Tackling Non-linearity in Seismic Risk Estimation using Fuzzy Methods

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Abstract: Traditional approaches to measure risk to natural hazards considers the use of composite indices. However, most of the times such indices are built assuming linear interrelations (interdependencies) between the aggregated components in such a way that the final index value is based only on an accumulative or scalable structure. In this paper we propose the use of Fuzzy Inference Systems type Mamdami in order to aggregate physical seismic risk and social vulnerability indicators. The aggregation is made by establishing rules (if-then type) over the indicators in order to get an index. Finally a quantitative seismic risk estimation is made though the convolution of these two main factors by means of fuzzy inferences, in such a way that no linear assumptions are used along the estimation. We applied the fuzzy model over the city of Bogota Colombia.

We consider that this approach is a useful way to estimate a measure of an intangible reality such as seismic risk, by assuming the urban settlement’s complexity where the interrelations between the associated risk components are inherently non-linear. The proposed model possess a practical use over the risk management field, since the design of the logic rules uses a smooth application of risk management knowledge following a multidisciplinary approach, thus making the model easily adapted to a particular circumstance or context regardless the background of the final user.

1 INTRODUCTION

Holism (from greek: all, whole, entire) is an epistemology position which postulate that complex systems cannot be completely understood by taking under scope each of their components in a separate way. The holism defines then, the basis for a non-reductionism methodology for the study of systems. The idea behind holism is "the integration of the parts, through its synergies, to understand the whole" (Carbóna, 2001). According to a holistic approach, the "whole" is more complex than the sum of its constituent elements, therefore the total behavior of the system cannot be derived from its fundamental components without considering the trade off of information (energy) between them. If we intent to frame risk to natural hazards into a holistic or integral scheme, we need to take into account the complexities over an urban environment. In this terms, an important part of the urban complexities can be considered as a result of the non linear interrelationships between the multitude of components conforming the urban system.

The physical risk, defined as the seismic risk component that reflects the type of assets that can be damaged because an earthquake occurrence (including lost lives) have a solid framework of analysis and experimentation. Even if that the large majority of seismic hazard, vulnerability and exposure models considers a probabilistic approach, the engineering field has the possibility to compare their results with experimental data, that can be obtained either from simulations or practical experiments. Although physical risk models uses approximations, there is a mark of reference to compare. At the other hand, how can we estimate an intangible reality such as social vulnerability?

Social vulnerability is a crucial aspect of risk management. There can be no analysis or management without a social vulnerability understanding however, vulnerability have embedded confuse concepts that may leads towards many (and some times different) conclusions. Nevertheless, most of the approaches used to define social vulnerability, focuses over the susceptibility and capacities of urban elements to act against external influences, thus understanding vulnerability as a sort of detector capable to determine the state of the system. Therefore, social vulnerability becomes an essential source of information in or-
order to implement suitable hazard and mitigation assessments, reduction and disaster preparedness that requires first of all, the identification of the various dimensions of vulnerability over a society, either economic, institutional structure or environmental resources.

Carreño et al. (2012) proposed a seismic risk model from a holistic perspective, by considering that seismic risk is the result of physical risk (those elements susceptible to be damage or destroyed) and an aggravation coefficient that includes both, the resilience and the fragility of an urban environment. By describing physical risk and social aggravation by means of indices the final estimation of seismic risk is made by means of the so called Moncho’s equation.

In this paper, we built a holistic seismic risk fuzzy model considering Cardona-Carreño risk descriptors. By establishing fuzzy logic rules between such descriptors we were able to aggregate them all into a single seismic risk index without assuming a linear behavior between the descriptors. We found seismic risk tendencies and spatial distributions patterns over Bogota (Colombia) by performing a classical Mamdani fuzzy approach, supported by well established fuzzy theory, which is characterized by a high expressive power and an intuitive human-like manner.

