

Healthcare Data Visualization: Geospatial and Temporal Integration

Shenhui Jiang¹, Shiaofen Fang¹, Sam Bloomquist¹, Jeremy Keiper¹, Mathew Palakal^{1,2}, Yuni Xia¹
and Shaun Grannis³

¹Department of Computer & Information Science, Indiana University Purdue University Indianapolis,
IN 46202, Indianapolis, U.S.A.

²School of Informatics, Indiana University Purdue University Indianapolis, IN 46202, Indianapolis, U.S.A.

³Regenstrief Institute, Indiana University School of Medicine, Indianapolis, U.S.A.

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Abstract: Healthcare data visualization is challenging due to the needs for integrating geospatial information, temporal information, text information, and heterogeneous health attributes within a common visual context. We recently developed a web-based healthcare data visualization system, Health-Terrain, based on a Notifiable Condition Detector (NCD) use case. In this paper, we will describe this system, with emphasis on the visualization techniques developed specifically for healthcare data. Two new visualization techniques will be described: (1) A spatial texture based visualization approach for multi-dimensional attributes and time-series data; (2) A spiral theme plot technique for visualizing time-variant patient data.

1 INTRODUCTION

As electronic healthcare systems are being fully integrated nationally, the effective visualization of large and complex healthcare data becomes increasingly desirable for timely decision making (Grossman et al., 2011). The problem, however, is very challenging for several reasons:

- 1) Health data is a data-rich, information-poor domain. In Electronic Health Record (EHR) systems, data are almost always heterogeneous, unstructured, hierarchical, and longitudinal.
- 2) EHR systems are large. While it is possible to visualize an EHR system in small scales with a focused scope, high impact knowledge discoveries may come from population-wide visualization and knowledge mining.
- 3) Visualizing population-level health data often involves presenting geospatial and time-series data in a common visual context. This presents a challenge in visual encoding of the information space.

For heterogeneous and complex data, feature extraction through data mining is critical. For healthcare data, this feature space often consists of healthcare terms (ontology) and their relationships.

Therefore, the effective integration of data processing, data mining, and text mining is necessary in healthcare data visualization. Although healthcare data is very large, the visualization of aggregated features, combined with patient level visualization, can be very effective in revealing the patterns and trends of population health. It is therefore important to develop multiple visualization tools to be integrated within a common visual interface to allow users to visually explore the data through an easily accessible platform such as a web browser.

One of the unique challenges in healthcare data visualization is how to visualize multi-attributes and time-series data with associated geospatial information. In our approach, we embed multiple attributes and the time variable within a geospatial representation to take advantage of the available geographic space. This can be done by mapping texture images onto the geospatial surfaces. The key is then to properly represent the multi-attributes and time-series information in a texture image by constructing visually effective texture representations. While visualizing aggregated data for geospatial areas provides global trends and patterns in a geospatial context, we are often interested in visualizing individual patient records

and their development over time. To this end, we also developed a spiral theme plot technique for visualizing time-variant patient records and attributes. These new visualization techniques have been implemented in a web-based healthcare data visualization system called Health-Terrain, and tested on real healthcare databases.

2 RELATED WORK

There are several existing works and visualization systems that deal with the secondary use of electronic health record data in a limited scope. LifeLines (Plaisant et al., 1996) uses a traditional 2D time line visualization technique to visualize specific patient medical and health history. It emphasizes the visualization of temporal ordering of events with limited aggregation effect. An extension of LifeLine, LifeLine2 (Wang et al., 2008), enables multiple patient comparisons and aggregation for analysis, but the visualization design limited its scalability. A similar system, call TimeLine (Bui et al., 2007), re-organizes and re-groups multiple EHR content types in a layout of Y-axis to track multiple events along the same time line. A set of visualization tools are described for visualizing a patient's electronic health record to aid physicians' diagnosis and decision-making. The traditional matrix view and parallel coordinates are the main techniques applied. The VISITORS system (Klimov and Shahar, 2005; 2009) combines a clinical knowledge base with visualization to enable users to explore multiple clinical records. It relies on domain ontologies to define clinically meaningful higher abstractions given raw, temporal data. CLEF (Hallett, 2008) is a system enabling visual navigation through a patient's medical record using semantically and temporally organized networks to represent events throughout the patient's medical history. CLEF also supports limited text processing capabilities for generating textual summaries. Interactive techniques have also been developed for the navigation of space and time dimensions (Bade et al., 2004; Maciejewski et al., 2009). None of these existing systems is capable of visualizing large-scale integrated EHR datasets. A review paper on visualization tools for infectious diseases is given in (Carroll et al., 2014). A more general survey was given in (Chittaro, 2001) about information visualization in Medicine.

