Delineation of Rectangular Management Zones Under Uncertainty Conditions

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Keywords: OR in Agriculture, Stochastic Optimization, Management Zones, Precision Agriculture.

Abstract: In this article we cover the problem of generating a partition of an agricultural field into rectangular and homogeneous management zones or quarters according to a given soil property, which has variability in time that is presented as a number of possible scenarios. This problem combines aspects of precision agriculture and optimization with the purpose of achieving a site and time specific management of the field properties that is consistent and effective in time for a medium term horizon. More specifically, we propose a two stage integer stochastic linear programming model with recource that solves the problem of generating a partition facing a finite number of future scenarios, with a solution that gives satisfactory results to any possible value of the chosen soil property. We describe the proposed model, the adopted methodology and the results achieved with this methodology.

1 INTRODUCTION

In agriculture, spatial variability of the soil properties is a key aspect in yield and quality of crops. One of the problems in precision agriculture consists in dividing the field into site specific management zones or quarters, which based on a soil property such as: pH, organic matter, phosphorus, nitrogen, crop yield, etc., defines zones with relative homogeneous characteristics. Delineating rectangular zones allows better agricultural machines performance and eases the design of irrigation systems, it is also important to consider the zones size and the total amount of management zones from field partition.

The problem of defining management zones in presence of site specific variability has been studied in (Albornoz et al., 2013) and (Albornoz et al., 2015), where a linear programming model for determining rectangular zones is defined, this problem considers spatial variability of an specific soil property and choose the best field partition. The main idea is to define homogeneous management zones to optimize the use of inputs for crops. The model is solved by the complete enumeration of the variables, but it is possible only to solve small and medium size instances due to the problem is np-hard. To deal with this problem, a column generation algorithm was proposed in (Albornoz and Nanco, 2015) which allows to efficiently solve large instances of the problem. Recently, this problem has been applied for irrigation systems design, see (Haghverdi et al., 2015) where linear programming is used as one of the methods for delineating management zones among others as K-means and Isodata, methods that are still being used. These other methods are classified as clustering methods, see (Ortega et al.,2002), (Jaynes et al.,2005) and (Jiang et al. (2011), but their major drawback is the resulting fragmentation of the zones, because these methods generate oval shaped and disjoint zones.
Although the problem of defining management zones in presence of site specific variability has been studied in previous works, to the best of our knowledge, an important characteristic that has not been considered yet is variability in time of the chosen soil property. Based on cited works we propose a two stage stochastic linear programming model with recource that solves field partition problem considering the chosen soil property as a random variable which can be modeled by a finite number of scenarios.

Stochastic programming is chosen in these situations because deterministic models are not capable of adding the effect of uncertainty to the solutions. Stochastic programming is based on considering random variables that are described by a number of possible scenarios; see e.g. (Birge and Loveaux, 2011), (Ramos et al., 2008) and (Ruszczynski and Shapiro,

Saez, J. and Albornoz, V.
Delineation of Rectangular Management Zones Under Uncertainty Conditions.
DOI: 10.5220/0005708202710278
In *Proceedings of 5th the International Conference on Operations Research and Enterprise Systems (ICORES 2016)*, pages 271-278
ISBN: 978-989-758-171-7

2003).

In the last few years, stochastic programming is being used more often in a wide variety of applications due to its capacity of solving problems increasingly large, thus more realistic models, see e.g. (Gassmann and Ziemba, 2012) and (Wallace and Ziemba, 2005) for general applications.

In agriculture, Stochastic programming is being used to solve many different problems related with situations where uncertainty is a key aspect in the decision making process. Besides delineation decision there are other important decisions to make, as crop planning, water planning, food supply chain and agricultural raw materials supply planning, among others. Crop planning is a decision where a crop pattern must be chosen for each management zone, this pattern last a specific number of crop cycles and thus must face future weather scenarios and prices, see (Itoh et al., 2003), (Zeng et al., 2010) and (Li et al., 2015). Water planning is important because the need for more agricultutal production requires large amounts of water for irrigation purposes, making water resources scarce, thus surface water resources must be allocated among farmers and also plan for the use of this water, see (Bravo and Gonzalez, 2009) and (Liu et al. 2014). Stochastic programming is also applied in agricultural supply chain problems, as food supply chain where a growing and distribution plan must be made, and raw materials supply where a raw material acquisition plan must be made considering that some raw materials are seasonal, in these problems variability appears in the form of weather conditions and product demands, see (Ahumada et al., 2012) and (Wiedenmann and Geldermann,2015).

