m,n}|^2 I^{\mathrm{sqr},m,n}_{t,b,s} - V^{\mathrm{sqr},n}_{t,b,s} = 0, \end{aligned} \tag{23}$$

$$V^{\mathrm{sqr},m}_{t,b,s} I^{\mathrm{sqr},m,n}_{t,b,s} = P^{m,n\,2}_{t,b,s} + Q^{m,n\,2}_{t,b,s}. \tag{24}$$

To transform the non-linear equation (24) into the linear equation, the piecewise linear approximation described in (Zou et al., 2010) is used in this paper. The equation is linearized as follows:

$$\begin{aligned} V^{\mathrm{nom}2} I^{\mathrm{sqr},m,n}_{t,b,s} = \sum_{h\in\Omega_H} \left(k^{m,n,h}_{t,b,s}\Delta P^{m,n,h}_{t,b,s}\right) \\ + \sum_{h\in\Omega_H} \left(k^{m,n,h}_{t,b,s}\Delta Q^{m,n,h}_{t,b,s}\right), \end{aligned} \tag{25}$$

$$P^{m,n}_{t,b,s} = P^{+,m,n}_{t,b,s} - P^{-,m,n}_{t,b,s}, \tag{26}$$

$$Q^{m,n}_{t,b,s} = Q^{+,m,n}_{t,b,s} - Q^{-,m,n}_{t,b,s}, \tag{27}$$

$$X^{P+,m,n}_{t,b,s} + X^{P-,m,n}_{t,b,s} \le 1, \tag{28}$$

$$X^{Q+,m,n}_{t,b,s} + X^{Q-,m,n}_{t,b,s} \le 1. \tag{29}$$

$$P^{+,m,n}_{t,b,s} + P^{-,m,n}_{t,b,s} = \sum_{h\in\Omega_H} \Delta P^{m,n,h}_{t,b,s}, \tag{30}$$

$$Q^{+,m,n}_{t,b,s} + Q^{-,m,n}_{t,b,s} = \sum_{h\in\Omega_H} \Delta Q^{m,n,h}_{t,b,s}, \tag{31}$$

$$0 \le \Delta P^{m,n,h}_{t,b,s} \le \Delta S^{m,n,h}_{t,b,s}, \tag{32}$$

$$0 \le \Delta Q^{m,n,h}_{t,b,s} \le \Delta S^{m,n,h}_{t,b,s}, \tag{33}$$

$$k^{m,n,h}_{t,b,s} = (2h-1)\Delta S^{m,n,h}_{t,b,s}, \tag{34}$$

$$\Delta S^{m,n,h}_{t,b,s} = \frac{V^{\mathrm{nom}}\overline{I^{m,n}}}{\overline{H}}. \tag{35}$$

3.2.3 Current, Voltage, and Power Limits

The current on branches, voltage of buses, and power flow on branches should be limited in the allowable range:

$$0 \le V^{\mathrm{nom}2} I^{\mathrm{sqr},m,n}_{t,b,s} \le \overline{S^{m,n}}^2, \tag{36}$$

$$\underline{V}^2 \le V^{\mathrm{sqr},m}_{t,b,s} \le \overline{V}^2, \tag{37}$$

$$0 \le P^{+,m,n}_{t,b,s} \le V^{\mathrm{nom}}\overline{I^{m,n}} X^{P+,m,n}_{t,b,s} \tag{38}$$

$$0 \le P^{-,m,n}_{t,b,s} \le \overline{P^{\mathrm{rev},m,n}} X^{P-,m,n}_{t,b,s}, \tag{39}$$

$$0 \le Q^{+,m,n}_{t,b,s} \le V^{\mathrm{nom}}\overline{I^{m,n}} X^{Q+,m,n}_{t,b,s}, \tag{40}$$

$$0 \le Q^{-,m,n}_{t,b,s} \le V^{\mathrm{nom}}\overline{I^{m,n}} X^{Q-,m,n}_{t,b,s}. \tag{41}$$

3.2.4 Maximum DG Size Limits

The following constraint defines the maximum DG installation capacity of each bus:

$$\sum_{t\in\Omega_T} (\overline{P^{\mathrm{WD}}} X^{\mathrm{WD},n}_t + \overline{P^{\mathrm{PV}}} X^{\mathrm{PV},n}_t) \le \overline{P^{\mathrm{node}}}. \tag{42}$$

