Copula Eigenfaces

Semiparametric Principal Component Analysis for Facial Appearance Modeling

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Abstract: Principal component analysis is a ubiquitous method in parametric appearance modeling for describing dependency and variance in a data set. The method requires that the observed data be Gaussian-distributed. We show that this requirement is not fulfilled in the context of analysis and synthesis of facial appearance. The model mismatch leads to unnatural artifacts which are severe to human perception. In order to prevent these artifacts, we propose to use a semiparametric Gaussian copula model, where dependency and variance are modeled separately. The Gaussian copula enables us to use arbitrary Gaussian and non-Gaussian marginal distributions. The new flexibility provides scale invariance and robustness to outliers as well as a higher specificity in generated images. Moreover, the new model makes possible a combined analysis of facial appearance and shape data. In practice, the proposed model can easily enhance the performance obtained by principal component analysis in existing pipelines: The steps for analysis and synthesis can be implemented as convenient pre- and post-processing steps.

1 INTRODUCTION

Parametric Appearance Models (PAM) describe objects in an image in terms of pixel intensities. In the context of faces, Active Appearance Models (Cootes et al., 1998) and 3D Morphable Models (Blanz and Vetter, 1999) are established PAMs to model appearance and shape. The dominant method for learning the parameters of a PAM is principal component analysis (PCA) (Jolliffe, 2002). PCA is used to describe the variance and dependency in the data. Usually, PAMs are generative models that can synthesize new random instances.

Using PCA to model facial appearance leads to models which are able to synthesize instances which appear unnaturally. This is due to the assumption that the color intensities or, in other words, the marginals at a pixel are Gaussian-distributed. We show that this is a severe simplification: The pixel intensities of new samples will follow a joint Gaussian distribution. This approximation is far from the actual observed distribution of the training data and leads to unnatural artifacts in appearance.

The ability to synthesize random *and* natural instances is important when generating new face instances (Mohammed et al., 2009) and in face manipulation (Walker and Vetter, 2009). This is because human perception is very sensitive to unnatural variability in a face. On the other hand, PCA face models are used as a strong prior in probabilistic facial image interpretation algorithms (Schönborn et al., 2013). Hence, such applications require a prior which follows the underlying distribution as closely as possible and, which is therefore, highly specific to faces.

In order to enhance the specificity of a PCA-based model, an obvious improvement would be the extension to a Gaussian mixture model (Rasmussen, 1999). Here, each color channel at a pixel is modeled with an (infinite) mixture of Gaussians. However, we skip this step and propose to use a semiparametric copula model directly.

A copula model provides the decomposition of the dependency and the marginal distributions such that the copula contains the dependency structure only. This separate modeling allows us to drop the parametric Gaussian assumption on the color channels and to replace them with nonparametric empirical distributions. We keep the parametric dependency structure; in particular, we use a Gaussian copula because of its

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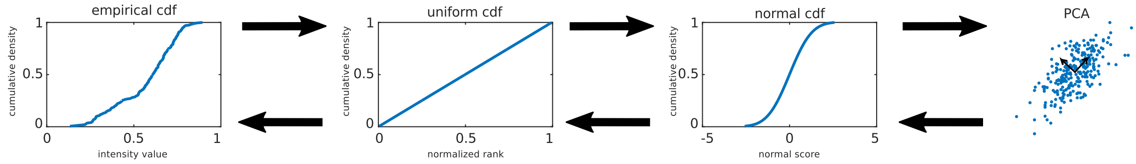


Figure 1: This figure shows the pre- and post-processing steps necessary to use a Gaussian copula before calculating PCA.

inherent Gaussian latent space. PCA can then be applied in the latent Gaussian space and is used to learn the dependencies of the data independently from the marginal distribution. The method is analytically analyzed in (Han and Liu, 2012) and is called Copula Component Analysis (COCA). Samples drawn from a COCA model follow the empirical marginal distribution of the training data and are, therefore, more specific to the modeled object.

The additional steps for using COCA can be implemented as simple pre- and post-processing before applying PCA. The data is mapped into a space where it is Gaussian-distributed. This mapping is obtained by first ranking the data and then transforming it by the standard normal distribution. We perform PCA on the transformed data to learn its underlying dependency structure. All necessary steps are visualized in Figure 1.