Within stochastic programming models exists the two stage models with recource. These models recognize two types of decisions that must be made sequentially. First stage decision or here-and-now must be made previously to the performing of the random variables. Then, second stage decision or wait-and-see, which must compensate the effects of the first stage decisions once the performance of the random variables are known, due to this, the variables in this stage are denoted as recource variables. The goal of these models consists in finding the optimal first stage decision that minimize total costs, defined by the sum of the first stage decision costs and the expected costs of the second stage decisions; see e.g. (Higle, 2005).

In this case, first stage decision chooses a field partition that minimizes the number of quarters; these zones must satisfy certain homogeneity level that depends on the performance value of the sample points which are the random variables in this case. On the other hand, second stage decision uses looseness variables that relax homogeneity constraints in exchange of a penalty. This penalty helps to achieve management zones homogeneity goal while minimizes the use of the looseness variables. In this problem, homogeneity is presented by relative variance concept; see (Ortega and Santibanez, 2007), which helps to measure the quality of the chosen partition.

In this article, problem formulation needs the generation of the total number of potential quarters; in other words, problem resolution considers the complete enumeration of zones is known. This is feasible for small and medium size instances as the ones used in this work, which represents a good starting point to approach to this problem. Although, proposed formulation can be extended to large instances by the application of a column generation algorithm, but its use exceeds the purpose of this work, see (Albornoz and Nanco, 2015).

In following sections, the article is organized as follows. Next section details the proposed model to solve this problem, from data collection to the solving process itself. After this, results obtained by the application of proposed methodology are presented. At last,future works and main conclusions from the application of the model are presented.

2 MATERIALS AND METHODS

As we mentioned before, this work consists in generating a field partition composed by a group of management zones or quarters based on a chosen soil property which has variability in space and time. The proposed methodology has three steps. First, the task is to model the soil property space variability by taking samples on the field, this process must be done several times in different periods to measure variability in time, with this data, instances are generated. Second step consists on the application of the two stage stochastic linear programming model that minimizes the number of quarters in its first stage and minimizes noncompliance of the homogeneity level in the second stage. Then, in the third step we solve the proposed model with appropriate software.

2.1 Instance Generation

In this step, we generate instances that will be solved by the model. To achieve this is necessary to use specialized software as MapInfo; this software creates thematic maps of the field that summarizes and shows spatial variability of the soil properties measured from the sample points. This includes sample coordinates, pH level, organic matter index, phosphorus, base sum, crop yield, etc. As an example, Figure 1 shows two thematic maps from the same field, one with organic matter (MO) and the other with phosphorus (P). In MO case, green zones represent normal levels of MO, while red and yellow zones represent zones with 34.8% and 3.97% above normal values of MO, also sky blue and blue zones presents values with 6.06% and 30.68% under normal MO values. On the other hand, in P case red and yellow zones are 279.31% y 10.34% above normal, and sky blue and blue zones are 13.79% y 48.27%, respectively. Both maps show spatial variability of these indices in a field, this proves the importance of dividing the field into management zones with uniform characteristics, to apply inputs needed in each zone through site specific farming.

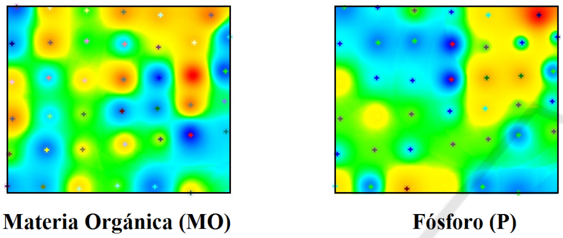

Figure 1: Organic matter and phosphorus map.