2 CARREÑO’S MODEL

Taking as a base Cardona’s original model (2001), Carreño et al. (2012) proposed a seismic risk model considering an integral (holistic) approach, regarding seismic risk as a function of the potential damage on assets (considering hazard intensities) plus the socio-economic fragilities and lack of resilience of the context. In this view, seismic risk would be the result of physical risk, aggravated by social conditions and lack of resilience capacities. Carreño et al. model relies in the use of descriptors for both: physical risk (see the 8 physical risk descriptors of Figure 1) and social aggravation (see the 11 aggravation descriptors of Figure 1).

A conceptualization of Carreño’s seismic risk model can be seen in Figure 1.

Carreño et al. (2012) obtained a seismic risk evaluation for Bogota city by means of indicators that leads to the calculation of a total risk index. This is obtained by direct application of Moncho’s equation described in 1:

\[ R_T = R_{ph} (1 + F) \]  

where \( R_T \) is the total risk, \( R_{ph} \) is the physical risk and \( F \) is a aggravation coefficient.

\[ R_{ph} = \sum_{i=1}^{p} w_{R_{ph}i} F_{R_{ph}i} \]  

where \( F_{R_{ph}i} \) are the physical risk descriptors, \( w_{R_{ph}i} \) are their weights assessed by an analytic hierarchy process (Carreño et al., 2007; Saaty and Vargas, 1991), and \( p \) the total number of considered descriptors in the estimation.

The physical risk descriptors values can be obtained from previous physical risk evaluations (damage scenarios) already made at the studied location.

The aggravation coefficient, \( F \), depends on a weighted sum of an aggravation descriptors set associated to socioeconomic fragility of the community \( (F_{SF}) \) and lack of resilience of exposed context \( (F_{LR}) \), according to Equation 3,

\[ F = \sum_{i=1}^{m} w_{SFi} F_{SFi} + \sum_{j=1}^{n} w_{LRj} F_{LRj} \]  

where \( w_{SFi} \) and \( w_{LRj} \) are the assessed weights on each factors and \( m \) and \( n \) the total number of descriptors for fragility and lack of resilience, respectively.
The descriptors values were obtained from existent databases and statistical data of the studied area.

The use of descriptors conforms an indirect technique to estimate a quantitative measure of change. The final aim is to describe intangible realities, hidden trends or different classes of information in a composite manner in order to present them all as quantifiable entities that can be compared across space and time scales. Basically, descriptors are an encapsulation of a more complex reality using a single construct, and they can be used solely as independent entities of measure, or they can be aggregated to form indices. Since an index is intended to describe a particular attribute, the attribute will determine a sort of causality structure between it and the descriptors that can be either reflective (the attribute influences the descriptors) or formative (the descriptors influence the attribute). The main difference is based in the internal correlation of the descriptors. In the case of Carreño’s indices, there is a strong formative causality structure and therefore, the attributes can be assumed to be interdependent. Therefore, the final outcome of descriptor’s aggregation would estimate the attribute considering only a linear influence between descriptors. Even if this assumption may be valid in some circumstances, the main objective of the index is lost since in fact, there is no a real measure of conditionality or causality between descriptors, since none of the indicators can be diminished or amplified by another indicator, and therefore there is no way to assess non-linearities or feedbacks that do exist in real systems.

In the same way Moncho’s equation presents a linear relation between its physical and aggravation components. Since the main assumption behind Moncho’s equation is to consider physical risk as the main seismic risk “driver”, a clear consequence of assuming linearity would be that the final estimation of the total seismic risk will rely in the existence, first, of a significative physical risk value. Therefore, if a region presents large social aggravation values, but the values for physical risk are small or medium for the same area, the final risk estimation will be small. This assumptions can lead to a underestimation in the final risk estimation that can be misleading. For example, an area with an important aggravation or social vulnerability, might be severely affected by an earthquake of less intensity and the effects could be even bigger and more spread, creating a series of unwanted consequences that cannot be estimated using a linear relationship between seismic risk components.