A semiparametric Gaussian copula model also provides additional benefits: First, learning is invariant to monotonic transformations of all marginals, including invariance to scaling. Second, the implementation can be done as simple pre- and post-processing steps. Third, the model also allows changing the color space. For facial-appearance modeling, the HSV color space is more appropriate than RGB. The HSV color space is motivated by the separation of the hue and saturation components and brightness value. On the other hand, without adaptions, PCA is not applicable to facial appearance in the HSV color space because of its sensitivity to differently-scaled color channels.

In summary, methods building on PCA can easily benefit from these advantages to improve their learned model.

1.1 Related Work

The Eigenfaces approach (Sirovich and Kirby, 1987; Turk et al., 1991) uses PCA on aligned facial images to analyze and synthesize faces. Active Appearance Models (Cootes et al., 1998) add a shape component which allows to model the shape independently from the appearance. The 3D Morphable Model (Blanz and Vetter, 1999) uses a dense registration, extends the shape model to 3D and adds camera and illumination parameters. The 3D Morphable Model allows handling appearance independently from pose, illumination and shape. These methods have a common core: They focus on analysis and synthesis of faces and all of them use a PCA model for color representation and can, therefore, benefit from COCA.

Photo-realistic face synthesis methods like Visiolization (Mohammed et al., 2009) use PCA as a basis for example-based photo-realistic appearance modeling.

1.2 Organization of the Paper

The remainder of the paper is organized as follows: The methods section explains the copula extension for PCA and presents the theoretical background for learning and inference. Additionally, practical information for an implementation is provided. In the experiments and results we demonstrate that facial appearance should be modeled using the copula extension. We qualitatively and quantitatively show that the proposed model leads to a facial appearance model which is more specific to faces.

2 METHODS

2.1 PCA for Facial Appearance Modeling

Let $x \in \mathbb{R}^{3n}$ describe a zero-mean vector representing 3 color channels of an image with n pixels. In an RGB image, the color channels and the pixels are stacked such that $x = (r_1, g_1, b_1, r_2, b_2, b_3, \ldots, r_n, g_n, b_n)^T$. We assume that the mean of every dimension is already subtracted. The training set of m images is arranged as the data matrix $X \in \mathbb{R}^{3n \times m}$.

PCA (Jolliffe, 2002) aims at diagonalizing the sample covariance $\Sigma = \frac{1}{m} XX^T$, such that

$$\Sigma = \frac{1}{m} US^2U^T \tag{1}$$

where S is a diagonal matrix and U contains the transformation to the new basis. The columns of matrix U are the eigenvectors of Σ and the corresponding eigenvalues are on the diagonal of S.

PCA is usually computed by a singular value decomposition (SVD). In case of a rank-deficient sample covariance with rank $m < n$ we cannot calculate

U^{-1}. Therefore, SVD leads to a compressed representation with a maximum of m dimensions. The eigenvectors in the transformation matrix U are ordered by the magnitude of the corresponding eigenvalues.

When computing PCA, the principal components are guided by the variance as well as the covariance in the data. While the variance captures the scattering of the intensity value of a pixel, the covariance describes which regions contain similar color. This mingling of factors leads to results which are sensitive to different scales and to outliers in the training set. Regions with large variance and outliers could influence the direction of the resulting principal components in an undesired manner.

We uncouple variance and dependency structure such that PCA is only influenced by the dependency in the data. Our approach for uncoupling is a copula model which provides an analytical decomposition of the aforementioned factors.

2.2 Copula Extension

Copulas (Nelsen, 2013; Joe, 1997) allow a detached analysis of the marginals and the dependency pattern for facial appearance models. We consider a relaxation to a semiparametric Gaussian copula model (Genest et al., 1995; Tsukahara, 2005). We keep the Gaussian copula for describing the dependency pattern, but we allow nonparametric marginals.

Let $x \in \mathbb{R}^{3n}$ describe the same zero-mean vector as used for PCA, representing 3 color channels of an image with n pixels. Sklar's theorem allows the decomposition of every continuous and multivariate cumulative probability distribution (cdf) into its marginals $F_i(X_i), i = 1, \dots, 3n$ and a copula C. The copula comprises the dependency structure, such that

$$F(X_1, \cdots, X_{3n}) = C(W_1, \dots, W_{3n}) \tag{2}$$

where $W_i = F_i(X_i)$. W_i are uniformly distributed and generated by the probability integral transformation[1].