Also, we need to include variability in time of the measured indices. For that, we use thematic map data sets from the same field for several time periods; these will be used either to generate the probability distribution function of the soil property or to create different scenarios with each one of these instances. A possible value of the random variable consist in assign a specific value to each of the sample points on the field, i.e., the random variable is represented by a vector that includes each one of the sample points; this vector has a finite number of possible values. Scenario probabilities are assigned depending on the number of instances and the time between each sampling process. It is important to notice that a field partition is a medium term decision, i.e. this partition will last a specific number of years and after that horizon is reached, another partition must be set, thus the model must take into account possible changes in soil properties during this time. This article uses only historical data for scenario creation, but it is also valid to consider forecasts for future periods in the scenario creation step, but this exceeds the purpose of this article.

Finally, potential management zones are generated (Z set) through an algorithm that uses all sample points (S set) as inputs. As an example, in Figure 2 there is an instance with 42 sample point field (6 rows and 7 columns) and three potential management zones from a total of 588, each one of them has rectangular form and includes at least one sample point.

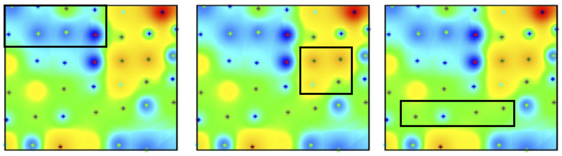

Figure 2: Potential management zones example.

A relationship matrix $C = (c_{sz})$ is created from potential zones generation, where $c_{sz} = 1$ means that potential zone z includes sample point s, and $c_{sz} = 0$ otherwise, for every $z \in Z$, $s \in S$. Besides, index variance $\sigma_{z\omega^2}$ is obtained for each potential quarter z and each scenario $\omega \in \Omega$, where Ω is the set of possible scenarios. Both parameters are used in the model presented in the following section.

2.2 Optimization Model

Proposed model consist in a two stage integer stochastic linear programming model with recource. In the first stage, the problem minimizes the number of management zones or quarters that cover the entire field. In the second stage, the problem minimizes noncompliance of the homogeneity level using looseness variables for each scenario but with a penalty cost for using them. This second stage is necessary because field partition must be chosen before knowing random variables performance, and it must satisfy the homogeneity constraint for any scenario, this is achieved by minimizing the expected value of the penalty for the noncompliance of the homogeneity level.

Sets, parameters and variables used in the model are described below:

Sets:
Z: set of potential quarters, with $z \in Z$.
S: set of sample points of the field, with $s \in S$.
Ω: set of possible scenarios, with $\sigma \in \Omega$.

Parameters:
c_{sz}: Coefficient that represents if quarter z covers sample point s or not.
M: Penalty cost per unit for noncompliance of the required homogeneity level.
n_z: Number of sample points in quarter or management zone z.
p_w: Probability of scenario ω.
$\sigma^2_{z\omega}$: Quarter variance z calculated from the soil property in scenario ω.
$\sigma^2_{T\omega}$: Total variance of the field calculated from the soil property data in scenario ω.

N: Total number of sample points.
UB: Upper bound for the number of quarters chosen.
α: Required homogeneity level.

Decision variables:

$$q_z = \begin{cases} 1, & \text{if quarter z is assigned to field partition} \\ 0, & \text{otherwise} \end{cases}$$

h_ω: Looseness for the homogeneity level in scenario ω.

The two stage stochastic model with recource is presented now:

$$Min \quad \sum_{z \in Z} q_z + \sum_{\omega \in \Omega} p_\omega Q(q, h_\omega) \tag{1}$$
$$s.t.$$
$$\sum_{z \in Z} c_{sz} q_z = 1 \quad \forall s \in S \tag{2}$$
$$\sum_{z \in Z} q_z \leqslant UB \tag{3}$$
$$q_z \in 0,1 \quad \forall z \in Z \tag{4}$$

$$Where \quad Q(q, h_\omega) = Min M_\omega h_\omega \tag{5}$$
$$s.t.$$
$$h_\omega \geqslant \sum_{z \in Z} [(n_z - k)\sigma^2_{z\omega} + (1-\alpha)\sigma^2_{T\omega}] q_z - (1-\alpha)\sigma^2_{T\omega} N \tag{6}$$
$$h_\omega \geqslant 0 \tag{7}$$

