

Comparison of GPU-based and CPU-based Algorithms for Determining the Minimum Distance between a CUSA Scalpel and Blood Vessels

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Keywords: Parallel Processing, GPGPU, Z-Buffering, STL, DICOM, CT/MRI, CUSA.

Abstract: In this study, we have designed a GPGPU (General-Purpose Graphics Processing Unit)-based algorithm for determining the minimum distance from the tip of a CUSA (Cavitron Ultrasonic Surgical Aspirator) scalpel to the closest point around three types of blood vessel STLs (STereo-Lithographies). The algorithm consists of the following two functions: First, we use z-buffering (depth buffering) as the classic matured function of the GPU in order to effectively obtain depths corresponding to image pixels. Second, we use multiple cores of the GPU for parallel processing so as to calculate the minimum Euclidean distance from the scalpel tip to the closest z-values of the depths. Therefore, the complexity of the GPU-based algorithm does not depend on the shape complexity (e.g., patch, edge, and vertex numbers) of the blood vessels.

1 INTRODUCTION

We are currently developing simulators and navigators for liver surgery. In general, in liver surgery, the tissue around the affected area (for example, cancer tissue) is fractured or emulsified with the CUSA scalpel used for ultrasound surgery and extracted. Simultaneously, small blood vessels having a diameter of 0.5 mm or less can be severed while in hemostasis by cauterizing with an electric scalpel, but severing larger vessels will cause significant bleeding and threaten the life of the patient. To avoid such an issue, it is ideal for the doctors to perform actual liver surgery as planned beforehand by regularly confirming the position of the blood vessels.

In general, DICOM (Digital Imaging and Communication in Medicine) data obtained by CT (Computed Tomography)/MRI (Magnetic Resonance Imaging) are used for recording the liver conditions. In this study, we first classify the cell tissue into the entire liver, portal veins, arteries, and veins through a special processing of the DICOM data. This is called liver segmentation (Zhang, 1994; Foruzan and Chen, 2013). Here, we represent the three types of blood vessels in an STL-format polyhedron. Further, the

CUSA, which is a device that incises the liver, is represented in the same STL with a 3D scanner.

This is because a representation in the STL format accurately maintains the normal vector of the object surface and the texture and feel of the distance of the shape can be accurately felt. However, if these are represented with a polyhedron (such as B-reps), the basic processing of surgical simulations will be relatively time consuming, such as the calculation of the embedded distance between the polyhedra and the embedded region as well as the calculation of the shortest distance in all directions including the operational directions of the polyhedral, as listed in Table 1.

To begin with, sensory information related to sight and touch is required for surgery. To obtain such information, we need to calculate the distance and/or the intersection between the CUSA and the liver or the three types of blood vessels. Since the 1980s, boundary structures (polyhedra such as B-reps [STL is a type of these structures] and set operations on primitives such as CSG [Constructive Solid Geometry]), volume representations such as voxel arrays and their hierarchical representations (such as Oct-Tree, OBB [Oriented Bounding Boxes], and AABB [Axis-Aligned Bounding Box]) have been

Table 1: Advantages and disadvantages of CPU-based models and GPU-based Z-buffer (BES: best, BET: better, NOR: normal, WOR: worst, *1: Normal vector, history, and shape convexity cannot be used. *2: Normal vector, history, and shape convexity can be used).

Feeling	Target area	Calculation target	CPU			GPU
			Boundary Representation	Volume Representation	Hierarchical Structure	
Tactile/Visual	Inside of the object	Penetration distance	NOR(*1)	NOR	NOR/BET	BES
Tactile/Visual	Inside of the object	Penetration volume	WOR	NOR	BET	BES
Visual	Outside of the object	Omnidirectional distance	BET(*2)	NOR	NOR	BET
Visual	Outside of the object	Directional distance	BET(*2)	NOR	NOR	BES

processed. Simultaneously, their distance has been extracted by many CPU-based algorithms (Canny, 1986; Gilbert et al., 1988; Quinlan, 1994; Noborio et al., 1989), and/or their intersection has been determined by other CPU-based algorithms (Noborio et al., 1989; Gottschalk et al., 1996; Bergen, 1997). However, irrespective of the method used, the computational time is proportional to (in the order of $O(\mathbf{n})$ and $O(\log(\mathbf{n}))$) the degree of complexity of the shape of the polyhedron representing the organ operated on (with \mathbf{n} being the number of surfaces).