3 FUZZY SEISMIC RISK MODEL

The integral frame that we followed was built in the believe that seismic risk can be viewed as the convolution of two principal components: the social aggravation and physical risk, which in turn forms the total seismic risk of an urban center. The Fuzzy Seismic Risk Model (FSRM) is divided in three main modules or sections: Social Aggravation, Physical Risk, and Total Risk. Each one of them is conformed by different submodules. The main objective is to be able to estimate seismic risk for an urban center considering social and physical aspects trough fuzzy inference modeling, therefore, not assuming a linear interdependency between seismic risk components.

3.1 Aggravation Coefficient

We built an aggravation coefficient by re-defining and grouping Carreño’s descriptors into three different Fuzzy Inference Systems (FIS) called: resilience, fragility and aggravation. Each subsystem is defined by a set of rules involving all proper descriptors. A conceptualization of the different steps followed to estimate the aggravation coefficient can be seen in Figure 2. The descriptors involved in each subsystem are presented in the left hand side of Figure 2. FIS 1, corresponds to the Fragility model and has as input variables the Marginal Slums (MS), the Social Disparity Index (SDI) and the Population Density (PD). The output of FIS 1 is the level of fragility. On the other hand, FIS 2 corresponds to the Resilience model which have as input variables the Human Health Resources (HHR), the Emergency Operability (EO) and the Development Level (DL). The output of FIS 2 is the resilience level. Finally, the Aggravation model (FIS 3) takes as inputs the fragility and resilience levels that are the output of FIS 1 and 2, respectively, and infers the aggravation coefficient. All the fuzzy inference systems proposed in this research are based on the Mamdani approach (Mamdani and Assilian, 1975), since it is the one that better represents the uncertainty associated to the inputs (antecedents) and the outputs (consequents) and allows to describe the expertise in an intuitive and human-like manner.

The original 11 aggravation descriptors, presented in Figure 1, were reduced to six variables by considering a subjective method which is based in the assumption that certain descriptors reflects similar attributes in terms of aggravation formation. For example, descriptors called: mortality rate and delinquency rate, are linked since they reflect negative social consequences produced by a social structure failure, i.e. a lack of access to certain social advantages, such as
an efficient public health program, a strong marginalization dynamics, no access to education or effective justice and law policies. Therefore, we consider that these particular descriptors could be described using only the descriptor called social disparity index, which is a fragility descriptor as well. In the case of descriptors related to resilience we consider that descriptors called: public space, hospital beds and emergency personnel, are already reflected by the descriptor named emergency operability, since the attributes of the former descriptors are related with the capacity of reaction when the emergency is being or has recently occurred. Descriptors called: marginal slums, population density, human health resources and development level remain the same.

We used three linguistic labels defined to qualify each descriptor: low, medium and high, along their respective universe of discourse. However, for the FIS outputs, i.e. resilience, fragility and aggravation, we decided to use five labels: low, medium-low, medium-high, high and very-high. We think that 3 classes is enough to accurately represent input variables of resilience and fragility models. Moreover, a reduced number of classes implies a more compacted and reduced set of fuzzy rules. To improve model’s sensibility, we design membership functions in order to consider the data variability. With these membership functions we build a set of fuzzy rules that could infer the behavior of the aggravation coefficient using the three Mamdani fuzzy inference systems mentioned before (see Figure 2).

The development of the fuzzy rule base considers all possible combinations of descriptors linguistic labels, therefore a total of 27 rules (3 descriptors) characterized by 3 linguistic labels each) where respectively used for infer fragility and resilience values. We think that the completeness of the fuzzy partition is of great importance in this application. These rules were intended to follow risk management literature which suggests possible outcomes when three of these elements interact to form resilience or fragility. At the other hand, the aggravation model, that has as inputs resilience and fragility inferred values, characterized by 5 classes each, is then composed of 25 fuzzy rules. In Table 1, the rules that conform the resilience FIS model are presented as an example. The use of classical fuzzy systems, with well established fuzzy inference theory, allow to build a solid model, easily understandable by experts which leads towards a deepest discussion over social vulnerability description and casuals non linear interrelations.

