

# Effect of Dispersive Reflectivity on the Dynamics of Interacting Solitons in Dual-Core Systems with Separated Bragg Grating and Nonlinearity

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**Abstract:** The interactions of in-phase and  $\pi$ -out-of-phase quiescent Bragg grating solitons in a dual-core system where one core has Kerr nonlinearity and other is linear and has a Bragg grating with dispersive reflectivity are systematically investigated. The effect of dispersive reflectivity on the outcomes of the interactions is analyzed. It is found that above a certain value of dispersive reflectivity solitons develop sidelobes. The presence of sidelobes has a significant effect on the outcomes of the interactions. In the absence of sidelobes, in-phase soliton-soliton interactions may result in several outcomes such as merger into a quiescent soliton, symmetric or asymmetric separation of solitons or destruction of both solitons. However, the interaction of solitons in the presence of sidelobes produces other outcomes such as repulsion of solitons or formation of a temporary bound state followed by separation of two solitons.  $\pi$ -out-of-phase solitons generally repel each other. However, in the presence of sidelobes, interactions of  $\pi$ -out-of-phase solitons may lead to the formation of temporary bound state and subsequent generation of two separating solitons.

## 1 INTRODUCTION

A key characteristic of fiber Bragg gratings (FBGs) is that they exhibit strong effective dispersion due to resonant reflection of light. This effective dispersion can be six orders of magnitude larger than the chromatic dispersion of silica fiber (de Sterke and Sipe, 1994; Eggleton et al., 1997). The interplay of FBG-induced dispersion and Kerr nonlinearity gives rise to the formation of gap solitons (GSs). Over the past few decades, gap solitons have been investigated extensively both theoretically (Aceves and Wabnitz, 1989; Christodoulides and Joseph, 1989; Malomed and Tasgal, 1994; Barashenkov et al., 1998; De Rossi et al., 1998; Neill and Atai, 2006) and experimentally (Eggleton et al., 1996; de Sterke C. M. and Krug, 1997; Eggleton et al., 1999) due to their potential applications in optical signal processing, optical buffer elements and logic gates (Krauss, 2008). A major property of gap solitons is that they can possess any velocity from zero (quiescent) to the speed of light in the medium. To date, gap solitons with a velocity in excess of 23% of the speed of light in the medium have been observed experimentally (Mok et al., 2006).

Gap solitons have been studied in more complex media and structures such as photonic crystals (Monat

et al., 2010; Skryabin, 2004; Neill and Atai, 2007), quadratic nonlinearity (Mak et al., 1998b; Conti et al., 1997; He and Drummond, 1997), waveguide arrays (Mandelik et al., 2004; Tan et al., 2009; Dong et al., 2011), dual core fibers (Atai and Malomed, 2000; Atai and Malomed, 2001; Mak et al., 1998a; Tsofe and Malomed, 2007), and Bragg gratings (BGs) with dispersive reflectivity (Atai and Malomed, 2005; Neill et al., 2008). Dual-core nonlinear couplers with dissimilar cores exhibit rich nonlinear dynamics and switching characteristics (Atai and Chen, 1992; Atai and Chen, 1993; Atai and Malomed, 1998). Therefore, slow GSs in grating-assisted couplers can be exploited to build novel optical devices for signal processing and switching.

In this paper, we investigate the effect of dispersive reflectivity on the interaction dynamics of gap solitons in a dual-core fiber where one core has only Kerr nonlinearity and the other core is linear and is equipped with a BG with dispersive reflectivity.