For our application, we consider the Gaussian copula because of its inherently implied latent space

$$\tilde{X}_i = \Phi^{-1}(W_i), \quad i = 1, \dots, 3n \tag{3}$$

where Φ is the standard normal cdf. The multivariate latent space is standard normal-distributed and fully parametrized by the sample correlation matrix $\tilde{\Sigma} = \frac{1}{m}\tilde{X}\tilde{X}^T$ only. PCA is then applied on the sample correlation in the latent space $\tilde{X}$.

[1]The copula literature uses U instead of W. We changed this convention due to the singular value decomposition which uses $X = USV^T$.

Algorithm 1: Learning.

Input: Training set $\{X\}$
Output: Projection matrices U, S
for *all dimensions* **do**
    for *all samples* **do**
        $\tilde{x}_{ij} = \Phi^{-1}\left(\frac{r_{ij}(x_{ij})}{m+1}\right)$
find $\tilde{U}, \tilde{S}$ such that $\tilde{\Sigma} = \frac{1}{m}\tilde{U}\tilde{S}^2\tilde{U}^T$ (via SVD)

The separation of dependency pattern and marginals provides multiple benefits: First, the Gaussian copula captures the dependency pattern invariant to the variance of the color space[2]. Second, whilst PCA is distorted by outliers and is generally inconsistent in high dimensions, the semiparametric copula extension solves this problem (Han and Liu, 2012). Third, the nonparametric marginals maintain the non-Gaussian nature of the color distribution. Especially when generating new samples from the trained distribution, the samples do not exceed the color space of the training set.

2.3 Inference

We learn the latent sample correlation matrix $\tilde{\Sigma} = \frac{1}{m}\tilde{X}\tilde{X}^T$ in a semiparametric fashion using nonparametric marginals and a parametric Gaussian copula. We compute $\hat{w}_{ij} = \hat{F}_{\text{emp},i}(x_{ij}) = \frac{r_{ij}(x_{ij})}{m+1}$ using empirical marginals $\hat{F}_{\text{emp},i}$, where $r_{ij}(x_{ij})$ is the rank of the data x_{ij} within the set $\{x_{i\bullet}\}$. Then, $\tilde{\Sigma}$ is simply the sample covariance of the normal scores

$$\tilde{x}_{ij} = \Phi^{-1}\left(\frac{r_{ij}(x_{ij})}{m+1}\right), \quad i = 1, \dots, 3n, \quad j = 1, \dots, m. \tag{4}$$

Equation (4) contains the nonparametric part, since $\tilde{\Sigma}$ is computed from the ranks $r_{ij}(x_{ij})$ solely and contains no information about the marginal distribution of the x's. Note, $\tilde{x} \sim \mathcal{N}(0, \tilde{\Sigma})$ is standard normal distributed with correlation matrix $\tilde{\Sigma}$. Subsequently, an eigendecomposition is applied on the latent correlation matrix $\tilde{\Sigma}$.

Generating a sample using PCA then simply requires a sample from the model parameters

$$h \sim \mathcal{N}(0, I) \tag{5}$$

which is projected to the latent space

$$\tilde{x} = \tilde{U}\frac{\tilde{S}}{\sqrt{m}}h \tag{6}$$

[2]More general, a copula model is invariant against all monotonic transformations of the marginals.

Algorithm 2: Sampling.

Output: Random sample x

$h \sim \mathcal{N}(0, I)$

$\tilde{x} = \tilde{U} \frac{\tilde{S}}{\sqrt{m}} h$

for *all dimensions i* **do**

  $w_i = \Phi(\tilde{x}_i)$

  $x_i = \hat{F}_{\text{emp},i}(w_i)$

and further projected component-wise to

$$w_i = \Phi(\tilde{x}_i), \quad i = 1, \ldots, 3n. \tag{7}$$

Finally, the projection to the color space requires the empirical marginals

$$x_i = \hat{F}_{\text{emp},i}(w_i), \quad i = 1, \ldots, 3n. \tag{8}$$

All necessary steps are summarized in Algorithms 1 and 2 and visualized in Figure 1.