Problem (1)-(4) correspond to the first stage decision, while (5)-(7) correspond to the second stage decision. Objective function (1) minimizes the sum of quarters chosen and minimizes the expected value of the penalty cost for noncompliance of the required homogeneity level, these are first and second stage objective functions respectively. Constraint (2) is typical for set partition models, guarantee that each sample point on the field is assigned only to one quarter. Constraint (3) establishes an upper bound to the number of quarters chosen to divide the field. Constraint (4) defines that quarter variables must be binary. Objective function (5) represents second stage decision for each scenario. Constraint (6) states that a required homogeneity level must be accomplished; this constraint is made from the linear version of the relative variance concept and a looseness variable for each scenario. Finally, constraint (7) states nature of second stage variables.

It is important to notice that this model, as in (Albornoz et al., 2013), uses an equivalent linear version of the constraint related to the relative variance concept. However in this case, as we have different possible scenarios, we must meet homogeneity level in each one of these scenarios, thus we will have a relative variance constraint for each scenario. As we have to choose only one field partition we need a way to deal with uncertainty because otherwise we will have to choose the best field partition for worst possible scenario in terms of relative variance. We propose to add new variables named as looseness variables as part of the second stage decision to get a solution that considers all possible scenarios, meeting the required homogeneity level in each one of these, and without being forced to solve the problem for the worst scenario.

Constraint (6) is created from the following non-linear constraint used in (Albornoz et al., 2013) :

$$1 - \frac{\sum_{z \in Z}(n_z - k)\sigma^2_z q_z}{\sigma^2_T [N - \sum_{z \in Z} q_z]} \geqslant \alpha \tag{8}$$

This constraint uses relative variance concept, presented in (Ortega and Santibanez, 2007), is a widely used criteria to measure effectiveness of chosen management zones and it must be equal or higher to a given value, which is the required homogeneity level, that should be at least 0.5 to validate an ANOVA test hypothesis assuming k degrees of freedom. To create constraint (6) first we need to linearize equation (8) obtaining the following expression:

$$(1-\alpha)\sigma^2_T [N - \sum_{z \in Z} q_z] \geqslant \sum_{z \in Z} (n_z - k)\sigma^2_z q_z \tag{9}$$

Then if we reorder equation (9) we obtain:

$$\sum_{z \in Z} [(n_z - k)\sigma^2_z + (1-\alpha)\sigma^2_T] q_z \leqslant (1-\alpha)\sigma^2_T N \tag{10}$$

As we have a number of possible scenarios we define a relative variance constraint for each one of these, and also different parameters for each scenario ω:

$$\sum_{z \in Z} [(n_z - k)\sigma^2_{z\omega} + (1-\alpha)\sigma^2_{T\omega}] q_z \leqslant (1-\alpha)\sigma^2_{T\omega} N \tag{11}$$

Here is when we add the looseness variables h_ω to the right side of equation (11):

$$\sum_{z \in Z} [(n_z - k)\sigma^2_{z\omega} + (1-\alpha)\sigma^2_{T\omega}] q_z \leqslant (1-\alpha)\sigma^2_{T\omega} N + h_\omega \tag{12}$$

These variables allow the problem to choose a field partition that considers all possible scenarios and meet all relative variance constraints by relaxing the right side of equation (11) for each scenario, thus finally obtaining constraint (6). It is important to notice that looseness variables are added to the linear version of this constraint to have only linear constraints in the model.