As contrasted with the above CPU-based high-speed algorithms, GPGPU (General-Purpose GPU) has been recently used for accelerating algorithms of computer vision, 3D structure modeling, 3D simulators, sorting, databases, and so on (Hubert, 2007; Miura et al., 2013; Pelletier, 2008; Cederman, 2008; Green et al., 2012; Yasuda, 2008; Taylor et al., 2008; Lee et al., 2014; Modat et al., 2010). With respect to the calculation of the distance and the intersection between a point and an object or multiple objects, the GPGPU has two advantages. One is fast digitalization (to digitalize all the objects) by z-buffering, which is the classic matured function of a GPU. The other is the fast parallel calculation (to calculate the minimum Euclidean distance or volume intersection between a point and an object and/or multiple objects) by using multicores of a GPU in parallel.

When an STL is to be processed by z-buffering, a z-value is calculated for each pixel that lies within the boundary of the STL. If the z-value at a pixel indicates that the STL is closer to the viewer than the z-value in the z-buffer, the z-value recorded in the buffer is replaced by the STL's value (Joy, 1996). Further, the Z value of the fastest patch can be preserved through the GPU background removal function. Therefore, a cuboid can be obtained with the width and pixilation calculated using the Z value of the surface and the

reverse side of the polyhedron, resulting in a cuboid digital approximation of the polyhedra.

Furthermore, we calculate Euclidean distances from the tip of the scalpel to rectangular parallelepipeds in parallel by using multicores of the GPU in order to select the minimum value as the shortest distance. Therefore, the computational time is basically in inverse proportion to the core count. Note that the GPU core count is still enormous and increases rapidly. However, the conversion time will no longer depend on the number of surfaces of the polyhedron.

Therefore, the superimposition calculation of the liver and the three types of blood vessel cuboids and the CUSA scalpel cuboids can be performed instantly by the GPU, and this enables a rapid calculation, by the GPU, of the embedded distance and the embedded regions. From this embedding, for example, an artificial sense of touch is constructed with the Kelvin–Voigt model, and this can be experienced through a tactile feedback device. Further, the polyhedra can be rapidly transformed in response to the embedded region, and the concave region becomes visible (Noborio et al., 2013; Onishi et al., 2014; Onishi et al., 2015).

On the basis of the abovementioned preprocessing, in this study, we calculate the shortest distance from the CUSA tip to the three types of blood vessels (portal veins, arteries, and veins). Herein, we design a GPU-based algorithm and, by using a CPU, compare it with a CPU-based algorithm that calculates the shortest distance from the CUSA tip to the three types of blood vessel STLs.

The rest of this paper is organized as follows: Section 2 describes the classic CPU-based algorithm and the proposed GPU-based algorithm for calculating the shortest distance from the CUSA tip to the three types of blood vessels. With respect to the advantages and disadvantages of GPU-based and

CPU-based algorithms, Section 3 investigates the effects of the required bit count for a parallel processing approach, not using the increasing trends of the computational time when the surface count increases, actual time shifts in certain liver incision simulations, and distance errors. Finally, Section 4 summarizes this study.

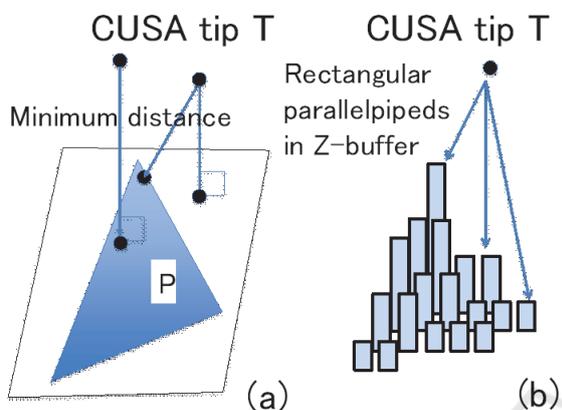


Figure 1: (a) Minimum distance between a point T and a patch P in CPU, and (b) minimum distance between a point T and a set of rectangular parallelepipeds of P in GPU.

2 ALGORITHM TO CALCULATE THE SHORTEST DISTANCE

In this section, we discuss the CPU-based and GPU-based algorithms to calculate the shortest (Euclidean) distance from the CUSA tip to the three types of blood vessel STLs.

2.1 CPU-based Algorithm

First, we have the tip of the scalpel and the three types of blood vessel (portal veins, arteries, and veins) STLs. Next, patch Ps are sequentially chosen from the three types of blood vessels, and the Euclidean distance to these is calculated. Finally, the minimum distance to all the patches is selected, and this is considered to be the shortest distance to the three types of blood vessel STLs.