It is possible to smoothen the empirical marginals with a kernel k and replace Equation (8) by $x_i = k(w_i, X_{i\bullet}), \quad i = 1, \ldots, 3n.$

2.4 Implementation

The additional steps for using COCA can be implemented as simple pre- and post-processing before applying PCA. Basically the data is mapped into a latent space where it is Gaussian-distributed. The mapping is performed in two steps. First, the data is transformed to an uniform distribution by ranking the intensity values. Then it is transformed to a standard normal distribution. On the transformed data, we perform PCA to learn the dependency structure in the data.

To generate new instances from the model, all steps have to be reversed. Figure 1 gives an overview of all necessary transformations. The following steps have to be performed, e.g. in MATLAB, to calculate COCA:

```
% calculate empirical cdf
[empCDFs, indexX] = sort(X, 2);

% transform emp. cdf to uniform
[~, rank] = sort(indexX, 2);
uniformCDFs = rank / (size(rank, 2)+1);

% transform uni. cdf to std. normal cdf
normCDFs = norminv(uniformCDFs',0,1)';

% calculate PCA
[U,S,V] = svd(normCDFs, 'econ');
```

Listing 1: Learning.

To generate an image from model parameters, the following steps are necessary:

```
% random sample
m = size(normCDFs, 2);
h = random('norm' ,0 ,1 ,m ,1);
sample = U * S / sqrt(m) * h;

% std. normal to uniform
uniformSample = normcdf(sample, 0, ...
    1) * (m - 1) + 1;

% uniform to emp. cdf
empSample = ...
    empCDFs(sub2ind(size(empCDFs), ...
    1:size(data, 1), ...
    round(uniformSample')))';
```

Listing 2: Sampling.

These are the additional steps which have to be performed as pre- and post-processing for the analysis of the data and the synthesis of new random samples. In terms of computing resources we have to consider the following: The empirical marginal distributions F_{emp} are now part of the model and have to be kept in memory. In the learning part, the complexity of sorting the input data is added. In the sampling part, we have to transform the data back by looking up their values in the empirical distribution.

The copula extension comes with low additional effort: it is easy to implement and has only slightly higher computing costs. We encourage the reader to implement these few steps since the increased flexibility in the modeling provides a valuable extension.

3 EXPERIMENTS AND RESULTS

For all our experiments, we used the texture of 200 face scans used for building the Basel Face Model (BFM) (Paysan et al., 2009). The scans are in dense correspondence and were captured under an identical illumination setting. We work on texture images and use a resolution of 1024x512 pixels. Our experiments are based on the appearance information only, the last experiment merging the appearance and shape to a combined model. We used the empirical data directly as marginal distribution. The results are rendered with an ambient illumination on the mean face shape of the BFM.

3.1 Facial Appearance Distribution

In a first experiment we investigate if the color intensities in our face data set are Gaussian-distributed. We followed the protocol of the Kolmogorov-Smirnov Test (Massey Jr, 1951). We estimate a Gaussian distribution for every color channel per pixel and compare it to the observed data. The null hypothesis of the test is that the observed data is drawn by the estimated Gaussian distribution. The test measures the maximum distance of the cumulative density function of the estimated Gaussian $\Phi_{\hat{\mu},\hat{\sigma}^2}$ and the empirical marginal distribution F_{emp} of the observed data:

$$d = \sup_x \left\| F_{\text{emp}}(x) - \Phi_{\hat{\mu},\hat{\sigma}^2}(x) \right\| \qquad (9)$$

Here, $\hat{\mu}$ and $\hat{\sigma}^2$ are maximum-likelihood estimates for the mean and variance of a Gaussian distribution respectively. In Figure 2 we visualize the maximal distance value over all color channels per point on the surface.