3.2.5 DG & CB Generation Limits

$$0 \le P^{\mathrm{WD},n}_{t,b,s} \le \eta^{\mathrm{WD}}_{b,s} P^{\mathrm{avl,WD},n}_t, \tag{43}$$

$$0 \le P^{\mathrm{PV},n}_{t,b,s} \le \eta^{\mathrm{PV}}_{b,s} P^{\mathrm{avl,PV},n}_t, \tag{44}$$

$$0 \le Q^{\mathrm{CB},n}_{t,b,s} \le Q^{\mathrm{avl,CB},n}_t. \tag{45}$$

Constraints (43) – (45) express the minimum and maximum generation of DGs and CB. Note that the scenarios of the wind power and PV, i.e., production factors $\eta^{\mathrm{WD}}_{b,s}$ and $\eta^{\mathrm{PV}}_{b,s}$, are used by multiplying the maximum available output of each installed DG. The following constraints show the maximum available output in each year:

$$P^{\mathrm{avl,WD},n}_t = \overline{P^{\mathrm{WD}}} X^{\mathrm{WD},n}_t C^{\mathrm{WD},n}; t = 1, \tag{46}$$

$$P^{\mathrm{avl,WD},n}_t = \overline{P^{\mathrm{WD}}} X^{\mathrm{WD},n}_t C^{\mathrm{WD},n} + P^{\mathrm{avl,WD},n}_{t-1}; t > 1 \tag{47}$$

$$P^{\mathrm{avl,PV},n}_t = \overline{P^{\mathrm{PV}}} X^{\mathrm{PV},n}_t C^{\mathrm{PV},n}; t = 1, \tag{48}$$

$$P^{\mathrm{avl,PV},n}_t = \overline{P^{\mathrm{PV}}} X^{\mathrm{PV},n}_t C^{\mathrm{PV},n} + P^{\mathrm{avl,PV},n}_{t-1}; t > 1, \tag{49}$$

$$Q^{\mathrm{avl,CB},n}_t = \overline{Q^{\mathrm{CB}}} X^{\mathrm{CB},n}_t C^{\mathrm{CB},n}; t = 1, \tag{50}$$

$$Q^{\mathrm{avl,CB},n}_t = \overline{Q^{\mathrm{CB}}} X^{\mathrm{CB},n}_t C^{\mathrm{CB},n} + Q^{\mathrm{avl,CB},n}_{t-1}; t > 1. \tag{51}$$

The number of installations of DG and CB in each bus is limited as,

$$\sum_{t\in\Omega_T} X^{\mathrm{WD},n}_t \le \overline{X^{\mathrm{WD}}_n}, \tag{52}$$

$$\sum_{t\in\Omega_T} X^{\mathrm{PV},n}_t \le \overline{X^{\mathrm{PV}}_n}, \tag{53}$$

$$\sum_{t\in\Omega_T} X^{\mathrm{CB},n}_t \le \overline{X^{\mathrm{CB}}_n}. \tag{54}$$

The constraints of the reactive power produced by DGs are expressed by using leading/lagging power factor:

$$\begin{aligned} -\tan(\cos^{-1}(\lambda^{\mathrm{WD}}_{\mathrm{lead}}))P^{\mathrm{WD},n}_{t,b,s} \le Q^{\mathrm{WD},n}_{t,b,s} \\ \le \tan(\cos^{-1}(\lambda^{\mathrm{WD}}_{\mathrm{lag}}))P^{\mathrm{WD},n}_{t,b,s}, \end{aligned} \tag{55}$$

$$\begin{aligned} -\tan(\cos^{-1}(\lambda^{\mathrm{PV}}_{\mathrm{lead}}))P^{\mathrm{PV},n}_{t,b,s} \le Q^{\mathrm{PV},n}_{t,b,s} \\ \le \tan(\cos^{-1}(\lambda^{\mathrm{PV}}_{\mathrm{lag}}))P^{\mathrm{PV},n}_{t,b,s}. \end{aligned} \tag{56}$$

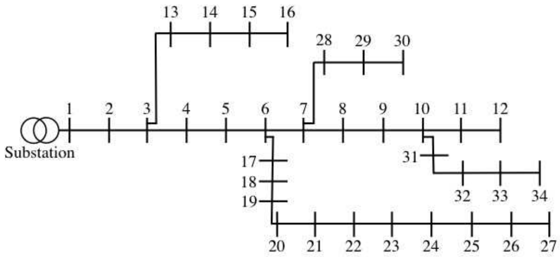

Figure 3: Distribution system configuration.