In general, the minimum values of the Euclidean distance of T and P are obtained from any of the distances of an infinite plane including Tip T and Patch P, an infinite straight line including side E of P and the distance of T, as well as the top V of both tips of side E and the distance of T. Accordingly, if the leg of a perpendicular line from coordinate T to a plane including Patch P falls within Patch P, the length of the perpendicular line is the shortest distance (Figure

1(a)). Otherwise, the distance to the side including the top is the shortest distance (Figure 1(a)). Therefore, the algorithm to calculate the shortest distance is as follows:

[Step1] Calculate the normal vector n (size normalized at 1) from the three top points of the blood vessel's STL triangular surface (Patch) P_i .

[Step2] Calculate the vector v from any top point of the triangular surface to the scalpel tip T.

[Step3] Get the inner product of normal vector n and vector v to find the size of the perpendicular vector from scalpel tip T to the plane.

[Step4] Only for the distance found in **[Step3]**, find the point going (on the infinite plane) in the opposite direction of the normal vector from the scalpel tip.

[Step5] Since the intersection in **[Step4]** is the intersection with the infinite plane crossing the three top points, use an outer product for deciding whether this intersection is within (including sides and top points) the triangular surface P_i .

[Step6] If the perpendicular line intersects with the infinite plane outside of the triangular surface P_i , it is not the shortest distance from the scalpel tip to the triangular surface P_i . Therefore, reject the perpendicular line distance to the plane and proceed to **[Step7]**. Otherwise, keep it as the shortest distance candidate d_i and proceed to **[Step8]**.

[Step7] Of the distances from the tip of the scalpel to the three sides, keep the shortest distance as the shortest distance candidate d_i .

[Step8] Select the smallest value from all triangular surfaces (Patch) P_i (i : from 1 to n , n : total surface count) as d_i . This is the shortest distance to the three types of blood vessel STLs from the CUSA tip T.

Here, the distance calculation of "points and surfaces," "points and sides," and "points and tops" in the existing algorithm is checked in a likely order (Lin and Canny, 1991).

2.2 GPU-based Algorithm

First, using the z-buffering (depth buffering), we obtain a set of rectangular parallelepipeds along the Z (depth) axis (Figures 1(b) and 2) in a two-dimensional XY array. This is a useful function of the GPU, and therefore, we can obtain the set of rectangular parallelepipeds very rapidly. For the three types of blood vessel STLs, we obtain three sets of rectangular parallelepipeds around the tip T. A large rectangular region where a doctor operates on the liver by using CUSA can be freely selected in the world coordinate

system. In other words, the coordinate system of depth buffering is defined as a rectangular region, which implies that rectangular parallelepipeds and the tip T are transformed into the camera coordinate system of the depth buffering (Figure 2).

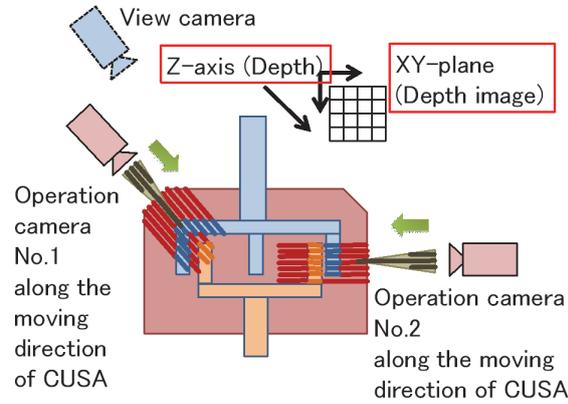


Figure 2: Z-buffer for visible ability and several Z-buffers for cutting the liver are independently allocated in the world coordinate (3-D) system.

Next, in the camera coordinate system, we can calculate the minimum Euclidean distance from the tip T to the rectangular parallelepipeds in parallel as the shortest distance by using the many cores of the GPU (Figure 3). In the assignment to the parallel processing GPU calculation unit, the thread count is 16×16 and the block count is width/16 \times height/16. That is, when the image resolution is 2048×2048 , a parallel calculation is performed with a thread count of 16×16 and a block count of 128×128 .

Finally, in the proposed GPU-based algorithm, we calculate the distance precision by using two types of image and depth quantization methods. First, a cuboid region of X: 106 mm \times Y: 106 mm \times Z: 213 mm is established around the scalpel tip to obtain the Z value of the three types of blood vessel STL surfaces. In such cases, a 32-bit variable records the Z value, and the XY image resolution is 2048×2048 (Figure 3). As a result, the quantization error for the Z axis is about 0.05 nm ($=213 \text{ mm}/2^{32}$), and the quantization error for the XY image is about 52 μm ($=106 \text{ mm}/2048$). Therefore, the maximum quantization error of the Euclidean distance used for calculating the XYZ value by using the three squares theorem is 73.5 μm . From the perspective of a doctor's requirements, this error can be sufficiently ignored. Moreover, the minimum distance to all pixels is the shortest distance between the scalpel tip T and the three types of blood vessel STLs (Figure 3).