3 RESULTS

To analyze the model behavior we used 10 instances for the problem, each one of them with a different number of sample points and using crop yield as soil property because this index has strong variability in time. In each instance, there are six possible scenarios, all of them with similar probabilities, where the two latest scenarios have are more likely to occur. Chosen parameter values for problem (1)-(7) are:

$M_\omega = 1.5 \quad \forall \omega \in \Omega \; UB = 40$
$\alpha = 0.9$
$p_\omega = 0.15 \quad \omega \in 1..4$
$p_\omega = 0.2 \quad \omega \in 5..6$

The rest of the parameters are calculated from crop yield data for each scenario. The number of potential quarters is obtained by the formula $\frac{((n+1)n(m+1)m)}{4}$ presented in (Albornoz and Nanco, 2015), where n is the number of sample points in length and m is the number of sample points in width. Instances used have a different number of potential quarters, starting from 588 to 13915.

3.1 Instance Solving

Instances were solved with the parameter values defined in this section and using compact equivalent deterministic reformulation of the model (1)-(7) using Cplex 12.4 as a solver in a Lenovo with Intel core i3-2310M processor CPU 2.10 GHz and a 4 GB RAM memory. Results are showed in Table 1.

First column indicates the instance number. Second column shows total number of sample points on the field. Third column indicates the number of potential quarters. Fourth column shows stochastic solution of the model. Fifth column is related to the expected value of perfect information (EVPI), which is the maximum willingness to pay for knowing all the information related to random variables performance. More precisely, this can be calculated using the following formula:

$$EVPI = RP - WS$$

Where RP is the optimum value of model (1)-(7) and WS is the wait-and-see solution, which considers solving the model for each scenario separately and then compute the expected value of this solution. At last, sixth column shows the percentage that EVPI represents from objective value function. It is important to notice that EVPI represents between 50% and 60% of objective value function, this means that willingness to pay for knowing random variables performance is really high, all of this due to the difference between scenario solutions and stochastic solution. This is because stochastic solution must face any possible scenario so it needs more quarters than the individual solutions.

3.2 Sensitivity Analysis of Penalty Cost

Penalty cost is the coefficient related to the looseness variables in the objective function in the second stage of the problem, thus if this cost is high or low, looseness variables will have a lower o higher value, and more or less quarters will be chosen respectively. For sensitivity analysis we chose instance 9, using the same parameters as we defined at the beginning of section 3. In this analysis we use the same value for every penalty cost M_ω in each scenario. Table 2 summarizes results achieved.

First column of Table 2 indicates the defined value of penalty cost. Second column shows total number of quarters chosen for the optimal solution (quarters are related to the first stage decision). From third to eighth column looseness variables from the second stage for each scenario are showed. Finally ninth column indicates the stochastic solution of the model (1)-(7).

Figure 3 shows behavior of the number of quarters chosen while penalty cost changes, it also shows looseness variables behavior.

It is important to notice when penalty cost raises,

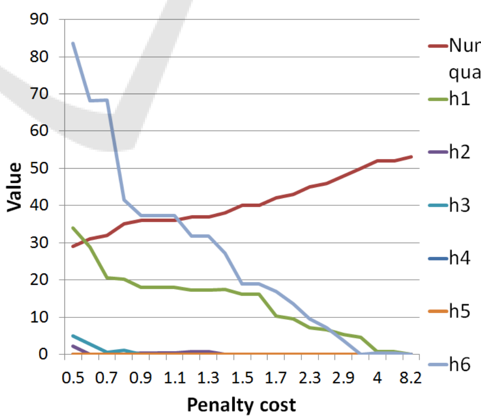

Figure 3: Solution behavior for different penalty costs M_ω.

looseness variables value decrease and at the same time the number of quarters increases to satisfy homogeneity level constraints. Looseness variables value decreases at a higher rate than number of quarters increase, this is because when penalty cost is getting higher it increases the impact of looseness variables in the objective function. There is a breaking point

Table 1: Instance solution results.

Instance	Sample Points	Potential Quarters	RP	EVPI	Percentage of O.F.
1	42	588	17.483	8.663	49.6 %
2	80	1980	29.952	15.991	53.4 %
3	100	3025	37.989	21.656	57.0 %
4	120	4290	45.531	27.307	60 %
5	140	5775	49.946	30.327	60.7 %
6	150	6600	52.586	33.125	63 %
7	160	7480	51.458	32.377	62.9 %
8	180	9405	50.817	31.974	62.9 %
9	200	11550	47.931	30.235	63.1 %
10	220	13915	43.999	27.49	62.5 %

Table 2: Sensitivity analysis for the penalty cost M_ω.