## 2 THE MODEL

Propagation of light in a dual-core fiber where one core has only Kerr nonlinearity and other one is linear and contains a BG with dispersive reflectivity is de-

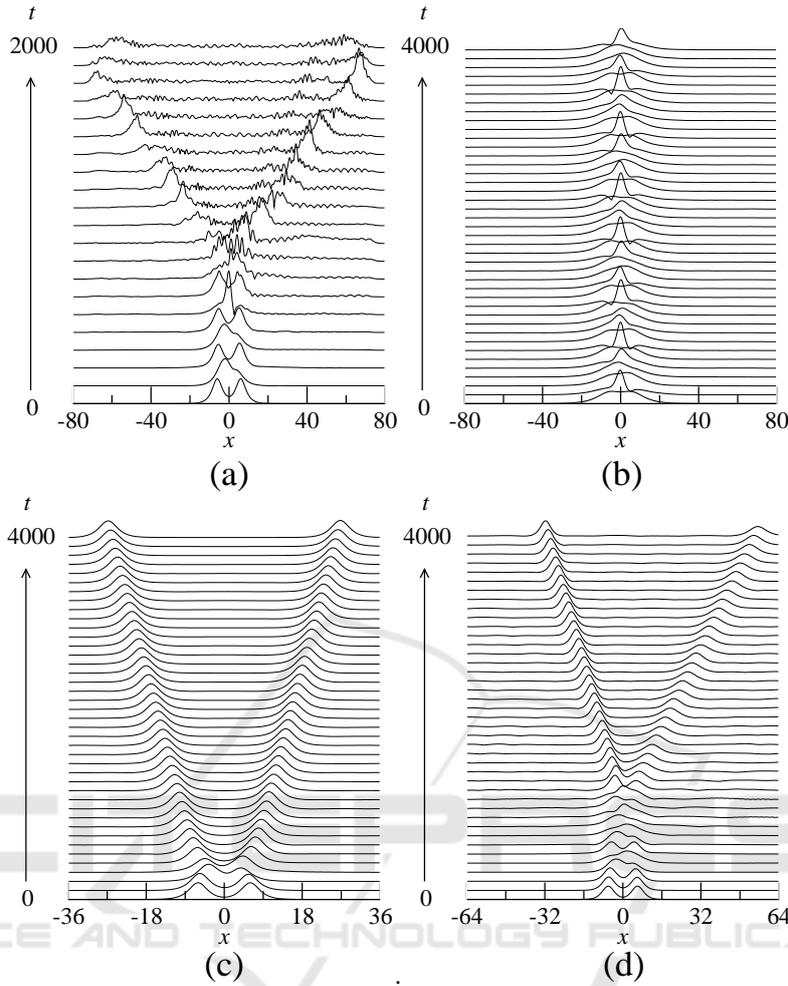


Figure 1: Examples of the outcomes of soliton-soliton interaction for in-phase quiescent solitons without sidelobes for  $\lambda = 1.0$ ,  $c = 0.2$  and  $\Delta x = 12$ . (a) Destruction for  $\omega = 1.570$ ,  $m = 0.020$  (b) Merger for  $\omega = 1.610$ ,  $m = 0.060$  (c) Symmetric separation for  $\omega = 1.570$ ,  $m = 0.160$  (d) Asymmetric separation for  $\omega = 1.590$ ,  $m = 0.260$ . Only  $|u|$  component is shown here.

scribed by the following set of normalized equations:

$$\begin{aligned} iu_t + iu_x + \left[ |v|^2 + \frac{1}{2}|u|^2 \right] u + \phi &= 0, \\ iv_t - iv_x + \left[ |u|^2 + \frac{1}{2}|v|^2 \right] v + \psi &= 0, \\ i\phi_t + ic\phi_x + u + \lambda\psi + m\psi_{xx} &= 0, \\ i\psi_t - ic\psi_x + v + \lambda\phi + m\phi_{xx} &= 0, \end{aligned} \quad (1)$$

where  $u(x,t)$  and  $v(x,t)$  are the forward- and backward-propagating waves in the nonlinear core, and  $\phi(x,t)$  and  $\psi(x,t)$  are their counterparts in the linear core which is equipped with a Bragg grating. The coefficient of linear coupling between the cores is normalized to 1 and the FBG-induced linear coupling coefficient between the forward- and backward-propagating waves is represented by a real parameter  $\lambda > 0$ . The group velocity in the nonlinear core is

set equal to 1 and  $c$  represents the relative group velocity in the linear core. The real parameter  $m > 0$  controls the strength of dispersive reflectivity. Substituting  $u, v, \phi, \psi \sim \exp(ikx - i\omega t)$  into Eqs. (1) and linearizing results in the following form of dispersion relation:

$$\begin{aligned} \omega^4 - \left( 2 + (\lambda - mk^2)^2 + (1 + c^2)k^2 \right) \omega^2 + \\ (\lambda k - mk^3)^2 + (ck^2 - 1)^2 = 0 \end{aligned} \quad (2)$$