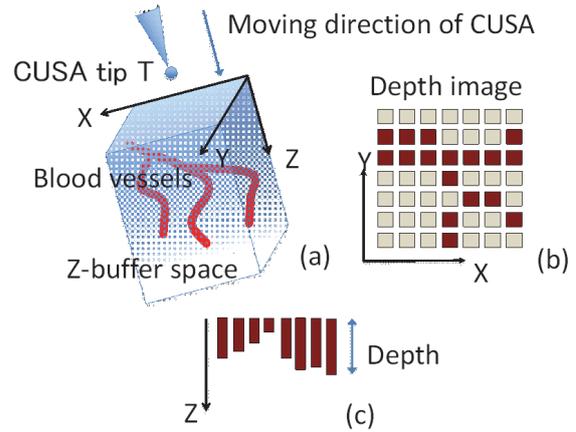


Figure 3: Many depths (2 values) in the XY image (many pixels), which are converted from STL blood vessels.

An array z-buffer[x,y] initialized to ∞

```

begin /* z-buffering in GPU
  for each patch P of three types of vessels do {
    for each pixel (x,y) that intersects P do {
      calculate z-depth of P at (x,y)
      if z-depth < z-buffer[x,y] then {
        z-buffer[x,y] = z-depth
      }
    }
  }

```

A vector d-bits[z] initialized to 0

```

begin /* many cores in GPU
  parallel for all pairs of pixels (x,y) and
  their z-depth do {
    calculate Euclidean distance D between (x,y, z-
    buffer[x,y]) and (xp,yp,zp) of tip T of CUSA scalpel
    raise a flag at d-bits[z] by D
  }

```

scan d-bits[z] sequentially from left to right in order to find an initial bit

calculate the distance corresponding to the bit

As shown in Figure 4, even if multiple parallel processing tasks simultaneously change specific bits, the existence of their distances is established. In such cases, if the bit set and the closest XYZ value are recorded at the same time, multiple proximate vectors (vectors expressed at the proximity point with the scalpel tip), which achieve this distance, are separately recorded. Because all the bits are initialized, a bit value of 0 implies that a distance could not be found within the scope corresponding to any parallel process.

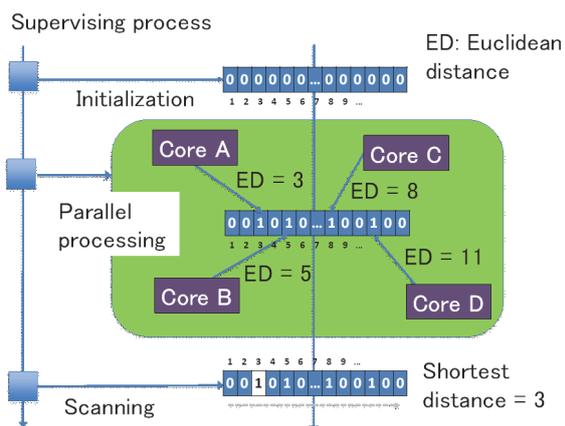


Figure 4: Parallel processing the shortest distance by using many of the cores in the GPU.

Here, we discuss the relationship between the bit number and the distance precision in the proposed algorithm. On the basis of the opinions of doctors, the ranging scope is set to 0–50 mm, and this is divided to the extent of $16 \times i$ (variable i is arbitrarily set). Further, there is a bit for each division, and if there are any pixels in which the distance from the scalpel tip to the three types of blood vessels fits within the bounds of the divisions, this bit is set to 1 (Figure 4). For example, if $i = 1$, and the distance error from this division is $50 \text{ mm}/16 = \text{about } 3 \text{ mm}$, $i = 16$, then the distance error from the division is $50 \text{ mm}/256 = \text{about } 0.2 \text{ mm}$. Further, if all arrays are initialized at 0 before starting the parallel processing for all pixels, when searching the arrays from 0 after the parallel processing is finished, in the array in which 1 is first located, we find the shortest distance by the following calculation: shortest distance = intermediate value of the distance range corresponding to the array number (Figure 4).

Lastly, errors related to this parallel processing are added to the quantization errors from when the polyhedra are transformed into Z-buffers. Of course, in CPU-based algorithms, for parallel processing errors, there are no quantization errors when the polyhedra are converted into Z-buffers; there are only limitations of floating-point numbers (single precision and double precision). Accordingly, compared to GPU-based algorithms, there are relatively few mistakes (Table 2).

