

# The Quasi-Triangle Array of Rectangular Holes with the Completely Suppression of High Order Diffractions

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**Abstract:** We propose the quasi-triangle array of rectangular holes with the completely suppression of high order diffractions. The membrane with holes can be free-standing and scalable from X-rays to far infrared wavelengths. Both numerical and experimental results demonstrate the completely suppression of high order diffractions. The desired diffraction pattern only containing the 0<sup>th</sup> and +1<sup>st</sup>/-1<sup>st</sup> order diffractions results from the constructive interference of lights from different holes according to some statistical law distribution. The suppression effect depends on the number of holes. Our results should be of great interest in a wide spectrum unscrambling for any wavelength range.

## 1 INTRODUCTION

Gratings are the key component of the spectrometers. Spectrum unscrambling only needs the first order diffraction of the traditional black-white grating. However, unwanted higher order diffraction always overlaps the first diffraction, and thus greatly degrade precision of analysis (Palmer, 2005). The sinusoidal transmission gratings only have 0<sup>th</sup> and +1<sup>st</sup>/-1<sup>st</sup> order diffractions (Born, and Wolf, 1980), but they are much more difficult to fabricate than the black-white ones (Jin, Gao, Liu, Li, and Tan, 2010, Vincent, Haidar, Collin, Guérineau, Primot, Cambriel, and Pelouard, 2008). The high order diffractions can become evanescent waves with a grating period in the range of the wavelength  $\lambda$  (Clausnitzer, Kämpfe, Kley, Tünnermann, Tishchenko, and Parriaux, 2008, Zhou, Seki, Kitamura, Kuramoto, Sukegawa, Ishii, Kanai, Itatani, Kobayashi, and Watanabe, 2014, Warren, Smith, Vawter, and Wendt, 1995). Unfortunately, for short wavelengths less than 100 nm, it's difficult to scale the grating period down to the wavelength size by the current nanofabrication technology (Pease, Deshpande, Wang, Russe, and Chou, 2007). Therefore, it has been a goal to design the black-white structure much larger than the wavelength with the suppression of high order diffractions.

Several structures with the suppression of high

order diffractions have been developed, and the points are to modulate the groove position or to introduce structures with complicated shapes (Gao, and Xie, 2011, Fan, et al, 2015, Cao, Förster, Fuhrmann, Wang, Kuang, Liu, and Ding, 2007). Unfortunately, the reported works can't obtain complete suppression of high order diffractions since it's difficult to realize the complex shapes or the very small gaps between the two adjacent grooves. Moreover, these structures based on the one dimensional grating can't be free-standing. Unfortunately, the supporting membrane will absorb 80% energy of the soft X-ray.

Recently, photon sieves with aperiodic distributed holes have drawn great attention owing to their novel properties, super-resolution focusing and imaging beyond the evanescent region (Kipp, Skibowski, Johnson, Berndt, Adelung, Harm, and Seemann, 2001, Huang, Liu, Garcia-Vidal, Hong, Luk'yanchuk, Teng, and Qiu, 2015, Huang, Kao, Fedotov, Chen, and Zheludev, 2008, Huang, Zheludev, Chen, and Abajo, 2007). The numerous holes can be designed that creates constructive interference, leading to a subwavelength focus of prescribed size and shape. Photon sieves with aperiodic distributed holes can acquire rich degrees of freedom (spatial position and geometric shape of holes) to realize complex functionalities, which are not achievable through periodic features with limited control in geometry.

Triggered by photon sieves, we propose the quasi-triangle array of rectangular holes with the completely suppression of high order diffractions. The membrane with holes can be free-standing and scalable from X-rays to far infrared wavelengths. The location distribution of holes according to some statistical law results in a desired diffraction pattern. We numerically and experimentally demonstrate the diffraction pattern of the quasi-periodic hole array with only 0<sup>th</sup> and +1<sup>st</sup>/-1<sup>st</sup> orders. Furthermore, we investigate the effect of the hole number on the diffraction property.