The dispersion relation results a genuine central band gap along with upper and lower band gaps (which are not genuine band gaps). No stationary solutions are available in the central bang gap. The soliton solutions are sought numerically in both upper and lower band gaps by relaxation algorithm in the range  $0 \leq m \leq 0.5$ .

### 3 INTERACTIONS OF GAP SOLITONS

We have investigated the interactions between two identical stable quiescent GSs by numerically solving Eqs. (1) using a symmetrized split-step Fourier method for in-phase and  $\pi$ -out-of-phase solitons subject to the following initial conditions:

$$\begin{aligned} u(x,0) &= u\left(x - \frac{\Delta x}{2}, 0\right) + u\left(x + \frac{\Delta x}{2}, 0\right) \exp(i\Delta\theta), \\ v(x,0) &= v\left(x - \frac{\Delta x}{2}, 0\right) + v\left(x + \frac{\Delta x}{2}, 0\right) \exp(i\Delta\theta), \\ \phi(x,0) &= \phi\left(x - \frac{\Delta x}{2}, 0\right) + \phi\left(x + \frac{\Delta x}{2}, 0\right) \exp(i\Delta\theta), \\ \psi(x,0) &= \psi\left(x - \frac{\Delta x}{2}, 0\right) + \psi\left(x + \frac{\Delta x}{2}, 0\right) \exp(i\Delta\theta), \end{aligned} \quad (3)$$

where  $\Delta x$  and  $\Delta\theta$  represent the initial separation and phase difference between the solitons respectively, and  $u$ ,  $v$ ,  $\phi$  and  $\psi$  belong to the stable regions. As is shown in Fig. 1, for the in-phase solitons, the interactions may result in several outcomes such as merger into a single soliton, destruction of both solitons, symmetric and asymmetric separation of solitons for low to moderate values of dispersive reflectivity. Fig. 2 summarizes the outcomes of in-phase and  $\pi$ -out-of-phase soliton-soliton interactions in the upper bandgap in the plane  $(m, \omega)$  for  $\lambda = 1$ ,  $c = 0.2$  and initial separation of  $\Delta x = 12$ . The simulations demonstrate that the outcomes of interactions are significantly influenced by strong dispersive reflectivity. This is mainly due to the fact that beyond a certain value of  $m$  (i.e. the region to the right of the dash-dotted curve in Fig. 2) solitons develop sidelobes. The presence of sidelobes drastically alters the interaction dynamics. As is shown in Fig. 2(a), the bound state formation and repulsion are generally observed when sidelobes are present. Typical examples of repulsion and formation a temporary bound state followed by generation of two separating solitons are shown in Fig 3(a) and 3(b), respectively.

In the case of  $\pi$ -out-of-phase solitons, in the absence of sidelobes solitons repel each other (see Fig. 2(b)). However, when solitons have sidelobes the interactions may also result in the formation of a bound state followed by two separating solitons. Figs. 3(c) and 3(d) respectively show examples of repulsion and formation of a bound state followed by generation of two moving solitons for initially  $\pi$ -out-of-phase solitons.