3.2.6 Investment Limits

The following constraints refer to the annualized and actual investment cost limits considering the lifetime.

$$\pi_t^{\mathrm{inv}} \leq \pi_{\mathrm{inv}}^{\mathrm{bgt}}, \tag{57}$$

$$\sum_{t\in\Omega_T} \alpha_t \Big[\sum_{n\in\Omega_{SS}} \pi_{\mathrm{inv}}^{\mathrm{SS}} X_t^{\mathrm{SS},n} + \sum_{n\in\Omega_L} (\pi_{\mathrm{inv}}^{\mathrm{PV}} X_t^{\mathrm{PV},n} + \pi_{\mathrm{inv}}^{\mathrm{WD}} X_t^{\mathrm{WD},n} + \pi_{\mathrm{inv}}^{\mathrm{CB}} X_t^{\mathrm{CB},n}) \Big] \leq \pi_{\mathrm{LT}}^{\mathrm{bgt}}. \tag{58}$$

3.2.7 Energy Not Supplied Limits

The unserved power must be less than the demand:

$$0 \leq P_{t,b,s}^{\mathrm{ENS},n} \leq \eta_t \eta_{b,s}^{\mathrm{load}} P^{\mathrm{load},n}, \tag{59}$$

$$0 \leq Q_{t,b,s}^{\mathrm{ENS},n} \leq \eta_t \eta_{b,s}^{\mathrm{load}} Q^{\mathrm{load},n}. \tag{60}$$

3.2.8 Substation Limits

The following constraints show the generation limit of the substation.

$$P_{t,b,s}^{\mathrm{SS},n} \leq \frac{S_t^{\mathrm{avl,SS},n}}{\sqrt{1+\tan(\cos^{-1}(\lambda^{\mathrm{SS}}))^2}}, \tag{61}$$

$$0 \leq Q_{t,b,s}^{\mathrm{SS},n} \leq \tan(\cos^{-1}(\lambda^{\mathrm{SS}})) P_{t,b,s}^{\mathrm{SS},n}, \tag{62}$$

$$S_t^{\mathrm{avl,SS},n} = S^{\mathrm{SS},n} + S_t^{\mathrm{new},n}, \tag{63}$$

$$S_t^{\mathrm{new},n} = X_t^{\mathrm{SS},n} \overline{S^{\mathrm{SS}}}; t=1, \tag{64}$$

$$S_t^{\mathrm{new},n} = X_t^{\mathrm{SS},n} \overline{S^{\mathrm{SS}}} + S_{t-1}^{\mathrm{new},n}; t>1. \tag{65}$$

The substation expansion is allowed up to the maximum power:

$$S_t^{\mathrm{new},n} \leq \overline{S^{\mathrm{new},n}}. \tag{66}$$

4 NUMERICAL SIMULATION

4.1 Distribution System

The 34-bus three-phase radial feeder, shown in Figure 3, is used to test the proposed scenario generation and allocation problem. The system has 1 substation and 33 buses with/without load. Details of the network are given in (Chis et al., 1997).

Table 1: Simulation parameters.

Total peak load power	5.45 (MVA)	Initial available substation power	5.50 (MVA)
Capacity of wind turbine and PV	100, 2.5 (kW)	Capacity of CB	100 (kVar)
Base power	10 (MVA)	Base voltage	11 (kV)
Maximum power that can be installed at each bus	250 (kW)	Maximum numbers of wind turbine, PV modules, and CB	2, 85, 5
Thermal capacity	6.5 (MVA)	Substation voltage	1.04 (p.u.)
Annual demand growth	2 (%)	Price increasing factor	1 (%)
Minimum/ maximum limits of voltage magnitude	±5% (0.95 and 1.05 p.u.)	Number of segments used in the piecewise linearization	2
Increasing factor of emission cost	2 (%)	Lifetime of devices	20 (years)
Investment cost of transformer, wind turbine, PV module, and CB	20000,125155, 3455,38500(€)	O&M costs of wind turbine, PV, and CB	0.0079,0.0064 (€/kWh) ,0.003(€/kVarh)
Subsidy rate of wind turbine and PV	10, 5 (%)	Power factor at the substation	0.9013
Discount rate	12.5 (%)	Lagging/leading power factor of DGs	0.9013, 0.0
Interest rate	8 (%)	Cost of CO_2 emission	30 (€/tCO_2)
Investment budget per year	350000 (€)	Emission rate of purchased energy	0.55 (tCO_2/MWh)
Investment budget throughout the life cycle of devices	5500000 (€)	Emission rate of wind turbine and PV	0.25 and 0.26 (tCO_2/MWh)
Cost of not supplied energy	15000 (€/MWh)	Maximum expansion of the substation	5 (MVA)
Candidate buses of wind turbines	13-16, 21-27	Candidate buses of PV	11, 12, 24-27, 31-34

Table 2: Model information.