Population-level healthcare data visualization involves both geospatial information and time-variant attributes. The geospatial visualization of time-series data is challenging because it is difficult

to encode the time axis in a geospatial context. Animation based techniques (e.g. Gemmell et al., 2005) do not provide a good space-time overview. Other techniques, such as color-coding of time (The New York Times, 2013), connecting time-lines (Google, 2013), and time-curves (Eccles et al., 2007), often introduces visual clutter and occlusion, which are infeasible for large scale datasets. A well-known technique in geospatial time-series visualization is Space-Time-Cube (Kraak et al., 2003; 2007; 2004; Kwan, 2000; Andrienko et al., 2003). It is a 3D representation of a combination of time axis (Z-axis) and a 2D geographic map (X-Y plane). Time-lines or time-curves are used to depict data evolution over time. While time and spatial information are integrated in a 3D visual representation in a space-time-cube, the sense of space-time embedding diminishes as the data moves up in the time axis. Visual clutter will also be a problem with large datasets. Similar 3D representation of spatio-temporal data using 3D icons have also been presented in (Tominski et al., 2005). Many other techniques have been developed for the visualization of time-series data without explicit geospatial information such as time-series plot (Tufte, 1983) and ThemeRiver (Havre et al., 2000). Many variations of ThemeRiver styled techniques have been applied in different time-series visualization applications, in particular text visualization (Cui et al., 2011). Spiral patterns have also been used in visualizing time-series data (Weber et al., 2001; Tominski et al., 2008) to provide better identification of periodic structures in the data.

Texture-based visualization techniques have been widely used for vector field data, in particular, flow visualization. Typically, a grayscale texture is smeared in the direction of the vector field by a convolution filter, for example, the Line Integral Convolution (LIC), such that the texture reflects the properties of the vector field (Cabral and Leedom, 1993; Stalling and Hege, 1995; Laramée et al., 2004). Similar techniques have also been applied to tensor fields (McGraw and Nadar, 2007; Auer et al., 2012).

3 THE HEALTH-TERRAIN SYSTEM

3.1 System Overview and Use Case

Our goal is to develop a prototype system, Health-

Terrain, to support visual exploration of large healthcare data sets on a browser based interface. The system integrates information visualization, web-based user interaction, and text and data mining techniques. A concept space approach is used to unify data representation unified data representation through data and text mining.

To test our visualization system we used a large public health notifiable disease reporting system. The Regenstrief Institute implemented and maintains an unparalleled HIE-based, automated electronic lab reporting (ELR) and case-notification system for over ten years in the State of Indiana. The Notifiable Condition Detector (NCD) System uses a standards-based messaging and vocabulary infrastructure that includes Health Level Seven (HL7) and Logical Observation Identifiers Names and Codes (LOINC) (Overhage, et al., 2008). The NCD receives real-time HL7 version 2 clinical transactions daily, including diagnoses, laboratory studies, and transcriptions from hospitals, national labs and local ancillary service organizations. The NCD dataset contains 833,710 public health notifiable cases spanning more than 10 years from among 439,547 unique patients. An additional dataset containing 325,791 unstructured clinical discharge summaries, laboratory reports, and patient histories were extracted. In order to comply with the patient privacy policies and protocols of the institutes where the datasets came from, the actual data visualized in this paper has been altered or perturbed.

3.2 Concept Space

The “concept space” represents a uniform layer of clinical observations and their associations, and enables users to explore data using various visualization and analysis methods. Concept terms are derived from data mining and text-mining processes applied to the use case datasets. Disease concepts were extracted from the NCD dataset. Text mining algorithms were then applied to additional linked text dataset (unstructured clinical summaries) to construct ontologies for different concept types, including disease, symptom, mental behaviour, and risky behaviour.