Table 2: Comparison of calculation time and distance precision of CPU- and GPU-based algorithms.

Calculation time	CPU-based algorithm	GPU-based algorithm
Theoretical and experimental evaluation	Dependent on the number of patches	Independent of the number of patches
Doctor's proposal	Unsatisfactory	Satisfactory
Distance precision	CPU-based algorithm	GPU-based algorithm
Theoretical and experimental evaluation	Accuracy of a floating-point number	Digital error of Z-buffer plus (measuring range)/($16 \times i$) ($16 \times i$: number of prepared bits, i : arbitrary number)
Doctor's proposal	Satisfactory	Satisfactory

3 EXPERIMENTAL COMPARISON OF CPU-BASED ALGORITHMS AND GPU-BASED ALGORITHM

In this section, we investigate the accuracy of and the required time for measuring the shortest distance between the scalpel tip and the three types of blood vessels. First, we evaluate the computational time for the two algorithms by monotonically adding the item count of polyhedral patches that constitute the three types of blood vessels. Next, we compare the computational time and the measurement accuracy when the liver is virtually incised by CUSA.

3.1 PC and Development Environments

Here, the PC environment and the development environment related to the CPU and the GPU are explained in detail.

CPU-related specifications:

- (1) PC
 - OS Windows 8.1 Professional 64-bit
 - CPU Intel Core i5-2500K CPU 3.30 GHz
 - Memory 16 GB
- (2) Development environment
 - IDE Microsoft Visual Studio 2012
 - Renderer Direct 3D 11.0

GPU-related specifications:

- (1) PC
 - VGA GeForce GTX480 CUDA processor core count 480
 - Graphics clock 700 MHz
 - Processor clock 1401 MHz
 - Memory 1536 MB GDDR5
- (2) Development environment
 - Language C++/HLSL
 - GPGPU Direct Compute

Since the development environments of CPU-based and GPU-based algorithms are different, the

following comparisons may not be precisely fair. However, our computer has sufficient memory and the STL's data size is not very large. Therefore, the comparisons are practically useful for designing a surgical simulator/navigator in the future.

3.2 Evaluation of Computational times of CPU-based and GPU-based Algorithms

Here, we investigated how the computational time for each algorithm increases depending on the polyhedra patch count. First, we place a number of patches for the three types of blood vessels on the horizontal axis and monotonically increase the patch count while increasing the approximate accuracy of the blood vessel shape to 357992. The computational time for the CPU- and GPU-based algorithms is shown on the vertical axis of the graph (unit: milliseconds) (Figure 5).

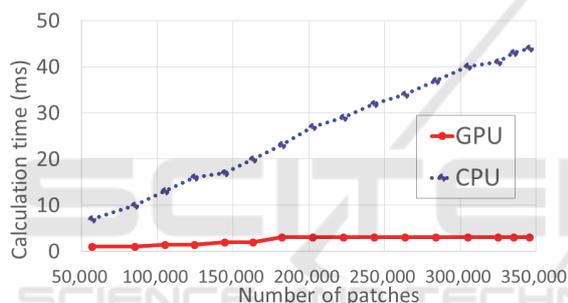


Figure 5: Computational time of CPU- and GPU-based algorithms in proportion to the number of patches.

From these results, we found that the computational time order for algorithms using CPU is $O(n)$ for surface count n ; further, we confirmed that the computational time increases in proportion to n (Since this is the shortest distance from the tip of the scalpel, it is not dependent on the degree of complexity of the scalpel shape.). This is the same characteristic as that of conventional algorithms using CPUs (Lin 1991).

Further, if the accuracy of the polyhedra shape is increased, the actual computational time will exceed 40 ms. Hence, it will be difficult to obtain not only the sense of touch but also the reality of sight (The normal video rate is exceeded.). On the other hand, the calculation time order of algorithms using the GPU is $O(1)$, which is a fixed value irrespective of an increase in n . In the future, since improvement in the liver segmentation function will allow the recognition of smaller blood vessels and more accurate shapes, the polyhedra patch count representing the three types

of blood vessels is expected to increase.

Accordingly, we can state that the computational time of the GPU-based algorithm, which is independent of the degree of complexity of the subject shape, is more desirable than that of the CPU-based algorithm, which is dependent on the degree of complexity of the subject shape.