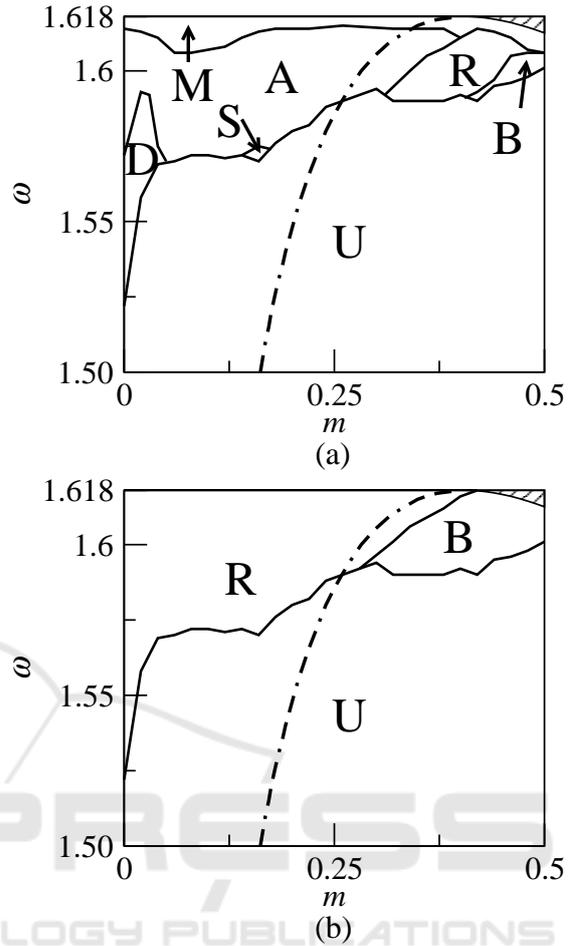


Figure 2: The outcomes of soliton-soliton interactions for  $\lambda = 1.0$ ,  $c = 0.2$  and  $\Delta x = 12$  in the upper bandgap for (a) in-phase and (b)  $\pi$ -out-of-phase solitons on the  $(m, \omega)$  plane. The labeled regions are merger (M), symmetric separation (S), asymmetric separation (A), destruction (D), repulsion (R), and formation of bound state followed by separation (B). In the region U, GSs are unstable. GSs have sidelobes in the region to the right of the dashed-dotted curve.

### 4 CONCLUSIONS

The interaction dynamics of quiescent Bragg grating solitons are investigated in a systematic way to test the effect of dispersive reflectivity in a dual-core system where one core contains Kerr nonlinearity and another core is equipped with a Bragg grating and dispersive reflectivity. It is found that, for low to moderate dispersive reflectivity (i.e. in the absence of sidelobes), in-phase quiescent Bragg grating solitons attract each other and generate different outcomes such as merger into a quiescent soliton, separation of solitons with equal and unequal velocities after initial attraction or destruction of both solitons. For high dis-