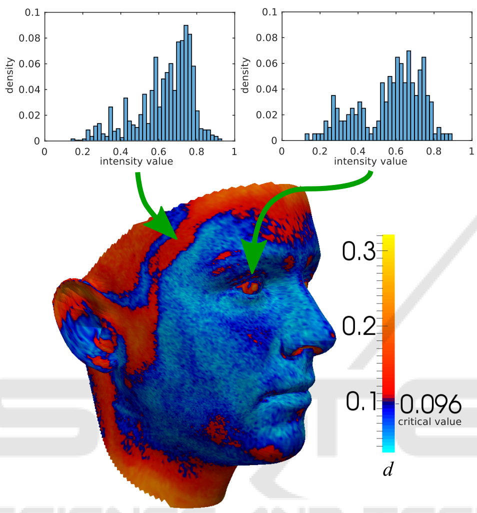


Figure 2: The result of the Kolmogorov-Smirnov Test to compare the empirical marginal distribution of color values from our 200 face scans with a Gaussian-reference probability distribution. We plot the highest value of the three color channels per pixel, because the values for the individual color channels are very similar. We show two exemplary marginal distributions in the eye and temple region. They are not only non-Gaussian but also not similar.

We assume a significance level of $1-\alpha = 0.05$. The critical value d_α is approximated using the following formula (Lothar Sachs, 2006):

$$d_\alpha = \frac{\sqrt{\ln(\frac{2}{\alpha})}}{\sqrt{2n}} \qquad (10)$$

With $n = 200$ training samples we get a critical value of 0.096. Non-Gaussian marginal distributions of color intensities are present in the region of the eyebrows, eyes, chin and hair, where multi-modal appearance is present. In total for 49% of the pixels over all color channels, the null hypothesis has to be rejected. In simple monotonic regions, like the cheek, the marginal distributions are close to a Gaussian distribution. In more structured regions like the eye, eyebrow or the temple region, the appearance is highly non-Gaussian. This leads to strong artifacts when modeling facial color appearance using PCA (see Figure 3 and Figure 4). Since those more structured regions are fundamental components of a face, it is important to model them properly.

3.2 Appearance Modeling

We evaluate our facial appearance model by its capability to synthesize new instances. We measured this capability by comparing the major eigenmodes, random model instances, the sample marginal distributions and the specificity of both models. The specificity is measured qualitatively by visual examples and quantitatively by a model metric.

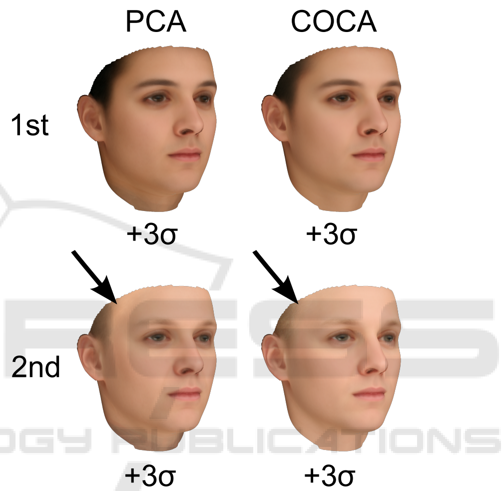


Figure 3: PCA and COCA are compared by visualizing the first two eigenvectors with 3 standard deviations on the mean. The components look very similar, except that the PCA artifacts on the temple (arrows) in the second eigenvector do not appear using COCA.

3.2.1 Model Parameters

The first few principal components store the strongest dependencies. We visualize the first two components by setting their value h_i to $\sigma = 3$ standard deviations and show the result in Figure 3. The first parameters of PCA and COCA appear very similar in the variation of the data they model. The second principal component of PCA causes artifacts in the temple region. These artifacts are caused by the linearity of PCA. COCA is a nonlinear method and therefore, the artifacts are not present.