3.3 Changes in Computational Time for the Shortest Distance between the Scalpel Tip and the Blood Vessels in Actual Surgery

Here, we show the computational time required to derive the shortest distance from the scalpel tip to the three types of blood vessels when a virtual liver is actually incised by using a virtual CUSA scalpel. Figure 6 shows a strobe shot of such an instance, and Figure 7 shows the time changes exclusively for the distance calculation of the CPU- and GPU-based algorithms for each of the motions (the cycle of moving the scalpel, incising the liver, and measuring the distance to the three types of blood vessels). Here, the computational time for the CPU-based algorithm is more than 40 ms. However, the computational time for GPU-based algorithms is usually about 3 ms.

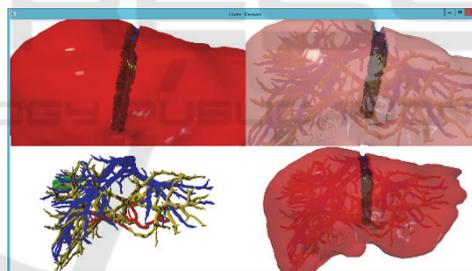


Figure 6: Surgery for polyhedral liver and its three types of blood vessels by using a polyhedron of the CUSA scalpel.

A simulation/navigation of liver surgery needs many calculation functions such as liver deformation and real liver sensing and following. The calculation of the shortest distance is one of them. Therefore, if the calculation (40 ms) is over the video rate, the surgical animation of simulation/navigation is sometimes frozen. Therefore, a faster calculation (2–3 ms) using a GPU is suitable for real-time simulation/navigation. A doctor can comfortably follow the instructions for the surgery when they are provided using smooth animation. As a result, this real-time animation reduces the possibility of mistakes made by the doctor, such as blood vessel injuries in surgery performed using augmented reality.

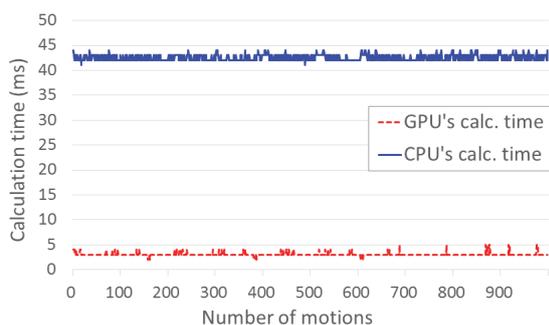


Figure 7: Computational time of CPU- and GPU-based algorithms in the surgery described in Figure 6.

These results were obtained with the regular-edition GPU (core count: 480). The current business-use GPU with thousands of cores will probably be about five times faster. In contrast, the CPU operating frequency has hit a plateau since 2007 (around 3 GHz), and Moore’s Law (a three-fold increase in speed every 2 years) is no longer applicable to the calculation of computing power. As a countermeasure, the CPU operating unit counts have been increased to boost computing power, but the GPU computing power has improved to a far greater extent. Therefore, we believe that the GPU-based algorithm has more of a future than the CPU-based version.

3.4 Comparison of the Shortest Distance from the Tip of the Scalpel to the Three Types of Blood Vessels Calculated by CPU- and GPU-based Algorithms in Actual Surgery

As explained in Section 2 (particularly, the final paragraph of Section 2.2), GPU-based algorithms realize parallel processing and maintain its speed. As a result, certain distance measurement ranges can only be measured with a certain degree of accuracy for the distance. On the other hand, in the case of CPU-based algorithms, no special consideration is needed for distance measurement ranges or distance accuracy (naturally, this is still conditioned by the calculation accuracy of floating-point numbers) (Table 2).

Here, we actually validated these characteristics. To begin with, Figure 8 shows the scalpel motion in a validation experiment, and Figure 9 shows the trends in distance measurement ranges and distance accuracy for CPU- and GPU-based algorithms in such cases. Here, CPU-based algorithms calculated all the shortest distances without any restrictions, but GPU-

based algorithms calculated the shortest distances for fixed distance measurement ranges (0–50 mm) with fixed distance accuracy ($50/(16 \times i)$ mm, i : optional).

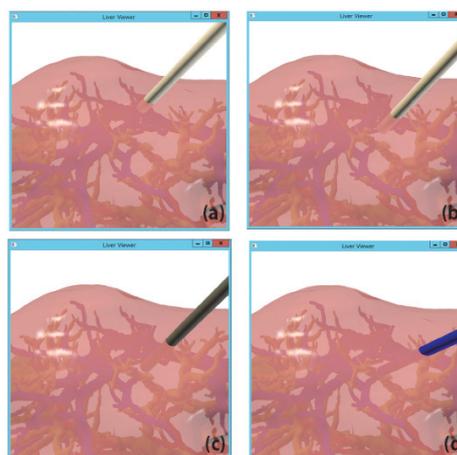


Figure 8: Motion example in which the CUSA scalpel accesses three types of blood vessels. In this figure, as long as the distance decreases monotonously, the color of the CUSA scalpel changes from white to light gray, dark gray, and then blue (black). From this color change, a doctor understands the present state of danger.