3.2 Physical Risk Coefficient

In the holistic model presented in Figure 1, 8 descriptors are associated with physical risk formation. Nevertheless, we consider important to include another descriptor already used in previous studies (Cardona, 2001; Carreño, 2006) called damage in main roads, due to its significance for the analysis of seismic risk. We categorized these new collection of physical risk descriptors into three different models called: Property Damage, Life Lines Sources Damage and Network Damage, each of those was later framed into a
FIS structure. Each model contains as inputs three of the original descriptors. The structure of the physical risk model can be seen in Figure 3. FIS 1, corresponds to the Property Damage model and have as input variables: Damage Area (DA), Dead People (DP) and Injured People (INJ). The output of FIS 1 corresponds to the level of property damage. The FIS 2 represents the Life Lines Sources Damage model and have as input variables: Telephonic Substation Affected (TSA), Electrical Substation Affected (ESBA), and Damage in Water Mains (DWM). The output of FIS 2 is the level of damage to life lines sources. The FIS 3 corresponds to the Network Damage model and have as input variables: Damage in Gas Network (DGN), Fallen Length of Electrical Lines (FLEN) and Damage in Mains Roads (DMR). The output of FIS 3 is the network damage level. FIS 4 corresponds to the Physical Risk model, which takes as inputs the outputs of all previous models, i.e. FIS 1, FIS 2 and FIS 3, and then infers the physical risk coefficient.

We decided to characterize each input variable (descriptor) into 3 labels: low, medium and high, and into five labels: low, medium-low, medium-high, high and very-high, each FIS output, i.e. property damage, life lines sources damage and network damage. Therefore, as before, 27 fuzzy rules were obtained for FIS 1, FIS 2 and FIS 3, and 25 for FIS 4. We also designed the membership functions in order to reach current data variability.

The rules of the Mamdani physical risk model are presented in Table 2 as an example.

### 3.3 Total Risk Coefficient

The FIS called Total Risk will perform the convolution of all the previous FIS already developed, embodied in one main structure that have as inputs the variables representing the inferred values of physical risk and social aggravation. In this case, both, inputs and outputs, are characterized by the 5 labels mentioned in the previous section, obtaining a Mamdani model composed of a set of 25 fuzzy rules to be used in the inference process. As previously, the membership functions are designed to represent the data variability. The Total Risk model structure can be seen in Figure 4.

### 4 RESULTS: CITY OF BOGOTA

Colombia’s Capital is divided since 1992 into 20 administrative Localities. However in our study we took into account only 19 on these because the locality called Sumapaz corresponds basically to the rural area of the city. For the social aggravation coefficient estimation on each district we used statistical and demographic data from 2001 (Carreño et al., 2012).

In Figures 5 and 6 we can see the aggravation values obtained by the proposed fuzzy model and the index method, respectively. The general aggravation
The physical risk coefficient values are presented in Figures 7 and 8 were the results of the fuzzy and the index models are showed. Although the spatial patterns changes, the highest values are encountered in the north part of the city on both models, thus containing the districts of: Usaquen, Suba Barrios Unidos and Chapinero.

For the rest of the city, the fuzzy model estimates homogeneous medium-high physical risk values, while in the index method map, this level of risk is given only for the south part of the city.

The highest levels of physical risk are alike in the two models. However, the change between higher and lower physical risk values is more smooth in the fuzzy model along 5 districts (Tunjuelito, Bosa, Ciudad Kennedy, Fontibón, Engativá and Antonio Nariño), when compared with the index model. Last, the total risk values were obtained for the 19 districts of Bogotá. Figures 9, 10 and 12 show the results for the totally fuzzy, the Moncho’s-Index and the Moncho’s-Fuzzy methods, respectively.