## 2 DESIGN AND SIMULATIONS

We start the design from the triangle array of rectangular holes, and the holes are shifted by  $s$  along the  $\xi$  axis (Fig. 1(a)) according to the probability distribution  $\rho(s) = (\pi/P_\xi) \cdot \cos(2\pi s/P_\xi)$ ,  $|s| \leq P_\xi/4$ , where  $2P_\xi$  is the period of the triangle array along the  $\xi$  axis. Here, we only consider the shift  $s$  along the  $\xi$  axis since spectral measurement is usually at one direction. Moreover, we choose the triangle array rather than the square array since the spacing between any two adjacent holes of the triangle array is larger than that of the square array for the same period and hole size. This will benefit

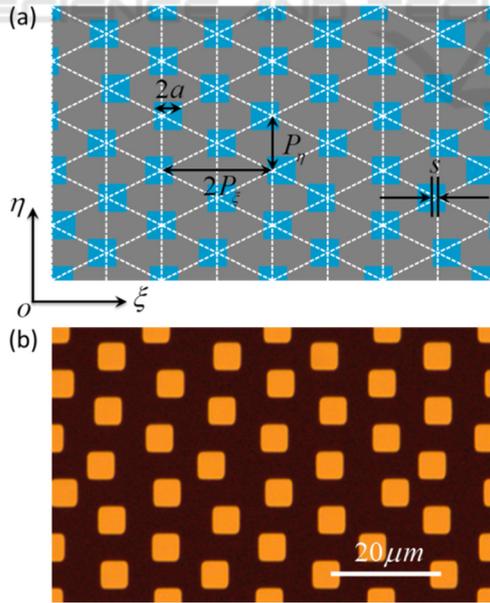


Figure 1: (a) The quasi-triangle array of rectangular holes: each hole shifts  $s$  along the  $\xi$  axis according to the probability distribution  $\rho(s)$ . (b) Microphotograph of the fabricated quasi-triangle array with  $4\text{cm} \times 4\text{cm}$  area.

the fabrication, and lead to more stable free standing structure. In addition, rectangular hole is selected since its diffraction intensity pattern is beneficial to the suppression of high order diffractions.

For a membrane that contains a large number of identical and similarly oriented holes, the light distribution in the Fraunhofer diffraction pattern is given by [3]

$$U(p, q) = C \sum_n e^{-ik(p\xi_n + q\eta_n)} \iint_A e^{-ik(p\xi' + q\eta')} d\xi' d\eta'. \quad (1)$$

Here  $C = \sqrt{P}/(\lambda z_0)$ ,  $P$  is the power density incident on the hole array,  $\lambda$  is the incident light wavelength,  $z_0$  is the distance between the hole array plane and the diffraction plane. The coordinates of the hole center are  $(\xi_1, \eta_1)$ ,  $(\xi_2, \eta_2)$ ,  $\dots$ ,  $(\xi_N, \eta_N)$ , and  $k$  is the wavevector.  $p = x/z_0$ ,  $q = y/z_0$ . The integration extends over the hole area and the integral expresses the effect of a single hole. The sum represents the superposition of the coherent diffraction patterns.

For the quasi-triangle array of  $2N_\xi N_\eta$  rectangular holes of sides  $2a = P_\xi/2$  and  $2b = P_\eta/2$  as shown in Fig. 1, the diffraction intensity pattern is

$$\begin{aligned} I(p, q) &= U(p, q) * U^*(p, q) \\ &= \frac{P \cdot (4ab)^2}{\lambda^2 z_0^2} \cdot \left( \frac{\sin kpa}{kpa} \right)^2 \left( \frac{\sin kqb}{kqb} \right)^2 \\ &\quad \cdot \left| \sum_n \int_s \rho(s) \exp(-ikp(\xi_n + s) - ikq\eta_n) ds \right|^2 \\ &= I_0 \cdot \frac{\cos^2(kpP_\xi/4) \cdot \cos^2(kp2P_\xi/4 + kqP_\eta/4)}{(1 - kpP_\xi/2/\pi)^2 (1 + kpP_\xi/2/\pi)^2} \\ &\quad \cdot \left( \frac{\sin kpa}{kpa} \right)^2 \left( \frac{\sin kqb}{kqb} \right)^2 \cdot \left( \frac{\sin N_\xi kp2P_\xi}{N_\xi \sin kp2P_\xi} \right)^2 \left( \frac{\sin N_\eta kqP_\eta}{N_\eta \sin kqP_\eta} \right)^2. \end{aligned} \quad (2)$$

Here  $I_0 = C^2 \cdot (2N_\xi N_\eta \cdot 4ab)^2$  is the peak irradiance of the diffraction pattern. For simplicity, in this letter we set  $I_0 = 1$ .  $2P_\xi$  and  $P_\eta$  are respectively the periods along the  $\xi$  and  $\eta$  axes.

Figure 2 presents the diffraction intensity pattern according to Eq. (2). As expected, the 0<sup>th</sup> and 1<sup>st</sup> order diffractions are kept along x axis, and the high-order diffractions disappear. The logarithm of diffraction intensity along x axis in Fig. 2(b) presents clearly the complete suppression of the high order diffractions. Insets in Fig. 2 shows clearly intensity distributions of the 0<sup>th</sup> and 1<sup>st</sup> order diffractions. Figure 2 shows that the diffraction pattern of the quasi-triangle array of rectangular holes along x axis is the same as the ideal sinusoidal transmission grating.