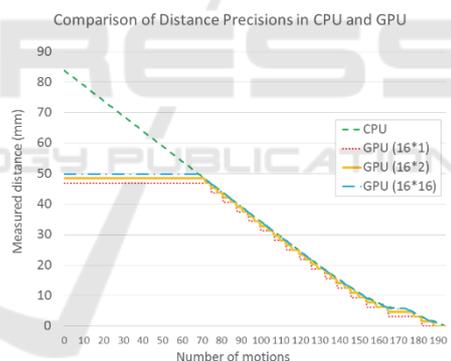


Figure 9: Range and precision of distances change when the CUSA is close to the blood vessels along the sequence of motions shown in Figure 8.

As compared to the CPU-based algorithm, the GPU-based algorithm is very fast and its complexity does not depend on the shape of the blood vessels. On the other hand, the CPU-based algorithm deals with a continuous boundary, and therefore, its distance precision is very good. Unfortunately, the GPU-based algorithm always digitalizes the shape of blood vessels by image pixels and their z-buffering. Consequently, it decreases the distance precision.

4 CONCLUSIONS

In this paper, we proposed an algorithm to calculate the shortest distance from the scalpel tip to the blood vessels independent of the degree of complexity of the shape of the three types of blood vessels. This satisfies the specifications required by doctors (with respect to the distance measurement range, distance accuracy, and computational time).

To begin with, the opinion among liver surgeons (as opposed to cerebrovascular and cardiovascular surgeons) is that a positioning accuracy of about 0.5 cm is sufficient for liver surgery (conversely, human behavioral functions cannot adjust to any higher degree of accuracy). It is reportedly the case that the direction of distance measurement, movement of the scalpel (incision mistakes only occur in this direction), and distance measurement range are acceptable at about 10 cm from the tip of the scalpel (only around the liver cancer to be cut out; the size of the entire liver is about 20 cm). Moreover, the surgeons voiced their desire for a real-time nature (within a range of several milliseconds in all situations) that calculates the senses of vision and touch to be emphasized. The algorithm proposed in this paper meets this request even when the shapes of the three types of blood vessels inside the liver are complex.

Currently, the number of cores in a GPU is increasing every year. The proposed algorithm can keep pace with this increase. Further, the technology for blood vessel imaging and its segmentation is improving every year, with smaller blood vessels as well as their more detailed shapes being recognized. Since this results in an enormous surface count for the STL expressing the three types of blood vessels, the value of the GPU-based algorithm proposed here that calculates the shortest distance to the blood vessels is expected to increase.

Lastly, an issue to consider in the future is the development of an algorithm with the guidance control of the scalpel tip to incise about 0.5 cm around cancer cells in order to remove them. Toward this end, it is necessary to enable the calculation of the shortest distance from all directions of the scalpel tip and calculate the proximity vector from the scalpel tip to the closest point on the cancer tissue surface in order to calculate the operational vector of the scalpel. Further, we developed a CUSA tip for the cutting region, but if the kidneys or the other organs, and not the liver, were the target, then CUSA cannot be used; blades or scissors would have to be employed to make the incision. In such cases, we would need to represent the cutting area with a line and not a point.

In the future, we would like to consider expanding the proposed algorithm to such instances.

ACKNOWLEDGEMENTS

We would like to express our sincere gratitude to Professor Masanori Kon and Associate Professor Masaki Kaibori of Kansai Medical University, who provided us with advice concerning liver surgery, and Professor Yen-Wei Chen of Ritsumeikan University, who provided the segmented liver DICOM data. Further, we would like to note that this research has been partially supported by the Collaborative Research Fund for Graduate Schools (A) of the Osaka Electro-Communication University and a Grant-in-Aid for Scientific Research of the Ministry of Education, Culture, Sports, Science and Technology (Research Project Number: 26289069).