Number of continuous variables	2,810,473
Number of general integer variables	444,960
Number of binary variables	570,240
Number of linear constraints	4,578,985
Number of non zero coefficients	14,070,169

4.2 Data and Parameters

The simulation parameters are shown in Table 1. Actual load data of Tokyo Electric Power Company (TEPCO) are used as demand. The wind speed and solar radiation are the meteorological observation data of Miyakojima Island in Japan from Jan. 1, 2015 to Dec. 31, 2015. A twenty-year period is used as a planning horizon. Demand, wind, PV, and price levels are described in Table 3. The problem is solved using Gurobi 6.5.0 (Gurobi 6.5.0, 2016) on a Linux-based computer with 4-core Intel ®Core i7-4770 at 3.4 GHz and 24 GB of RAM. The information about the overall model is described in Table 2.

4.3 Simulation Cases

The following three cases are consided:

Case A: The investment is only allowed for the expansion of the substation, i.e., the right-hand side of Eq. (42) is zero.

Table 3: Scenario factors of each time block. The values in parentheses represent the factor's probabilities.

Time Blocks	Hours	Price (€/MWh)	Demand factors	Wind factors	PV factors
1	1370	97.63	0.61 (0.328)	0.41 (0.370)	0.65 (0.163)
		98.04	0.58 (0.328)	0.16 (0.207)	0.29 (0.602)
		98.28	0.54 (0.344)	0.00 (0.423)	0.02 (0.235)
2	420	103.08	0.93 (0.433)	0.41 (0.329)	0.68 (0.321)
		103.01	0.84 (0.236)	0.20 (0.443)	0.41 (0.371)
		102.44	0.78 (0.331)	0.00 (0.229)	0.06 (0.307)
3	1316	97.84	0.51 (0.212)	1.00 (0.423)	0.62 (0.910)
		97.68	0.48 (0.444)	0.23 (0.572)	0.28 (0.066)
		97.19	0.44 (0.344)	0.00 (0.005)	0.01 (0.024)
4	1286	100.60	0.73 (0.381)	0.92 (0.428)	0.68 (0.451)
		99.20	0.68 (0.171)	0.28 (0.022)	0.37 (0.271)
		98.26	0.64 (0.448)	0.00 (0.550)	0.05 (0.278)
5	960	94.15	0.52 (0.245)	0.69 (0.109)	0.59 (0.070)
		94.15	0.48 (0.453)	0.33 (0.529)	0.29 (0.060)
		94.15	0.45 (0.302)	0.01 (0.361)	0.01 (0.870)
6	1205	94.15	0.73 (0.324)	0.46 (0.236)	0.58 (0.155)
		94.15	0.69 (0.381)	0.24 (0.441)	0.28 (0.207)
		94.15	0.66 (0.295)	0.00 (0.323)	0.03 (0.638)
7	1590	94.15	0.62 (0.370)	0.50 (0.248)	0.57 (0.675)
		94.15	0.59 (0.304)	0.25 (0.354)	0.27 (0.143)
		94.15	0.56 (0.326)	0.00 (0.397)	0.01 (0.182)
8	613	94.15	0.88 (0.502)	0.49 (0.323)	0.56 (0.726)
		94.15	0.81 (0.168)	0.22 (0.393)	0.26 (0.096)
		94.15	0.77 (0.330)	0.00 (0.284)	0.02 (0.178)

Table 4: O&M costs (€).

Cases	A	B	C
Losses cost	1,163,844	1,012,212	786,011
Not supplied energy cost	45,320	67,221	4,937
Purchased energy cost	24,975,772	22,841,032	16,196,578
DG O&M cost	0	141,731	572,780
Capacitor bank cost	218,980	193,093	134,986
Emission cost	4,567,235	4,196,637	3,043,670
O&M system cost	30,971,152	28,451,927	20,738,962

Table 5: Total system costs (€).