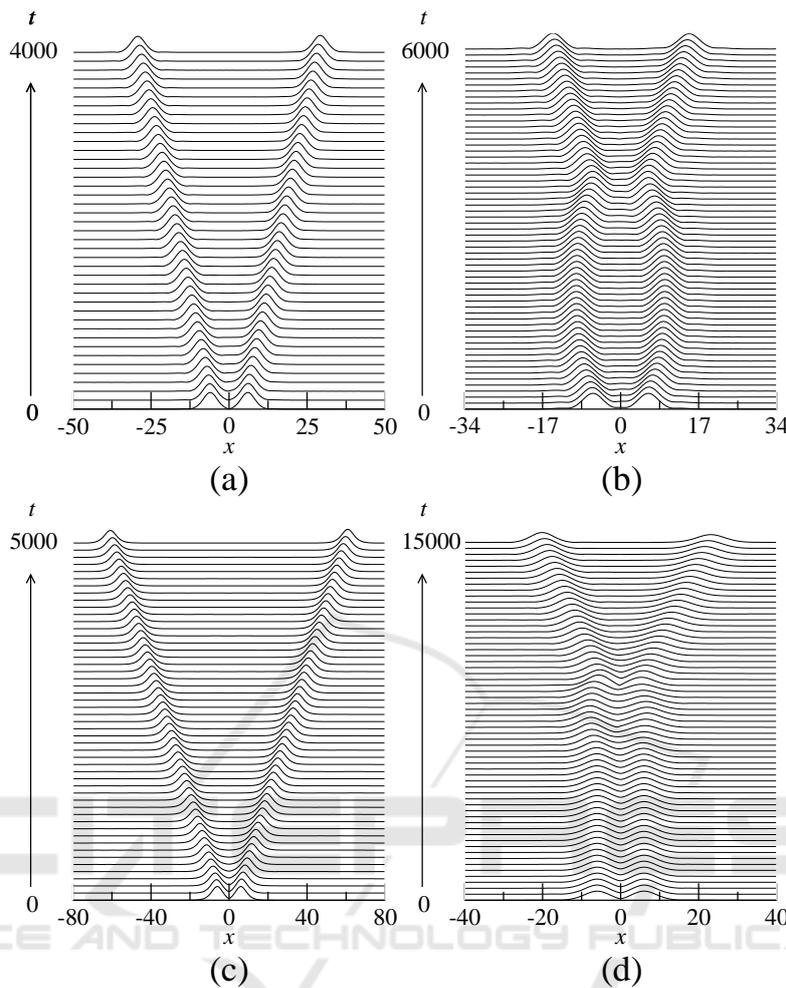


Figure 3: Examples of the outcomes of soliton-soliton interaction for  $\lambda = 1.0$ ,  $c = 0.2$  and  $\Delta x = 12$  for solitons with sidelobes. (a) Repulsion for  $\Delta\theta = 0$ ,  $\omega = 1.590$ ,  $m = 0.360$ , (b) Formation of temporary bound state followed by separation for  $\Delta\theta = 0$ ,  $\omega = 1.590$ ,  $m = 0.420$ , (c) Repulsion for  $\Delta\theta = \pi$ ,  $\omega = 1.601$ ,  $m = 0.280$  and (d) Formation of a bound state followed by separating solitons for  $\Delta\theta = \pi$ ,  $\omega = 1.610$ ,  $m = 0.380$ . Only  $|u|$  component is shown here.

persive reflectivity solitons have side-lobes in their profile. The presence of sidelobes significantly affects the interaction dynamics. In particular, in-phase solitons with sidelobes may repel each other or form of a temporary bound state followed by generation of two separating solitons. In the case of  $\pi$ -out-of-phase solitons, solitons without sidelobes always repel each other. However, in the region where solitons have sidelobes, the interactions may also give rise to the formation of a bound state that subsequently evolves in to two separating solitons.

## REFERENCES

- Aceves, A. B. and Wabnitz, S. (1989). Self-induced transparency solitons in nonlinear refractive periodic media. *Phys. Lett. A*, 141(1):37–42.
- Atai, J. and Chen, Y. (1992). Nonlinear couplers composed of different nonlinear cores. *J. Appl. Phys.*, 72(1):24–27.
- Atai, J. and Chen, Y. (1993). Nonlinear mismatches between two cores of saturable nonlinear couplers. *IEEE J. Quantum Electron.*, 29(1):242–249.
- Atai, J. and Malomed, B. A. (1998). Bound states of solitary pulses in linearly coupled Ginzburg-Landau equations. *Phys. Lett. A*, 244:551–556.
- Atai, J. and Malomed, B. A. (2000). Bragg-grating solitons in a semilinear dual-core system. *Phys. Rev. E*, 62(6):8713–8718.
- Atai, J. and Malomed, B. A. (2001). Solitary waves in systems with separated bragg grating and nonlinearity. *Physical Rev. E*, 64(6 Pt 2):066617.
- Atai, J. and Malomed, B. A. (2005). Gap solitons in bragg gratings with dispersive reflectivity. *Phys. Lett. A*, 342(5):404–412.