REFERENCES

- Zhang Z. Iterative point matching for registration of free-form curves. *Int. J. Comput. Vision* 2, pp.119-152, 1994.
- Foruzan A.H., Chen Y.W. et al. Segmentation of liver in low-contrast images using K-means clustering and geodesic active contour algorithms. *IEICE Trans* 4, pp.798-807, 2013.
- Canny J.F. Collision detection for moving polyhedral. *IEEE Trans PAMI* 2, pp.200-209, 1986.
- Gilbert E., Johnson D., Keerthi S. A fast procedure for computing the distance between complex objects in three-dimensional space. *IEEE J. Robotic. Autom.* 2, pp.193-203, 1988.
- Quinlan S. Efficient distance computation between non-convex objects. *IEEE J. Robotic. Autom.* '94, pp.3324-3329, 1994.
- Noborio H., Hata H., Arimoto S. Algorithms searching for the nearest point of 3-D objects using octotree. *IPSP Trans* 3, pp.311-320, 1989 (in Japanese).
- Noborio H., Fukuda S., Arimoto S. Fast interference check method using octree representation. *Adv. Robotics* 3, pp.193-212, 1989.
- Gottschalk S., Lin M.C., Manocha D. OBBTree: A hierarchical structure for rapid interference detection. *SIGGRAPH '96*, New Orleans, pp.171-180, 1996.
- Bergen G. Efficient collision detection of complex deformable models using AABB trees. *Journal of Graphics Tools* 4, pp.1-13, 1997.
- Hubert N. *GPU Gems 3: Programming Techniques for High-Performance Graphics and General-Purpose Computation*, Addison-Wesley Professional; First version, 2007.
- Miura M., Fudano K., Ito K., Aoki T., Takizawa H., Kobayashi H. Performance evaluation of phase-based

- correspondence matching on GPUs. Proc. SPIE 8856, Applications of Digital Image Processing XXXVI, 885614, 26 September 2013.
- Pelletier M. G. Parallel algorithm for GPU processing for use in high speed machine vision sensing of cotton lint trash. *Sensors* 8(2), pp.817-829, 2008.
- Cederman D. On sorting and load balancing on GPUs. *ACM SIGARCH Computer Architecture News Archive* 36(5), pp.11-18, December 2008.
- Green O., McColl R., Bader D. A. GPU merge path - A GPU merging algorithm. Proc. of the 26th ACM International Conference on Supercomputing (ICS), San Servolo Island, Venice, Italy, June 25-29, pp.331-340, 2012.
- Yasuda K. Accelerating density-functional calculations with graphics processing units. *Journal of Chemical Theory and Computation* 4(8), pp.1230-1236, August 2008.
- Taylor Z. A, Cheng M., Ourselin S. High-speed nonlinear finite element analysis for surgical simulation using graphics processing units. *IEEE Trans Med Imaging* 27(5), pp.650-663, 2008. doi: 10.1109/TMI.2007.913112.
- Lee H.-P., Audette M., Joldes G. R. and Enquobahrica A. neurosurgery simulation using non-linear finite element modeling and haptic interaction. Proc. SPIE Int. Soc. Opt. Eng. Author manuscript; available in PMC 2014 Jan 22. Published in final edited form as: Proc. SPIE Int. Soc. Opt. Eng. 2012 Feb 23; 8316: 83160H. doi: 10.1117/12.911987.
- Modat M., Ridgway G. R., Taylor Z. A., Lehmann M., Barnes J., Hawkes D. J., Fox N.C., Ourselin S. Fast free-form deformation using graphics processing units. *Journal of Computer Methods and Programs in Biomedicine* 98(3), pp.278-284, June 2010.
- Joy K. The Depth-Buffer Visible Surface Algorithm, On-Line Computer Graphics Notes, <http://www.idav.ucdavis.edu/education/GraphicsNotes/Z-Buffer-Algorithm/Z-Buffer-Algorithm.html>, 1996.
- Lin M., Canny J. A. Fast algorithm for incremental distance calculation. *IEEE Robotics and Automation '91*, Sacramento, pp.1008-1014, 1991.
- Noborio H, Onishi K, Koeda M, Mizushino K, Kunii T, Kaibori M, Kwon M, Chen YW. A fast surgical algorithm operating polyhedrons using Z-buffer in GPU. Proc. of the 9th Asian Conference on Computer Aided Surgery, Tokyo, pp.110-111, 2013.
- Onishi K., Mizushino K., Noborio H., Koeda M. Haptic AR dental simulator using Z-buffer for object deformation. Proc. of the HCI International 2014 (16th International Conference on Human-Computer Interaction), Creta Maris, Heraklion, Crete Greece, pp.342-348, June 22-27, 2014.
- Onishi K., Noborio H., Koeda M., Watanabe K., Mizushino K., Kunii T., Kaibori M., Matsui K., Kwon M. Virtual liver surgical simulator by using Z-buffer for object deformation. *Universal Access in Human-Computer Interaction (Proc. of HCII 2015), Part III, LNCS 9177*, pp.345-351, Los Angeles, CA, USA, August 2015.