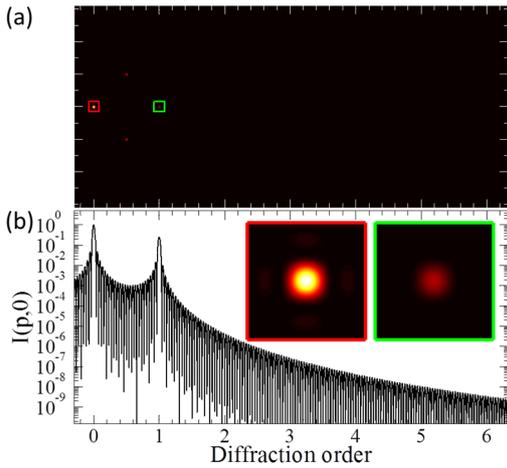


Figure 2: (a) The far-field diffraction intensity pattern of the quasi-triangle array of rectangular holes. (b) The diffraction intensity along the x axis. Insets: the 0<sup>th</sup> and 1<sup>st</sup> order diffractions.

Numerical simulation based on Eq. (2) is carried out to evaluate the diffraction property of the quasi-triangle array of 100000 rectangular holes. The logarithm of diffraction intensity along x axis is shown in Fig. 3, and high-order diffraction is much less than the noise of  $10^{-5}$  between 0th and 1st diffraction (red dash line in Fig. 3), which agrees well with the theoretical prediction of Eq. (2) and Fig. 2.

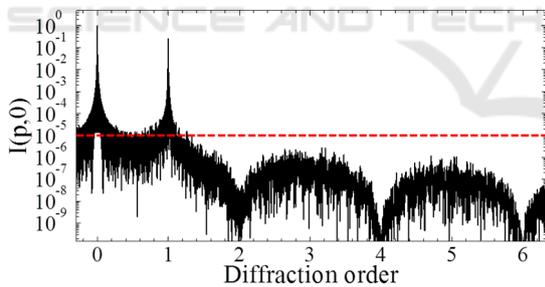


Figure 3: The diffraction intensity along x axis of the quasi-triangle array with 100000 rectangular holes.

### 3 EXPERIMENTAL RESULTS AND DISCUSSIONS

A proof-of-principle experiment is performed to confirm our theoretical and numerical results. The experimental setup is shown in Fig. 4. A plane wave is produced by expanding a 632 nm laser beam from Sprout (Lighthouse Photonics) and then illuminating the quasi-triangle array of holes. The far-field diffraction pattern from the array is focused by a

lens and recorded by a CCD (ANDOR DU920P-BU2). The quasi-triangle array of rectangular holes with  $4\text{cm} \times 4\text{cm}$  area is fabricated on a glass substrate by DESIGN WRITE LAZER 2000 from Heidelberg Instruments Mikrotechnik GmbH. The microphotograph of fabricated structure is illustrated in Fig. 1(b). Periods  $2P_\xi$  and  $P_\eta$  of the quasi-triangle array along the  $\xi$  and  $\eta$  axes are respectively  $20\mu\text{m}$  and  $10\mu\text{m}$ . The side of square holes is  $5\mu\text{m}$ .

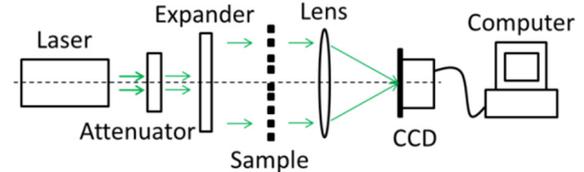


Figure 4: Experimental setup for the optical measurement.

Figure 5 presents the recorded diffraction pattern and it is obvious that high order diffractions of the quasi-triangle array of holes are effectively suppressed. The diffraction intensity along x axis in Fig. 5(b) is almost the same as the ideal sinusoidal transmission gratings. The ratio of 1st order diffraction intensity to the 0th order diffraction intensity is 73.56% and much larger than the theoretical prediction and numerical value 25%. This is because the CCD is saturated by the 0th order diffraction intensity. In addition, the red vertical lines in Fig. 5(a) are crosstalk along y direction due to our one-dimension CCD.