M_ω	Number of quarters	h_1	h_2	h_3	h_4	h_5	h_6	F.O.
0.5	29	34.050	2.172	4.951	0	0	83.679	38.363
0.6	31	28.834	0	2.832	0	0	68.119	39.980
0.7	32	20.531	0	0.627	0	0	68.374	41.400
0.8	35	20.292	0.033	1.150	0	0	41.425	42.548
0.9	36	17.924	0.321	0	0	0	37.280	43.496
1	36	17.924	0.321	0	0	0	37.280	44.329
1.1	36	17.924	0.321	0	0	0	37.280	45.162
1.2	37	17.244	0.704	0	0	0	31.714	45.939
1.3	37	17.244	0.704	0	0	0	31.714	46.684
1.4	38	17.430	0	0	0	0	27.182	47.368
1.5	40	16.245	0	0	0	0	19.005	47.931
1.6	40	16.245	0	0	0	0	19.005	48.460
1.7	42	10.198	0	0	0	0	16.861	48.900
2	43	9.590	0	0	0	0	13.567	49.947
2.3	45	7.1226	0	0	0	0	9.468	50.724
2.6	46	6.656	0	0	0	0	7.237	51.418
2.9	48	5.299	0	0	0	0	3.726	51.926
3	50	4.514	0	0	0	0	0	52.031
4	52	0.792	0	0	0	0	0.289	52.649
8	52	0.792	0	0	0	0	0.289	53.297
8.2	53	0	0	0	0	0	0	53

where penalty cost is 8.2, from this point it is not feasible to use looseness variables in problem solution because is more expensive than use more quarters, when this occurs then the problem chooses a field partition based only on the worst scenario in terms of space variability, this way, required homogeneity level is reached in every scenario in exchange of a field partition with a higher number of quarters compared to the other cases with lower penalty cost.

4 FUTURE WORKS

This article covers small and medium size instance solving by the complete enumeration of all potential quarters, this also needs computation of parameters described in section 2.1 for each potential quarter. This is not feasible for large instances due to the problem is np-hard and the number of variables increases really fast when the problem gets bigger, thus we need more computational effort to calculate all the parameters for each variable.

To deal with this issue, we propose to design a decomposition method based in column generation to solve large instances without using all the problem variables. This will be developed based on the decomposition of the deterministic version of the model presented in this article, see (Albornoz and Nanco, 2015), because structure is similar, and quarters can be added as columns in the algorithm as well. This work is currently being done and it will be included in a new article.

5 CONCLUSIONS

This work presents a two stage stochastic linear programming model with recource to approach the field partitioning problem facing uncertainty conditions represented by a soil property, which presents variability in time and is modeled by a set of possible scenarios. Proposed model solution defines an optimal field partition that considers every possible soil property value and as a recource it considers looseness variables that help to achieve the required homogeneity level. The model was applied to ten different instances, and it showed that stochastic solution is completely different from individual scenario solutions; this is validated by the EVPI value in each instance, concluding that a stochastic model is a better choice than a deterministic one. Besides, from sensitivity analysis we can conclude when penalty cost raises, looseness variables use decrease at a faster rate than the number of quarters increase.

Studied instances in this article were solved by the complete enumeration of all the potential quarters, but this is only feasible for small and medium size instances because the number of potential quarters grows exponentially as the number of sample points increases. For this, problem size increases faster than the sample points increase on the field and a column generation algorithm will be required to solve large instances. Also there is another complex situation when a greater number of scenarios are modeling the random variables, these aspects will be considered in future works.

ACKNOWLEDGEMENTS

This research was partially supported by Direccion General de Investigacion y Postgrado (DGIP) from Universidad Tecnica Federico Santa Maria, Grant USM 28.15.20. Jose Luis Saez wishes to acknowledge the Graduate Scholarship also from DGIP.

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