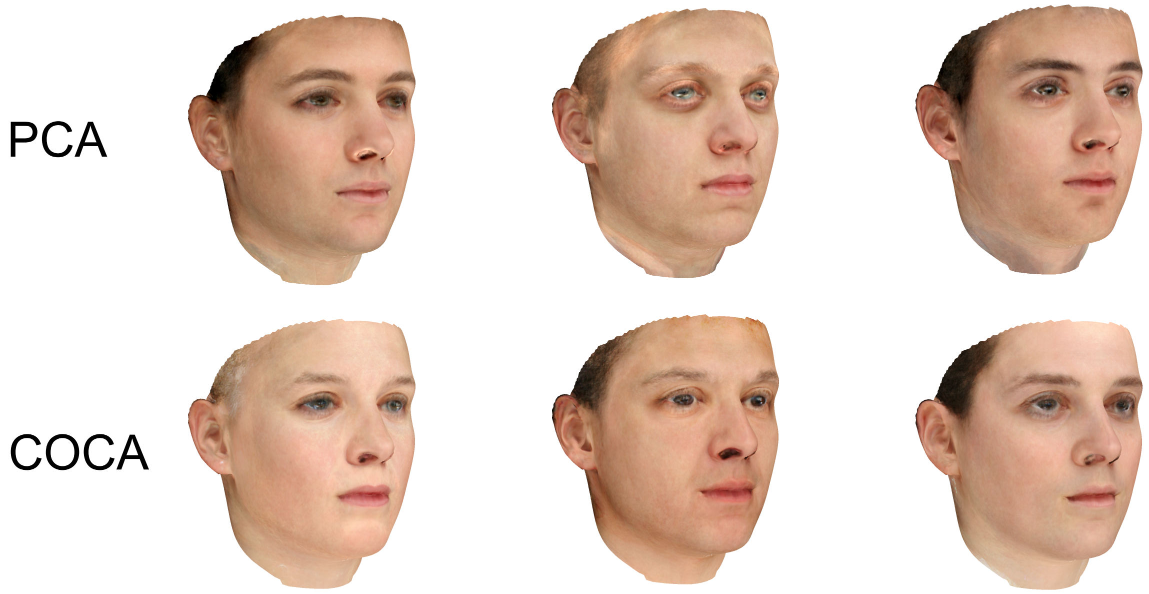


Figure 4: The first and second row show random samples projected by PCA and COCA respectively. Using PCA, we can observe strong artifacts in the regions where the marginal distribution is not Gaussian (see Figure 2). The improvement of COCA can be observed in the temple region, on the eyebrows, around the nostrils, the eyelids and at the border of the pupil. We chose representative samples for both methods.

3.2.2 Random Samples

The ability to generate new instances is a key feature for generative models. A model which can produce more realistic samples is desirable for various applications. For example, the Visio-lization method to generate high resolution appearances is based on a prototype generated with PCA (Mohammed et al., 2009).

Another field of application for the generative part of models are Analysis-by-Synthesis methods based on Active Appearance Models (AAM) or 3D Morphable Models (3DMM). They can profit from a stronger prior which is more specific to faces and reduces the search space (Schönborn et al., 2013).

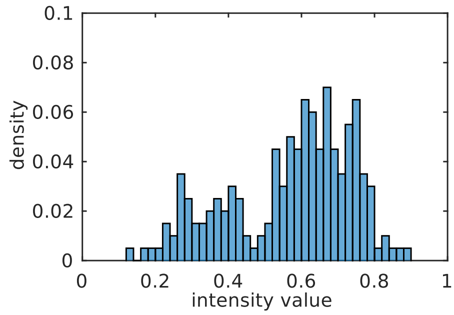
Generating a random parameter vector leads to a random face from our PCA or COCA model. We sample h according to Equation (5) independently for all 199 parameters and project them via PCA or COCA on the color space following Equation 6. Random samples using COCA contain fewer artifacts and, therefore, appear much more natural (see Figure 4). These artifacts are caused by the linearity of PCA. For non-Gaussian-distributed marginals, PCA does not only interpolate within the trained color distribution but also extrapolates to color intensities not supported by the training data.

The most obvious problem is the limited domain of the color channels: using PCA, color channels have to be clamped. The linearity constraint of PCA leads to much brighter or darker color appearance than those present in the training data in regions which are not Gaussian-distributed. In the next experiment, we show that the higher specificity is not only a qualitative result but can also be measured by a model metric.

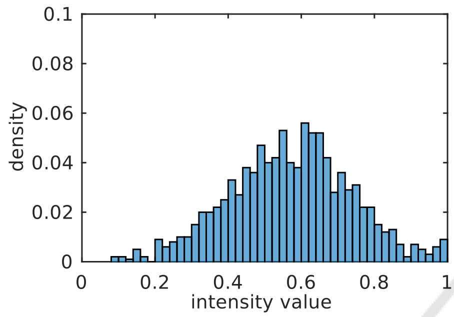
Few samples od COCA contain artifacts arising from outliers in the training data which appear at the borders of the empirical cdfs. Those artifacts can be removed by slightly cropping the marginal distributions (removing the outliers) or by applying COCA in the HSV color space.