Cases	A	B	C
O&M system cost	30,971,152	28,451,927	20,738,962
Investment costs	413,085	2,228,716	8,369,486
Incentive	0	185,179	697,443
Total costs	31,384,236	30,495,464	28,411,005
Computational time	25262 s	277680 s	34693 s

Case B: All the constraints are considered.

Case C: Case B without investment constraints (57) and (58).

4.4 Results and Discussions

Tables 4 and 5 show the O&M costs and total system costs. Optimal location, sizing, and timing are shown in Tables 6 and 7. The installation of DGs plays an important role to reduce the total system cost despite the fact that the investment costs are increasing. A significant contribution is that it drastically reduces the O&M costs (see Table 4). This is one of the general benefits of DG installment. From Table 4, the greatest cost savings occur in the emission cost because the emission rate of the purchased energy at the substation is two times higher than that of the DGs. Moreover, the losses cost and purchased energy cost are reduced since most DGs are allocated around the terminal buses of radial distribution system. As

Table 6: Optimal location and timing (bus).

Cases	A		B				C			
Years	SUB	CB	SUB	WD	PV	CB	SUB	WD	PV	CB
1		12 21 22 23 25 26 27 33		24 25 26	24 26 27 33 34	11 12 21 22 23 25 26 32		13 14 15 21 22 23 24 25 26 27	11 12 24 25 26 27 31 32 33 34	11 21 22 23 25 26
2		11						14		
3		24				24				
4								16		
5	1							16		
6	1		1							
7							1			23
8						21				
9		21								24
10										
11										
12						27				
13		21 31	1			21 31				21 31
14		14 21 23 25				11 23 24	1			12 21 27
15		11 14 22 26				22 31 33				11 21 25 33
16		11 15 31				21 22 24 31				11 21 22 24
17		13 22				22 25 31				14 15 22 31
18		12 13 24 32				12				11 13 26
19		13 14 21 31								13 24
20		16								15 22 31 32

Table 7: Optimal sizing (kW).

Cases	A		B				C			
Years	SUB	CB	SUB	WD	PV	CB	SUB	WD	PV	CB
1		800		600	262.5	800		1500	1875	600
2		100						100		
3		100				100				
4								100		
5	1000							100		
6	1000		1000							
7							1000			100
8						100				
9		100								100
10										
11										
12						100				
13		200	1000			200				200
14		500				300	1000			300
15		400				400				400
16		400				500				400
17		400				400				500
18		500				100				400
19		500								500
20		100								400
Total	2000	4100	2000	600	262.5	3000	2000	1800	1875	3900

shown in Table 6, the DGs allow the substation expansion to defer. However, the results imply that the expansion is not inevitable due to the intermittent nature of renewable DGs and the demand growth (see Table 7).

The O&M cost of CB decreases even if the number of CB increases (see Tables 4 and 7), implying that CB co-exists well with the large amount of the installed DGs. Without the budget constraints, nearly the same amount of wind turbine and PV are installed. However, in the consideration of the budgets, the wind power to be installed is larger than PV because it is affected by the high subsidy rate of wind.

In the same way, the simulations without the incentive were tested, i.e., the incentives of wind energy and PV are 0. The O&M and total system costs are shown in Tables 8 and 9. Tables 5 and 9 indi-

Table 8: O&M costs (€).

Cases	A	B	C
Losses cost	1,163,844	1,000,584	795,689
Not supplied energy cost	45,320	71,244	359
Purchased energy cost	24,975,772	22,808,383	16,558,486
DG O&M cost	0	133,506	537,774
Capacitor bank cost	218,980	196,581	133,903
Emission cost	4,567,235	4,191,123	3,105,423
O&M system cost	30,971,152	28,401,420	21,131,633

Table 9: Total system costs (€).

Cases	A	B	C
O&M system cost	30,971,152	28,401,420	21,131,633
Investment costs	413,085	2,262,126	7,914,809
Incentive	0	0	0
Total costs	31,384,236	30,663,546	29,046,443

Table 10: Optimal sizing under no incentive (kW).