The concept space uses a controlled vocabulary that can be pre-defined based on application needs, and enhanced by data/text mining algorithms. These terms and their relationships are represented in an association map, as a space of extracted partial knowledge. This association map is often the starting point of a visual exploration process. Figure 1 shows an example of the association map of

diseases. Association map is a graph visualization of the association relationships among the diseases and other terms in the concept space. It can serve as a platform supporting interactive selection of concepts to dynamically visualize data using a variety of tools in the visualization system. To draw an association graph, a spring-embedder algorithm (Kobourov, 2012) is used to layout the graph nodes. Nodes picked on the association map are then be visualized with geospatial information, possibly with time varying variables.

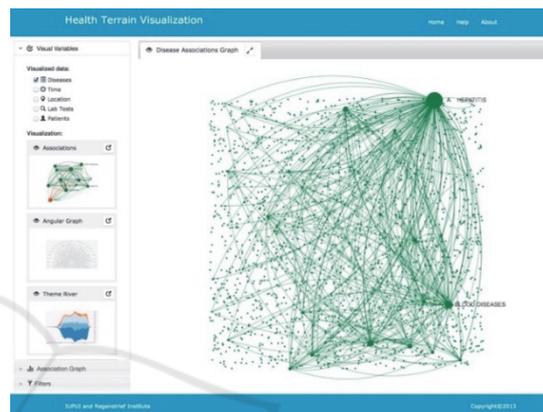


Figure 1: A Disease association map.

In text mining, we processed 325,791 unstructured clinical notes containing patient discharge summaries, laboratory reports, and medical histories. Advanced NLP was applied in the form of named entity recognition (NER) for extracting diseases and other terms, with the help of the Unified Medical Language System (UMLS) (Humphreys, et al., 1998). Stemming and concept clustering algorithms (Osinski and Weiss, 2005) were applied to normalize the lexical variants and duplications of the terms. Term correlations were computed using the tf-idf (term frequency – inverse document frequency) vector space model to identify the significantly co-occurring diseases. An association-mining algorithm was applied to the combined terms to generate an association graph among all the concepts terms. The resulting concept space, along with the processed NCD data, is represented in a data model designed to support our specific ontology.

4 SPATIAL TEXTURE BASED APPROACH

Population-level healthcare data and information are

often tightly coupled with geospatial regions. The visualization of this type data requires the integration of geo-visualization and multi-dimensional and time-variant information visualization. For this purpose, we propose a Spatial Texture based approach. In this approach, we encode multi-dimensional attributes or time-variant attributes for a geospatial region into a texture image, and then map the texture image to the surface of the geospatial region to provide an integrated visual representation. The key is the visual encoding of multiple attributes or a time-variant attribute in a texture image.

4.1 Noise Texture

We aim to represent multiple attributes for each geospatial region using color coded texture patterns so that the users can visually perceive the representations of different attributes, not only within one region, but also its overall geospatial distributions across many regions in a geographic area (e.g. a state).

We first construct noise patterns to create a random variation in color intensity, similar to the approach in (Gossett and Chen, 2004). Different color hues will be used to represent different types of attributes, for example the occurrences of different diseases. A turbulence function (Perlin, 1985) will be used to generate the noise patterns of different frequencies (sizes of the sub-regions of the noise pattern). These multi-scale patterns may be applied to different scales of geographic areas (e.g. counties vs zip-codes). Since the noise pattern involves the mixing and blending of different color hues, we choose to use an RYB color model instead of RGB model, as proposed in (Gossett and Chen, 2004), since RYB color model provides more intuitive representation of the weights of different colors after blending. Figure 2 shows two examples of the texture mapped views of three diseases, Diabetes, Hepatitis B, and Chlamydia, over the Indiana state map. For example, more reddish areas exhibits higher rate of Diabetes and bluish areas show higher occurrence of Chlamydia.

4.2 Offset Contours

Offset contouring is designed to represent attribute changes over time within a geographic region. It can also be used to represent multiple attributes by assigning each attribute to each contour. Similar to the Noise Pattern texture, we first construct a texture image using offset contour curves to form shape-

preserving sub-regions. We will then use varying color shades or hues to fill the sequence of sub-regions to represent the change of attribute values over time, or to simply fill the sub-regions with different color values to represent multiple attributes.

The offset contours are generated by offsetting the boundary curve toward the interior of the region, creating multiple offset boundary curves (Figure 3). There are several offset curve algorithms available in curve/surface modeling. But since in our application, the offset curves do not need to be very accurate, we opt to use a simple image erosion algorithm (Rosenfeld and Kak, 1982) directly on the 2D image of the map to generate the offset contours.