3.3 Appearance Marginal Distribution

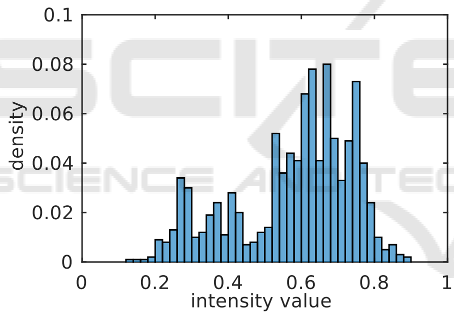
We analyze the marginal distributions of our random faces at a single point at the border between the pupil and the sclera of the eye. In this region the Kolmogorov-Smirnov Test rejected the null hypothesis. We analyze the empirical intensity distribution of a single color channel at this point (Figure 5a). The sample marginal distributions drawn from 1000 random instances generated by PCA and COCA are shown in Figure 5b and Figure 5c respectively. Whilst COCA is able to generate samples distributed similar to our input data, PCA is approximating a Gaussian distribution, which is inaccurate in a lot of facial regions.



(a) Empirical marginal distribution



(b) PCA sample marginal distribution



(c) COCA sample marginal distribution

Figure 5: The marginal distribution of the red color intensity of a single point in the eye region. (a) shows the distribution observed in the training data, (b) shows the distribution of samples drawn from a PCA model and (c) from a COCA model.

3.3.1 Specificity and Generalization

To measure the quality of the PCA and COCA models, we use model metrics motivated by the shape modeling community (Styner et al., 2003). The first metric is specificity: Instances generated by the model should be similar to instances in the training set. Therefore, we draw 1000 random samples from our model and compare each one to its nearest neighbor in the training data. We measure the distance using the mean absolute error over all pixels and color channels in the RGB-color space. The COCA model

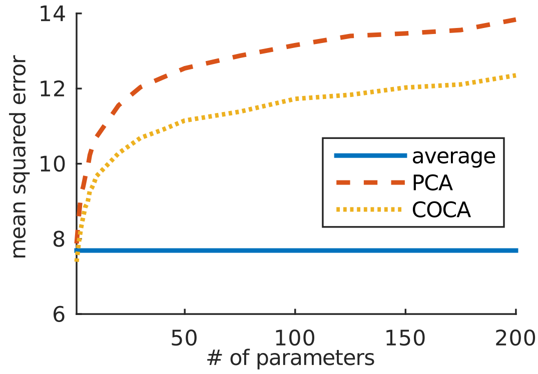


Figure 6: The specificity shows how close generated instances are to instances in the training data. The average distance of 1000 random samples to the training set (mean squared error per pixel and color channel) is shown. A model is more specific if the distance of the generated samples to the training set is smaller. We observe that COCA is more specific to faces (lower is better).

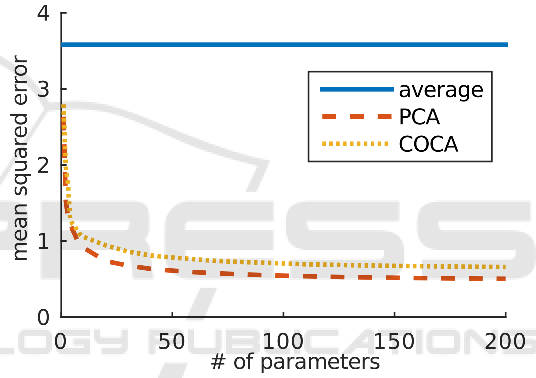


Figure 7: The generalization ability shows how exactly unseen instances can be represented by a model. The lower the error, the better a model generalizes. As a baseline, we present the generalization ability of the average face. We observe that PCA generalizes slightly better (lower is better).

is more specific to facial appearance (see Figure 6). This corresponds to our observation of a more realistic facial appearance (Figure 4).

Specificity should always be used in combination with the generalization model metric (Styner et al., 2003). The generalization measures how exactly the model can represent unseen instances. We measure the generalization ability of both models using a test set and use the same distance measure as for specificity. The test data consists of 25 additional face scans not contained in the training data. We observe that both models generalize well to unseen data. PCA generalizes slightly better, see Figure 7.