Cases	A		B				C			
Years	SUB	CB	SUB	WD	PV	CB	SUB	WD	PV	CB
1	0	800	0	300	522.5	800	0	1000	2055	600
2	0	100	0	0	0	0	0	0	7.5	0
3	0	100	0	0	0	100	0	100	0	0
4	0	0	0	0	0	0	0	100	0	0
5	1000	0	1000	0	0	0	0	0	0	0
6	1000	0	0	0	0	0	0	100	0	0
7	0	0	0	0	0	0	1000	0	0	100
8	0	0	0	0	0	100	0	100	0	0
9	0	100	0	0	0	0	0	100	0	0
10	0	0	0	0	0	100	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0
13	0	200	0	0	0	200	0	0	0	300
14	0	500	0	0	0	300	0	0	0	300
15	0	400	1000	0	0	400	1000	0	0	400
16	0	400	0	0	0	500	0	0	0	400
17	0	400	0	0	0	400	0	0	0	400
18	0	500	0	0	0	0	0	0	0	500
19	0	500	0	0	0	0	0	0	0	400
20	0	100	0	0	0	0	0	0	0	500
Total	2000	4100	2000	300	522.5	2900	2000	1500	2062.5	3900

cate that the incentive is helpful to decrease the total system costs, though the O&M costs of case B is increased slightly. The optimal sizing under no incentive is shown in Table 10. From this result, it is suggested that PV is installed more than wind power in the case that there are no incentives.

It is worth pointing out that the DGs have an important role in terms of system stability as well as cost minimization. The average of the voltage deviations of all scenarios in the first- and final- planning years are illustrated per case in Figure 4. The figure shows that the overall voltage drops as the demand increases for twenty years. Besides, the large installation of DGs makes the amplitude of the voltage more stable than no DGs.

5 CONCLUSIONS

The paper has presented a procedure for creating the demand and DG generation scenarios with K-means. Simultaneously, a long-term allocation problem of RES-based DGs has been formulated as a two-stage stochastic programming problem and tested on the 34-bus distribution system. The obtained results and

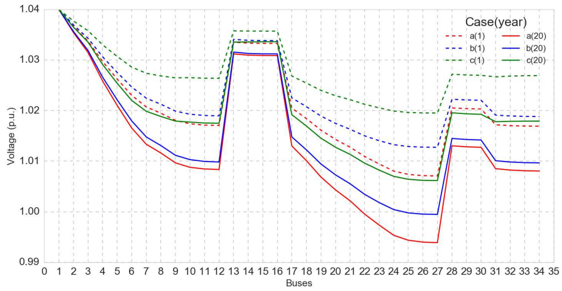

Figure 4: Average of the voltages of all scenarios per case in the first and final year.

insights are summarized as below:

- The long-term optimal solutions for the decision-making are obtained by solving the stochastic optimization problem with the created scenarios.
- The uncertainties of scenarios are well-represented because the substation expansions are inevitable due to the renewable energy intermittency, while the DG installation reduces the total distribution system cost.
- The proposed method with K-means can be easily implemented, improved to create many scenarios, and expanded to a multi-stage architecture.
- The proposed problem determines the optimal long-term siting, sizing, and timing of DGs, considering the variables and constraints with respect to the practical equipment and economics.
- The results show that an optimal DG allocation is quite important in order to reduce the system cost.

Future research include the following:

- Investigation of the planning results for a large distribution system.
- Comparison with the existing methodologies to analyze whether the results will be much different.
- Improvement of the scenario generation by means of the probability density function and time series model.
- Extension to a multi-stage stochastic programming problem and comparative evaluation of the validity of the solution.

ACKNOWLEDGEMENTS

We gratefully acknowledge the work of members of our laboratory. We are also grateful to the referees for useful comments. This research was supported by JST, CREST.

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APPENDIX

Nomenclature

Sets:

Ω_B	Set of time blocks
Ω_H	Set of blocks used for the piecewise linearization of quadratic power
Ω_L	Set of load buses
Ω_N	Set of branches
Ω_{SS}	Set of substation buses
Ω_T	Set of years
Ω_{S_b}	Set of scenarios in time block b

Indices:

b	Time block index
h	Index of the segment used for the linearization
n,m	Index of bus numbers
t	Time index
s	Scenario index

Parameters:

$\pi^{\text{SS}}_{\text{anu}}, \pi^{\text{WD}}_{\text{anu}}, \pi^{\text{PV}}_{\text{anu}}, \pi^{\text{CB}}_{\text{anu}}$	Annualized investment costs of transformer, wind turbine, PV module, and capacitor bank
$\pi^{\text{SS}}_{\text{inv}}, \pi^{\text{WD}}_{\text{inv}}, \pi^{\text{PV}}_{\text{inv}}, \pi^{\text{CB}}_{\text{inv}}$	Investment costs of transformer, wind turbine, PV module, and capacitor bank
$\pi^{\text{bgt}}_{\text{inv}}$	Annual investment budget
$\pi^{\text{bgt}}_{\text{LT}}$	Investment budget throughout the lifetime of the devices to be installed
$\pi^{\text{WD}}_{\text{om}}, \pi^{\text{PV}}_{\text{om}}, \pi^{\text{CB}}_{\text{om}}$	Operation and maintenance costs of wind turbine, PV module, capacitor bank
π^{loss}	Cost of power loss
π^{CO_2}	Cost of CO_2 emission
π^{ENS}	Cost of energy not supplied
$\pi^{\text{SS}}_{b,s}$	Cost of energy purchased from upper grid at substation in time block b and scenario s
$C^{\text{WD},n}, C^{\text{PV},n}, C^{\text{CB},n}$	Binary parameters whether bus n is the candidates to install wind turbines, PV modules, and capacitor banks
d	Discount rate
η^{emi}_t	Increasing factor of emission cost
η_t	Increasing factor of load
η^{SS}_t	Increasing factor of energy cost
$\eta^{\text{load}}_{b,s}$	Demand factor in time block b and scenario s
$\eta^{\text{WD}}_{b,s}, \eta^{\text{PV}}_{b,s}$	Production factors of wind turbine and PV module in time block b and scenario s
$\overline{I^{n,m}}$	Maximum current flow of branch n,m
$k^{n,m,h}_{t,b,s}$	Slope of the h-th block of the piecewise linearization for branch n,m in year t, time block b, and scenario s
i	Interest rate
$L^{\text{SS}}, L^{\text{WD}}, L^{\text{PV}}, L^{\text{CB}}$	Lifetimes of transformer, wind turbine, PV module, and capacitor bank
N^{hours}_b	Number of hours in time block b
$P^{\text{load},n}$	Active power of load in bus n
$\overline{P^{\text{WD}}}, \overline{P^{\text{PV}}}$	Maximum active power generations of wind turbine and PV module
$\overline{P^{\text{node}}}$	Maximum active power of RES that can be installed in each bus
$\overline{P^{\text{rev},m,n}}$	Maximum reverse active power flow in branch m,n
λ^{SS}	Power factor at substation
$\lambda^{\text{WD}}_{\text{lead}}, \lambda^{\text{WD}}_{\text{lag}}$	Leading/lagging power factors of wind turbine
$\lambda^{\text{PV}}_{\text{lead}}, \lambda^{\text{PV}}_{\text{lag}}$	Leading/lagging power factors of PV module
$\overline{Q^{\text{CB}}}$	Maximum reactive power generation per capacitor bank
$Q^{\text{load,n}}$	Reactive power of load in bus n
$r^{n,m}$	Resistance of branch n,m
$\nu^{\text{SS}}_{\text{emi}}, \nu^{\text{WD}}_{\text{emi}}, \nu^{\text{PV}}_{\text{emi}}$	Emission rates of purchased energy and distributed generation
$\gamma^{\text{WD}}_{\text{sub}}, \gamma^{\text{PV}}_{\text{sub}}$	Subsidy rates for investment of wind turbines and PV modules
$\overline{H}$	Number of segments used in the piecewise linearization
$\overline{S^{\text{SS}}}$	Maximum power generation of new transformers
$\overline{S^{n,m}}$	Maximum transmission capacity of branch n,m
$\overline{S^{\text{new},n}}$	Maximum new power allowed for investment in the substation n
$S^{\text{SS},n}$	Existing power in the substation n

S^{base}	Base power
$\underline{V}, \overline{V}$	Minimum/maximum voltage magnitudes of the distribution network
V^{nom}	Nominal voltage of the distribution network
$x^{n,m}$	Reactance of branch n,m
$\overline{X^{\text{WD},n}}, \overline{X^{\text{PV},n}}, \overline{X^{\text{CB},n}}$	Maximum number of wind turbines, PV modules, and capacitor banks to be installed in bus n
$z^{n,m}$	Impedance of branch n,m
$\text{Pr}_{b,s}$	Probability of scenario s in time block b
$\text{Pr}^{\text{load}}_{b,s}, \text{Pr}^{\text{WD}}_{b,s}, \text{Pr}^{\text{PV}}_{b,s}$	Probabilites of demand, wind power production, and PV production in time block b and scenario s
$\Delta S^{n,m,h}_{t,b,s}$	Upper bound of h-th block of the power flow of branch n,m in year t, time block b, and scenario s
α_t	Present value factor