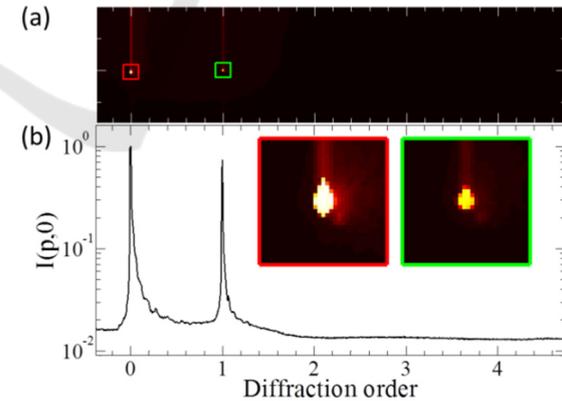


Figure 5: (a) The far-field diffraction intensity pattern of the quasi-triangle array of rectangular holes. (b) The diffraction intensity along the  $\xi$  axis.

Now we focus on the complete suppression of high order diffractions along the x axis. As  $N_\xi$  is large enough, the intensity according to Eq. (2) along the x axis is given by:

$$I(p) = \frac{I_0 \cdot \text{sinc}^2(N_\xi kp8a/\pi)}{(1 - kp2a/\pi)^2 (1 + kp2a/\pi)^2 \cos^2(kp4a)} \quad (3)$$

$$= \begin{cases} I_0, & p = 0 \\ \frac{1}{4} I_0, & p = \pm\pi/2ka \end{cases}$$

Equation (3) demonstrates that the quasi-triangle array of infinite rectangular holes can generate the same diffraction pattern as sinusoidal transmission gratings along the  $x$  axis. Only three diffraction peaks (the 0<sup>th</sup> order and +1<sup>st</sup> /-1<sup>st</sup> orders) appear on the  $x$ - $y$  plane. The above theoretical results are scalable from X-ray to far infrared wavelengths.

To obtain physical insight into the diffraction property of the quasi-triangle array of rectangular holes, the average transmission function along  $\xi$  axis is calculated by integrating the probability distribution over  $\eta$  axis:

$$T(\xi) = \int_{|\xi - P_\xi/4|}^{P_\xi/4} \rho(s) ds = \frac{1}{2} (1 + \cos(\frac{2\pi}{P_\xi} \xi)). \quad (4)$$

Equation (4) shows that the quasi-triangle array of infinite rectangular holes has the same transmission function along the  $\xi$  axis as sinusoidal transmission gratings. It is the average diffractive effect similar to sinusoidal grating that eliminates high-order diffractions.

We can also understand the suppression of high-order diffractions by the interference weakening or strengthening. It's known that diffraction peaks is from the constructive interference of lights from the different holes. The interference of lights from different rectangular holes is controlled by the hole position. The desired diffraction pattern only containing the 0<sup>th</sup> order and +1<sup>st</sup> /-1<sup>st</sup> order diffractions can be tailored by the location distribution of holes according to some statistical law.

In the above discussions, we assume the size of the quasi-triangle array is infinite. In practice, the number of holes is finite and thus the diffraction pattern will not be perfect as that described by Eq. (2) and (3). Numerical simulation based Eq. (2) is carried out and we obtain the noise intensity at 3rd order diffraction location for different number of holes in Fig. 6. It reveals that the noise intensity is approximately inversely proportional to the number of holes. This can be attributed to the approaching of the average transmittance function of quasi-triangle array to the ideal sinusoidal function with the hole number increasing. The noise intensity around the location of the 3rd order diffraction becomes less than  $10^{-5}$  as the hole number reaches to 30000.

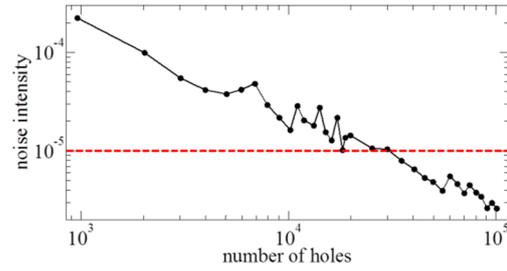


Figure 6: The maximum noise intensity around the location of the 3<sup>rd</sup> order diffraction versus the number of holes.

## 4 CONCLUSIONS

In conclusion, the binary quasi-triangle array of rectangular holes has been proposed to suppress the high-order diffractions which may lead to wavelength overlapping in spectral measurement. The new design of the membrane with holes can be free-standing and scalable from X-rays to far infrared wavelengths. Both numerical and experimental results have demonstrated the high-order diffractions are efficiently suppressed. The total number of holes illuminated by the light affects the suppression of the high-order diffractions and the noise intensity will be less than  $10^{-5}$  as the hole number larger than 30000. The binary quasi-periodic hole array offers an opportunity for high-accuracy spectral measurement and will possess broad potential applications in optical science and engineering fields.

## ACKNOWLEDGEMENTS

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