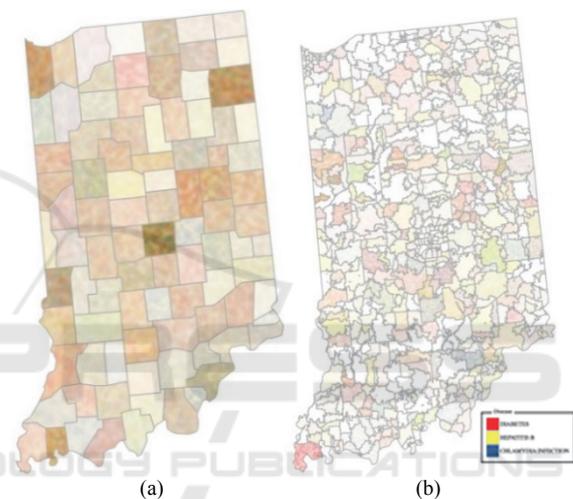


Figure 2: Noise textures mapped over the Indiana State map: (a) county based; (b) zip-code based.

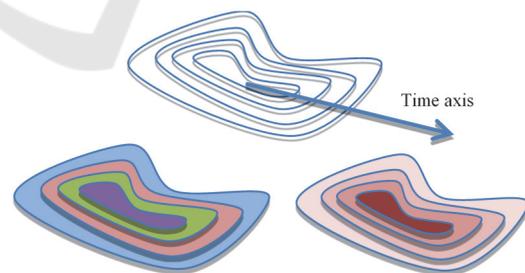


Figure 3: Offset contours with different colors or different shades of the same color.

In time-series data visualization, the time line can be divided into multiple time intervals and represented by the offset contours. Varying shades of a color hue can be used to represent the attribute changes (e.g. occurrence of a disease) over time. This approach, however, has two limitations. First, when the boundary shape of a region is highly

concave, the image erosion technique sometimes does not generate clean offset contours. This usually can be corrected using a geometric offset curve algorithm such as the one in (Hoschek, 1988). A second limitation of this approach is that it requires a certain amount of spatial area to layout the contours and color patterns. In public health data, however, these attributes are typically defined on geographic areas, which provides a perfect platform for texture based visual encoding. Figure 4 shows a few examples of the texture mapped views of offset contours over the Indiana state map. Figure 4 (a-b) show the time-series views of Influenza, from 2004 to 2012. The time interval is divided into 8 subintervals. Figure 4 (c-d) show three diseases, Influenza, Typhoid Fever, and Hepatitis B.

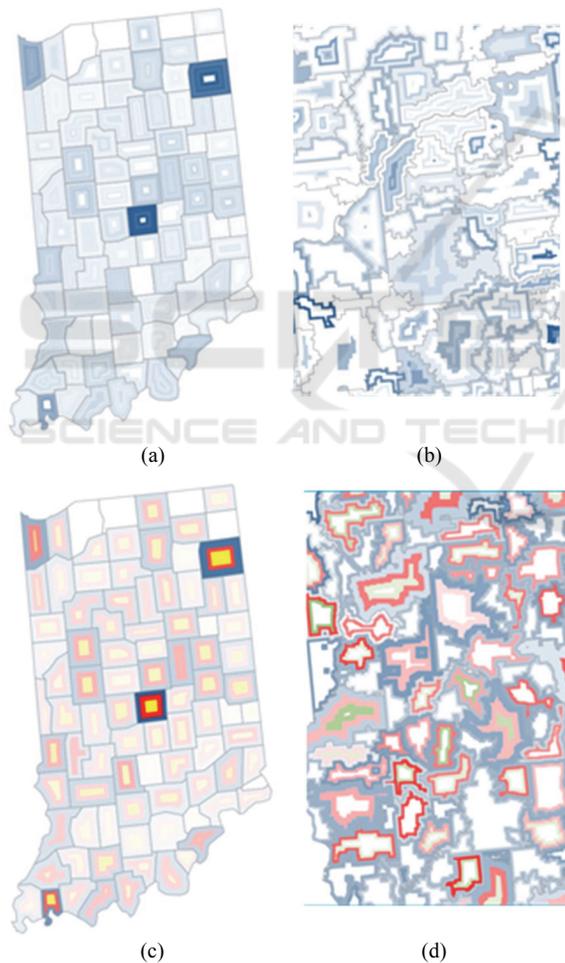


Figure 4: Texture mapped views of offset contours over the Indiana state map: (a) County based time-series data; (b) Zip-code based time-series data; (c) County based multi-diseases data; (d) Zip-code based multi-diseases data.