Variables:

$\pi^{\text{CB}}_{t,b,s}$	Operation and maintenance cost of capacitor banks in year t, time block b, and scenario s
$\pi^{\text{emi}}_{t,b,s}$	Costs of CO_2 emission in year t, time block b, and scenario s
$\pi^{\text{emi,SS}}_{t,b,s}, \pi^{\text{emi,DG}}_{t,b,s}$	Costs of CO_2 emission from purchased energy and DG in year t, time block b, and scenario s
π^{inv}_t	Cost of investment in year t
$\pi^{\text{loss}}_{t,b,s}$	Cost of power losses in year t, time block b, and scenario s
$\pi^{\text{ENS}}_{t,b,s}$	Penalty cost for energy not supplied in year t, time block b, and scenario s
$\pi^{\text{new}}_{t,b,s}$	Operation and maintenance costs of distributed generation in year t, time block b, and scenario s
$\pi^{\text{om}}_{t,b,s}$	Operation and maintenance costs of in year t, time block b, and scenario s
$\pi^{\text{SS}}_{t,b,s}$	Cost of energy purchased from upper grid at substation in year t, time block b, and scenario s
μ^{inc}_t	Incentive for new installation of the distributed generations in year t
$I^{\text{sqr},n,m}_{t,b,s}$	Square of the current flow magnitude of branch n,m in year t, time block b, and scenario s
$P^{\text{avl,WD},n}_t, P^{\text{avl,PV},n}_t$	Total active power available of wind turbines and PV modules to be installed in bus n and year t
$P^{\text{ENS},n}_{t,b,s}$	Not served active power in bus n, year t, time block b, and scenario s
$P^{\text{WD},n}_{t,b,s}, Q^{\text{WD},n}_{t,b,s}$	Active/reactive power generation of wind turbines in bus n, year t, time block b, and scenario s
$P^{\text{PV},n}_{t,b,s}, Q^{\text{PV},n}_{t,b,s}$	Active/reactive power generation of PV modules in bus n, year t, time block b, and scenario s
$P^{n,m}_{t,b,s}, Q^{n,m}_{t,b,s}$	Active/reactive power flow of branch n,m in year t, time block b, and scenario s
$P^{+,n,m}_{t,b,s}, Q^{+,n,m}_{t,b,s}$	Active/reactive power flow (forward) of branch n,m in year t, time block b, and scenario s
$P^{-,n,m}_{t,b,s}, Q^{-,n,m}_{t,b,s}$	Active/reactive power flow (backward) of branch n,m in year t, time block b, and scenario s
$P^{\text{SS},n}_{t,b,s}, Q^{\text{SS},n}_{t,b,s}$	Active/reactive power purchased from the grid at the substation in bus n, year t, time block b, and scenario s
$\Delta P^{n,m,h}_{t,b,s}, \Delta Q^{n,m,h}_{t,b,s}$	Value of the h-th block of the piecewise linearized active/reactive power of branch n,m in year t, time block b, and scenario s
$Q^{\text{avl,CB},n}_t$	Total reactive power available of capacitor banks to be installed in bus n and year t
$Q^{\text{CB},n}_{t,b,s}$	Reactive power compensated by capacitor banks in bus n, year t, time block b, and scenario s
$S^{\text{avl,SS},n}_t$	Total power available in the substation n and year t
$S^{\text{new},n}_t$	New transformers installed in the substation n and year t
$V^{\text{sqr},n}_{t,b,s}$	Square of voltage magnitude of bus n in year t, time block b, and scenario s
$X^{\text{SS},n}_t, X^{\text{WD},n}_t, X^{\text{PV},n}_t, X^{\text{CB},n}_t$	Number of transformers, wind turbines, PV modules, and capacitor banks to be installed in bus n and year t
$X^{P+,n,m}_{t,b,s}, X^{P-,n,m}_{t,b,s}$	Binary variable defined for forward/backward active power flow of branch n,m in year t, time block b, and scenario s
$X^{Q+,n,m}_{t,b,s}, X^{Q-,n,m}_{t,b,s}$	Binary variable defined for forward/backward reactive power flow of branch n,m in year t, time block b, and scenario s