The proposed fuzzy model estimate a higher total risk values for the city of Bogota and a more homogeneous spatial pattern. However, the areas with highest levels, correspond also to those areas with the highest
levels according to the index method, even if the current values are different. As the physical risk values obtained by the fuzzy model are higher for a number of Bogota localities, comparing with the same physical risk values obtained by the index method (see Figure 7 vs. Figure 8), the result is a more higher total risk when using the fuzzy model (see Figure 9 vs. Figure 10).

The total risk levels using the Moncho’s-Fuzzy model (see Figure 12), show the highest level of seismic risk at the northern part of the city, which is congruent with the results from the other two models. However, a direct application of Moncho’s equation...


gives an homogeneous medium-high level for almost all the city. As it can be seen, a direct application of the Moncho’s equation vanishes the effect of social aggravation to the total risk.

Figure 11 shows the total risk trend line over 19 districts of the city of Bogotá. The variability estimated by the fuzzy model resembles the one given by the Moncho’s-Index model, especially at the right hand side of the plot. However, the districts of: Santa Fe, San Cristóbal and Chapinero, show an opposite trend from the values estimated by the index method. The homogeneous values given by the Moncho’s-Fuzzy model are more clearly seen in this graph, although the highest risk values looks similar when comparing with the others two alternative models.

According to the previous analysis, with the use of classical fuzzy inference system methodology it is plausible to achieve similar results to those obtained from a more analytical method such as indexes, in terms of district classification, or in reproducing spatial patterns of aggravation and physical risk. Fuzzy logic inference capabilities can be exploited in a more suitable way since the outputs from each FIS used in the model are always fuzzy sets, giving the chance to connect them through a new FIS without losing consistency, allowing model completeness and avoiding the assumption of interdependency between descriptors, in order to calculate a final risk output. Therefore, one of the main advantages of the model is the assumption of connectivity to create a risk value that actually reflects the result of the correlation (both linear and non linear) between the components that were assumed to influence seismic risk. It is interesting to note how by not using Moncho’s equation for estimate total risk allows to have a more clear vision of the drivers of seismic risk in a non linear way. For example, Figures 6, 8 and 10 shows the three components of seismic risk estimated by index method. Moncho’s equation follows the idea that there can be only a high seismic risk if there is a high physical risk value. If not, no matter if there is a large area presenting a “very high” aggravation value (the southeastern part of the city in figure 6), the final index will say that the total risk in that area is only between “high and medium high”. The opposite behavior is estimated by using fuzzy inferences (Figures 5, 7 and 9) where an area presenting high aggravations values, corresponds a proportional value of total risk. As discussed, one of the steps needed to conform the designed FIS was in terms of the use of a subjective methodology in order to reduce the original dataset in order to avoid over correlation (double counting) between variables. In more general risk models, the number of variables involved, specially in the social vulnerability part, can be in the order of hundreds. Clearly a subjective scheme will not be enough to reduce the number of variables and more analytical methods are needed. The most common way to reduce a data set relies in the use of statistical
approaches such as linear correlation, principal component analysis, or factor analysis. Nevertheless, all of these methods assume as well, a linear correlation between variables. At the same time, at the moment of aggregate descriptors in order to build a composite index, most of the time linear assumptions are used which leads to lose information coming from the non-linear interdependency over descriptors that, in fact, are the most important. Therefore, the development of tools to perform variable selection that do not follow strictly linear assumptions are most needed.

5 CONCLUSIONS

We obtain a inference fuzzy model to make an estimation of social aggravation over Bogota city using the descriptors proposed in (Carreño et al., 2012). Building inference compositional rules over the selected descriptors, we were able to obtain a robust method that resembles the identification of relevant aspects and characteristics of seismic risk of cities already achieved by other consolidated method. The proposed model displays simplicity, flexibility and resolution capacities and does not assume linearity between the different components needed to obtain a final outcome.

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