- Barashenkov, I. V., Pelinovsky, D. E., and Zemlyanaya, E. V. (1998). Vibrations and oscillatory instabilities of gap solitons. *Phys. Rev. Lett.*, 80(23):5117–5120.
- Christodoulides, D. N. and Joseph, R. I. (1989). Slow bragg solitons in nonlinear periodic structures. *Phys. Rev. Lett.*, 62(15):1746–1749.
- Conti, C., Trillo, S., and Assanto, G. (1997). Doubly resonant Bragg simultons via second-harmonic generation. *Phys. Rev. Lett.*, 78:2341–2344.
- De Rossi, A., Conti, C., and Trillo, S. (1998). Stability, multistability, and wobbling of optical gap solitons. *Phys. Rev. Lett.*, 81(1):85–88.
- de Sterke, C. M. and Sipe, J. E. (1994). Gap solitons. *Progress in Optics*, 33:203–260.
- de Sterke C. M., Eggleton, B. J. and Krug, P. A. (1997). High-intensity pulse propagation in uniform gratings and grating superstructures. *J. Lightwave Technol.*, 15(8):1494–1502.
- Dong, R., Rüter, C. E., Kip, D., Cuevas, J., Kevrekidis, P. G., Song, D., and Xu, J. (2011). Dark-bright gap solitons in coupled-mode one-dimensional saturable waveguide arrays. *Phys. Rev. A*, 83(6):063816.
- Eggleton, B. J., de Sterke, C. M., and Slusher, R. E. (1997). Nonlinear pulse propagation in bragg gratings. *J. Opt. Soc. Am. B*, 14(11):2980–2993.
- Eggleton, B. J., de Sterke, C. M., and Slusher, R. E. (1999). Bragg solitons in the nonlinear schrödinger limit: experiment and theory. *J. Opt. Soc. Am B*, 16(4):587–599.
- Eggleton, B. J., Slusher, R. E., Krug, P. A., and Sipe, J. E. (1996). Bragg grating solitons. *Phys. Rev. Lett.*, 76(10):1627–1630.
- He, H. and Drummond, P. D. (1997). Ideal soliton environment using parametric band gaps. *Phys. Rev. Lett.*, 78:4311–4315.
- Krauss, T. F. (2008). Why do we need slow light? *Nature Photonics*, 2(8):448–450.
- Mak, W. C. K., Chu, P. L., and Malomed, B. A. (1998a). Solitary waves in coupled nonlinear waveguides with bragg gratings. *J. Opt. Soc. Am. B*, 15(6):1685–1692.
- Mak, W. C. K., Malomed, and Chu, P. L. (1998b). Asymmetric solitons in coupled second-harmonic-generating waveguides. *Phys. Rev. E*, 57:1092–1103.
- Malomed, B. A. and Tasgal, R. S. (1994). Vibration modes of a gap soliton in a nonlinear optical medium. *Phys. Rev. E*, 49(6):5787–5796.
- Mandelik, D., Morandotti, R., Aitchison, J. S., and Silberberg, Y. (2004). Gap solitons in waveguide arrays. *Phys. Rev. Lett.*, 92(9):093904.
- Mok, J. T., de Sterke, C. M., Littler, I. C. M., and Eggleton, B. J. (2006). Dispersionless slow light using gap solitons. *Nature Phys.*, 2(11):775–780.
- Monat, C., de Sterke, M., and Eggleton, B. J. (2010). Slow light enhanced nonlinear optics in periodic structures. *J. Opt.*, 12:104003.
- Neill, D. R. and Atai, J. (2006). Collision dynamics of gap solitons in kerr media. *Phys. Lett. A*, 353(5):416–421.
- Neill, D. R. and Atai, J. (2007). Gap solitons in a hollow optical fiber in the normal dispersion regime. *Phys. Lett. A*, 367(1-2):73–82.
- Neill, D. R., Atai, J., and Malomed, B. A. (2008). Dynamics and collisions of moving solitons in bragg gratings with dispersive reflectivity. *J. Opt. A: Pure Appl. Opt.*, 10:085105.
- Skryabin, D. V. (2004). Coupled core-surface solitons in photonic crystal fibers. *Opt. Express*, 12(20):4841–4846.
- Tan, Y., Chen, F., Beličev, P. P., Stepić, M., Maluckov, A., Rüter, C. E., and Kip, D. (2009). Gap solitons in defocusing lithium niobate binary waveguide arrays fabricated by proton implantation and selective light illumination. *Appl. Phys. B*, 95(3):531–535.
- Tsofe, Y. J. and Malomed, B. A. (2007). Quasisymmetric and asymmetric gap solitons in linearly coupled Bragg gratings with a phase shift. *Phys. Rev. E*, 75(5 Pt 2):056603.