The third model metric is compactness - the ability to use a minimal set of parameters (Styner et al., 2003). The compactness can be measured directly by

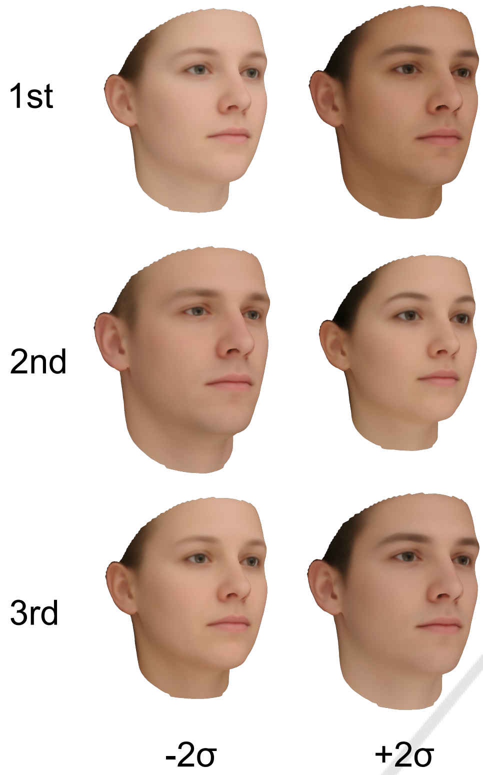


Figure 8: We learned a common shape and appearance model using COCA. We visualize the first eigenvectors with 2 standard deviations, which show the strongest dependencies in our training data. Whilst the first parameter is strongly dominated by appearance the later parameters are targeting shape and appearance. Since the model is built from 100 females and 100 males, the first components are strongly connected to sex.

the number of used parameters. In our experiments, the number of parameters is always the same for both models.

There is always a tradeoff between specificity and generalization. Whilst PCA performs slightly better in generalization, COCA performs better in terms of specificity. The better generalization ability of PCA comes at the price of a lower specificity and clearly visible artifacts.

3.3.2 Combined Shape and Color Model

Color appearance and shape are modeled independently in AAMs and 3DMMs. Recently, it was demonstrated that facial shape and appearance are correlated (Schumacher and Blanz, 2015) and those correlations were investigated using Canonical Correlation Analysis on separate shape and appearance PCA models.

The main reason to build separate models is a practical one - shape and color values are not in the same range. Some approaches accommodate this issue by normalization (Edwards et al., 1998). However, this approach is highly sensitive to outliers. Since Copula Component Analysis is scale invariant, we can directly apply it to the unscaled data.

We learned a COCA model combining the color and shape information (see Figure 8 and Figure 9). Shape and texture vectors are combined by simply concatenating them. By integrating this additional dependency information, the model becomes more specific (Edwards et al., 1998).

As a future extension, COCA allows us to also integrate attributes like age, weight and size or even social attributes like thrustworthiness or social competence directly into the model.

4 CONCLUSIONS

We showed that the marginal distribution of facial color is not Gaussian-distributed for large parts of the face and that PCA is not able to model facial appearance properly. In a statistical appearance model, this leads to unnatural artifacts which are easily detected by human perception. To avoid such artifacts, we propose to use PCA in a semiparametric Gaussian copula model (COCA) which allows to model the marginal color distribution separately from the dependency structure. In this model, the parametric Gaussian copula describes the dependency pattern in the data and the nonparametric marginals relax the restrictive Gaussian requirement of the data distribution.

The separation of marginals and dependency pattern enhances the model flexibility. We showed qualitatively that COCA models facial appearance better than PCA. This finding is also supported by a quantitative evaluation using specificity as a model metric.

Moreover, the COCA model enables to add further data to the model: Age, weight, size, and other data like social attributes living on different scales can be incorporated in the model in an unified way. To demonstrate this feature, we showed that the inclusion of shape also increased the specificity of the model.

The computer graphics and vision community is heavily modeling and working with color intensities. We believe that these intensities are most often not Gaussian-distributed and, therefore, our findings can be transferred to a lot of applications.

Finally, we again want to encourage the reader to replace PCA with a COCA model, since the additional model flexibility comes with almost no implementation effort.

Figure 9: Random samples projected by a common shape and appearance model using COCA.

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