5 SPIRAL THEME PLOT

Spatial texture provides overviews of health care data associated with geographic regions. It is however often desirable for health administrators and physicians to also see the details of individual patients and their medical history (over time). When this is done with a large population, the collective view of patient medical histories often exhibit identifiable patterns and trends that may not be easily detected from the visualization of statistical data over geographical regions.

We developed a new time-series visualization method called Spiral Theme Plot by integrating ThemeRiver (Havre et al., 2000) and spiral pattern (Weber et al., 2001) to plot patients as points in stacked spiral rings. Time is represented as a spiral base curve. Diseases (or any other term) are represented as stacked themes along a spiral base curve. Patients are plotted within the regions of the themes as points with proper visual attributes. One significant attribute, for example “age”, will be represented as radius. Other attributes of the patients, such as race and gender, are represented as color and shape of the dots. Spiral Theme Plot allows multiple years of patients data be plotted periodically such that seasonal patterns or abnormal patterns for seasonal diseases can be easily detected. For patients with multiple hospital visits at different times for the same or different conditions, curves are drawn to connect these multiple occurrences by the same patient.

The base spiral curve is:

$$\begin{cases} x = r(\theta) \sin \theta \\ y = r(\theta) \cos \theta \end{cases}$$

where $r(\theta)$ is a monotonic continuous radius function of angle θ . When $r(\theta)$ is a linear function $r(\theta) = a + b\theta$, the gap between the spirals is a constant $2\pi b$, which can be estimated based on the maximum cumulative width of the themes (Fig. 5).

When plotting patient data within each theme, the width of the theme at a particular angle is determined by the total occurrence of the disease at that particular time. The boundary curve of each theme can then be interpolated by spline curves. This interpolation is done by splitting the time axis into a fixed number of segments. The maximum width of each segment is used as an interpolation point. This leads to a discrete set of interpolation points from which the spline curve can be generated as the boundary curve of the theme. When plotting a point for each patient, the width of the theme needs to be computed first in order to determine the proper

radius of the point. Although this width information can theoretically be computed from the spline representations, we found that it is more efficient to simply check the color values along the normal direction of the spiral curve to estimate the width of a theme at each angle.

Lines are drawn between points representing multiple occurrences of the same patient. Such lines sometimes can become very dense leading to a cluttered image. We implemented an edge bundling strategy to bundle these connecting lines for each pre-defined time interval (Fig. 6). Figure 7 show a periodical (seasonal) pattern of Flu over 4 years.



Figure 7: Seasonal pattern of Flu.

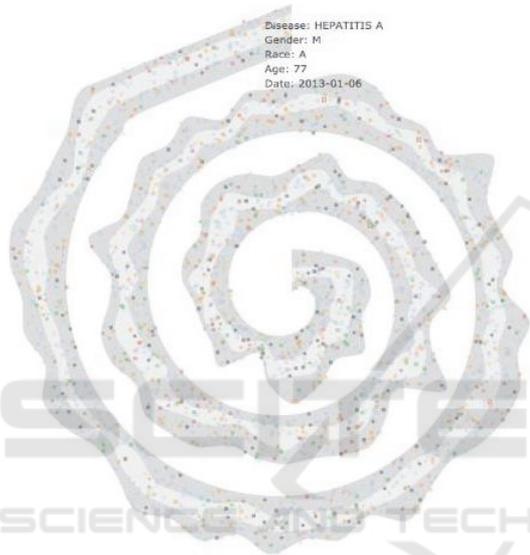


Figure 5: Spiral Theme Plot for Hepatitis A, B, and C over four years.

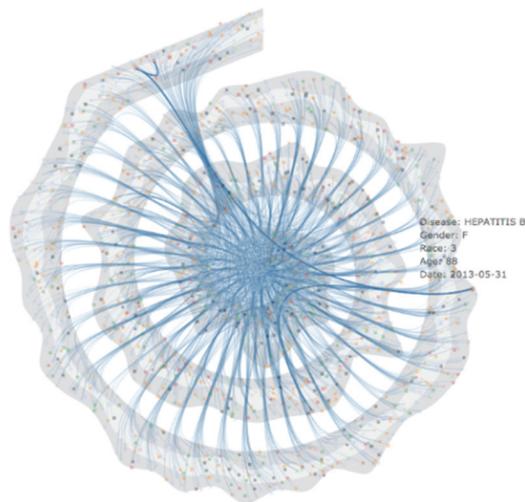


Figure 6: Spiral Theme Plot with bundled links.

6 SYSTEM INTERFACE AND EVALUATION

The system is implemented using Javascript in an HTML5 canvas. The visualization algorithms are implemented using HTML, CSS, SVG, and WebGL technologies with a number of open-source Javascript libraries.

The user interface uses multiple split windows so that multiple types of visualizations can be applied and compared for the same dataset. Fig. 8 show a screen shot of three visualizations for a dataset selected from an association map. Visualization results can also be saved into a slider bar, with time stamps, and be brought back later (Fig. 9). This provides a flexible workspace for health administrators or physicians to explore and compare different scenarios for health policy planning, decision making, resource management, etc.



Figure 8: A screen shot of a split window interface.

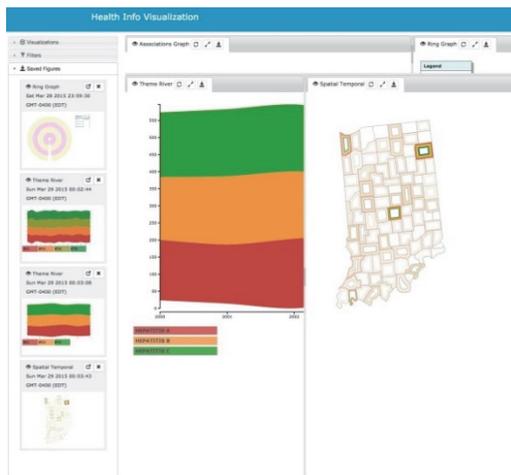


Figure 9: System interface with saved working windows.

To evaluate the system, we adopted the National Institute of Standards and Technology (2007) definition of usability for our participants. Using an unstructured qualitative interview process, we explored dimensions of effectiveness, efficiency, and satisfaction. Due to the data privacy policy provisions of the institutional review board research process, we used obfuscated de-identified clinical data for the usability assessment.

Prior to reviewing the interviewees were oriented to a few detailed dimensions of the application: The interviewees' responses can be summarized as follows:

- Users were pleased with the abilities to quickly identify associations of different terms and form subnetworks. Some felt that the visualization has the potential to make them think about things that they wouldn't otherwise, and that has value to them.
- Some users felt that they may not use visualization to identify disease outbreaks, but would instead use this visualization after an outbreak has been detected through other means in order to explore the relationships and characteristics of individuals within an outbreak in order to identify potential risk factors and target interventions.
- Users felt that this visualization system was very complex and exhibited high information density, which sometimes can obfuscate important information. More in-line guidance or pop-up descriptions (e.g., mouse-overs) would be helpful.
- For geospatial data visualization, some suggested adding a nonlinear scaling to highlight details in lower prevalence regions, or presenting the data

as incident rates. Epidemiologist interviewees requested extended functionality to visualize the highest prevalence diseases in each county.

7 CONCLUSIONS

We present a health data visualization system which emphasizes the integration of geospatial and temporal information in healthcare data. We focus on two new visualization methods we developed specifically for public health data: Spatial Textures, and Spiral Theme Plot. Spatial Texture approach is effective because geospatial visualization intrinsically provides additional screen space (surface areas) that can be taken advantage of to encode additional data and attributes. The Spiral Theme Plot technique is a combination of several information visualization methods including Theme River, Spiral Plot and Scatter Plot. For public health data with large patient databases, this particular combination satisfies several key requirements for visualizing time-variant patient records. With the rich set of tools available to support web based user interface, graphics, and data communications, we also feel that it is as efficient to develop a web based visualization system as in a traditional programming environment.

In the future, we would like to continue refining and expanding this visualization system by adding new visualization tools and improving the existing ones, in particular, the desired features and improvements suggested by the evaluators. We would also like to develop a configurable user and data interface so that the system can be easily configured for other types of use cases in public health applications.

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